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Johnson Series

SCHOOL ARITHMETIC

Practical Book

WITH ANSWERS

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SCHOOL ARITHMETIC

PRACTICAL BOOK

BY

JOHN M. COLAW

AND

FRANK W. DUKE

AUTHORS OF A SERIES OF MATHEMATICAL TEXT-BOOKS

B. F. JOHNSON PUBLISHING CO.

ATLANTA RICHMOND DALLAS

Ed...T 119.06.285

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PREFACE

THE common school course in arithmetic, which is offered in this book, together with the Primary Book of the same series, is intended to fit pupils for the high school or for the affairs of life. The Practical Book is designed to cover the subject-matter usually given in the grammar grades of the best public schools. In most schools this work will be taken in the fifth, sixth, and seventh school years. The plan of the book allows, however, such modifications in arranging the work by grades as the needs of different schools may require.

Every subject is so presented as to be readily mastered by pupils of the age and grade for which it is intended. The subjects embraced in the Primary are first reviewed and extended without duplicating. In a later chapter the fundamental operations are again reviewed and compared in such a manner as to give pupils a thorough working knowledge of these operations, and to impress upon them the advantages of a uniform scale as found in the decimal system.

The book contains a very large number of problems, both oral and written, which are for the most part new. These have been selected and graded with great care. It will be noted that apart from the mathematical calculations involved in them, the controlling factor in their selection has been a desire to arouse the interest and hold the attention of the pupil. This has been done by basing problems on facts with which the pupil is familiar or which appeal to him as worthy of investigation.

The various groups of "Related Problems" are intended to stimulate the study of the national resources and business life

of this country. In this way arithmetic is not only brought into proper contact with the chief phases of business and industrial life, but becomes an important factor in giving a clear quantitative interpretation to the numerous topics which have been made centers of special study in these groups.

The illustrations which accompany certain written exercises are used for the purpose of making the problems in these exercises more concrete, and, therefore, more easily comprehended by the pupil. Only those illustrations, however, have been admitted which are suggestive of some mathematical principle or which bear directly upon the proper interpretation of problems.

The chapters on Short Methods and The Equation have many features that will appeal to progressive teachers. By avoiding all technicalities in the treatment of the equation, the many advantages arising from its use in the solution of problems have been placed within the mental reach of pupils of the grade in which this subject is presented. Ratio and Proportion are also treated in a simple and practical way, and are used to give pupils another method of solving certain problems.

The chapters on the Solution of Problems contain many helpful suggestions. Great care has been taken to lead pupils to grasp clearly the conditions of problems, — to find out first what is given in a problem and what is to be found, and then to plan an outline of the work which will yield the required result. The illustrative solutions are intended not only to exhibit the best methods of dealing with different classes of problems, but to give the best arrangement of the actual work.

For criticisms and suggestions made during the preparation and publication of the manuscript of this book, the authors desire to acknowledge their indebtedness to Dr. James K. Powers of the B. F. Johnson Publishing Co.

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SCHOOL ARITHMETIC

PRACTICAL BOOK

1. Measure with a ruler the length of your book. How many inches long is it? *One inch* is the *unit of measure.*

Measure with a quart measure the water in a bucket. How many quarts are there? What is the unit of measure?

When we say we have ten apples, the unit used in counting them is *one apple.* *One* of any kind, or any group *regarded as one*, by comparison with which we count or measure is called a **Unit.**

READING AND WRITING NUMBERS

2. You have already learned to write numbers, at least to millions. You now need to learn something more about reading and writing numbers.

Our system of writing numbers by figures was first introduced into Europe by the Arabs. It is therefore called the Arabic System.

How many different figures are used in this system? Why is the zero needed?

3. Compare the numbers 1, 10, 100, 1000. How many ones does 1 express when it stands alone? When it stands at the left of a cipher? At the left of two ciphers?

In the number 22, which 2 expresses the greater value? How many times as great? In the number 222, what is the value of each 2?

Upon what does the value of each figure of a number depend?

Any figure represents ten times the value it would represent in the next place to the right.

4. In counting by *ones* the *unit* is *one*. In counting by *tens* we may regard *one ten* as the unit. Similarly, *one hundred* may be regarded as the unit for the *hundreds* of a number.

We speak of the different places, ones, tens, hundreds, as **orders**. Thus, the *ones* of a number are called *units of the first order*, or simply *units;* the *tens* are called *units of the second order;* the *hundreds* are called *units of the third order;* and so on.

How many units of any order make one unit of the next higher order?

How many units of any order make one unit of a higher order two places to the left?

5. When more than four figures are used to express a number, they are usually separated into groups of three as an aid in reading them. Each group is called a **period**. The right-hand group is called the *period of units;* the second group, the *period of thousands;* the third group, the *period of millions;* and so on.

6. The manner of writing numbers is shown in the following:

Orders:		Billions	Hundred-millions	Ten-millions	Millions	Hundred-thousands	Ten-thousands	Thousands	Hundreds	Tens	Ones (Units)
		9	0	7	5	8	4	0	5	3	2
Periods:		4th Billions		3d Millions			2d Thousands			1st Units	

The entire number is read nine billion, seventy-five million, eight hundred forty thousand, five hundred thirty-two.

In reading integers do not use the word "and" between orders or periods.

1. The left-hand period may have one, two, or three figures. How many must each of the other periods contain?

2. Each group contains *ones*, *tens*, and *hundreds* of that period and is read as if it stood alone, the *name* of the period being added except in the case of the last one. In writing numbers how are the periods usually separated?

3. In the number 324, what place or order is occupied by the 4? By the 2? By the 3? Show how 3 has not only the value *three*, but the place value *hundreds*.

4. In the number 113,840,532, what name is given to the period 532? To the period 840? To the period 113? Has each period a place value?

7. Read the following:

1. 305, 2450, 14,275, 235,600.
2. 888, 3065, 13,007, 451,736.
3. 739, 7008, 42,675, 295,001.

4. 3,250,190, 70,692,826, 125,470,321.
5. 5,009,010, 26,035,906, 950,150,100.
6. 1,928,374,765, 25,482,604,826.
7. 3,470,153,005, 124,514,826,720.

Read aloud:

8. In a certain year the value of the cotton crop of Texas was $154,989,000; of Mississippi, $85,957,000; of Georgia, $76,024,000; of Alabama, $57,000,000; of South Carolina, $47,219,000.

9. The total area of the United States is about 3,602,990 square miles. In the year 1900 the farm area was about 1,310,299 square miles, and the city area about 9237 square miles.

10. From 1880 to 1890 the wool production of the United States increased from 223,000,000 pounds to 277,000,000 pounds, and from 1890 to 1900 it increased from 277,000,000 pounds to 289,000,000 pounds.

8. In writing numbers the periods are written in regular order, beginning with the highest. If any order of units of a period is lacking, a cipher must be put in its place. How many ciphers must be supplied if any entire period is lacking?

9. Write the following in figures:

1. Two hundred two.
2. Three hundred thirty-three.
3. Two thousand, twenty.
4. Fifty thousand, two hundred fifty.
5. Five hundred thousand, three hundred twenty-five.
6. One million, one thousand, one.

7. Twenty-one million, three hundred sixty-seven.

8. Three hundred two million, two hundred four.

9. Seven billion, four hundred five million, eighty-eight.

10. Twenty billion, three hundred seventy-five.

11. Five million, five thousand, five.

12. Six million, three thousand, four hundred forty-six.

13. Thirty-seven billion, fifty million, fifty thousand.

14. Two hundred twenty-six million, four hundred thousand, twenty.

15. Six hundred seventy-five million, three hundred twenty-one thousand, five hundred thirty-six.

Write the following numbers in figures:

16. In a certain year the grain crop of the United States was: wheat, six hundred thirty-two million bushels; corn, two billion, thirty million bushels; oats, eight hundred eleven million bushels; barley, eighty-five million bushels; rye, twenty-seven million bushels; buckwheat, twelve million bushels.

17. The strawberry crop of Maryland in a certain year was twenty-four million, forty thousand quarts; of Michigan, eighteen million, three hundred eight thousand quarts; of Ohio, seventeen million, nine hundred twenty thousand quarts; of New York, thirteen million, three hundred eighty thousand quarts; of Tennessee, thirteen million, six hundred eight thousand quarts.

18. In a recent year the value of breadstuffs exported by the United States was: wheat and wheat flour, sixty-one million, two hundred thousand dollars; corn, twenty-five million, three hundred thousand dollars; barley, four million, six hundred thousand dollars; oats and oatmeal, one million, two hundred thousand dollars.

ADDITION

10. 1. In one pail I have 5 quarts of milk and in another 3. I pour all into one pail. How many quarts are then in that pail?

2. If Kate has $5 and earns $3 more, how much does she then have?

3. Increase each of the following numbers by 9, and read the results: 2, 4, 3, 7, 8, 6, 5.

11. The process of combining two or more numbers into one whole is called **Addition.**

12. The result obtained by adding is called the **Sum.**

1. What is the sum of 5 yards and 7 yards?

2. Can you find the sum of 3 feet and 5 pounds?

13. Numbers that have the same unit are called **Like Numbers.**

Only like numbers can be added.

1. Are 6 yards and 2 yards like numbers? Why are 6 yards and 2 quarts not like numbers?

2. The sum of 6 yards and 3 yards is 9 what? (Note that the sum is always like the numbers added.)

3. Is the sum of two numbers the same regardless of the order in which they are added? Show that the sum of 6 and 3 is the same as the sum of 3 and 6. Make other tests.

14. Add 7 to each number in column A, then to each in column B, and so on:

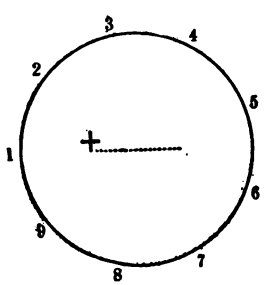
A	*B*	*C*	*D*	*E*	*F*	*G*
52	83	24	95	46	67	78
92	53	64	35	16	77	88
12	73	54	45	36	27	98
42	23	14	75	66	57	38
62	13	74	55	86	97	48

Vary the foregoing by substituting each of the other digits for 7.

15. 1. Add by 2's from 2 to 20. From 1 to 19.
2. Add by 3's from 3 to 30. From 2 to 32.
3. Add by 4's from 2 to 34. From 1 to 45.
4. Add by 5's from 1 to 31. From 3 to 43.
5. Add by 6's from 2 to 44. From 3 to 45.
6. Add by 7's from 2 to 51. From 7 to 49.
7. Add by 8's from 6 to 94. From 3 to 75.

16. Place these diagrams on the blackboard, and indicate with a pointer the numbers to be added:

8	1	6
3	5	7
4	9	2



1. In the diagram at the left add numbers (*a*) by rows, (*b*) by columns, and (*c*) by diagonals.

2. Add the number in the center to those in each outer square.

3. Using the diagram at the right begin at any number and add in either direction till 80 is reached or passed.

4. Write each figure in succession in the center and combine with each number on the circle. Continue this exercise until the pupil can give readily the results of each of the 45 combinations, as shown in the following table:

2=1+1									=2
3=1+2									=3
4=1+3=2+2									=4
5=1+4=2+3									=5
6=1+5=2+4=3+3									=6
7=1+6=2+5=3+4									=7
8=1+7=2+6=3+5=4+4									=8
9=1+8=2+7=3+6=4+5									=9
10=1+9=2+8=3+7=4+6=5+5									=10
11= 2+9=3+8=4+7=5+6									=11
12= 3+9=4+8=5+7=6+6									=12
13= 4+9=5+8=6+7									=13
14= 5+9=6+8=7+7									=14
15= 6+9=7+8									=15
16= 7+9=8+8									=16
17= 8+9									=17
18= 9+9=18									

17. Add each column from bottom to top, then from top to bottom; also add each line (as *a*) from left to right, then from right to left. *In adding give results only*, as 2, 8, 13, etc.

	1	2	3	4	5	6	7	8	9	10
a.	3	3	6	7	5	9	8	7	4	4
b.	5	6	5	6	7	6	4	6	8	4
c.	6	4	7	5	6	8	5	9	7	9
d.	2	7	3	9	4	7	9	5	6	5

18. Give sums quickly:

1. $31 + 16 =$	6. $51 + 29 =$
2. $46 + 24 =$	7. $34 + 66 =$
3. $18 + 72 =$	8. $43 + 29 =$
4. $33 + 27 =$	9. $93 + 27 =$
5. $32 + 28 =$	10. $125 + 175 =$

ORAL EXERCISES

19. 1. Tom walked due east 7 miles, then due north 9 miles. How far did he walk?

2. A boy who had 15 cents found 5 cents and earned 30 cents. How much had he then?

3. My house is 30 feet long and 20 feet wide. What is the distance around it?

4. If a man and his wife have 5 sons and 3 daughters, how many persons are there in the family?

5. Ella paid 12 cents for paper, 5 cents for ink, and 8 cents for pencils. How much did she pay for all?

6. *A* has \$5, *B* \$10, and *C* as much as both. How much have they together?

7. Twenty years ago Edgar's grandfather was 46 years old. How old will he be 5 years hence if still living?

8. Mary picked 5 quarts of strawberries, Kate 4 quarts, and Jane 7 quarts more than Kate. How many did Jane and Mary together pick?

9. There are 24 carnations in one vase, 8 roses in another, and 5 geraniums in a third. How many flowers are there in all?

10. In a freight train there are 5 coal cars, 20 box cars, and 10 stock cars. How many cars are there in the train?

11. Mr. Johnson has 12 cows, 6 horses, and 3 pairs of mules. How many animals has he?

12. Three boys paid 10 cents for trout flies, 30 cents for lines, and 50 cents for a lunch. How much did they spend?

13. For a meal the meat cost 25 cents, the vegetables 20 cents, and other food 30 cents. What was the total cost?

14. There are 40 pupils in room A, 25 pupils in room B, and 30 pupils in room C. How many pupils are there in the three rooms?

15. A boy spent 4 weeks of his vacation at the seashore, 3 weeks camping out, and the remaining 5 weeks in the city. How long was his vacation?

16. A freight car is loaded with 12,000 lb. of bran, 7000 lb. of corn, and 3000 lb. of oats. How many pounds of feed are in the car?

WRITTEN EXERCISES

20. 1. Find the sum of 273, 854, and 765.

$$\begin{array}{r} 273 \\ 854 \\ 765 \\ \hline 1892 \end{array}$$

Why are the numbers placed in vertical columns? What is the sum of the ones? Where is the 2 written? What is done with the 1 ten? What is the sum of the tens? Why do you write 9? How is the 18 hundreds obtained?

Write from dictation and add:

2.	3.	4.	5.
8618	6392	5728	9796
3376	3763	7276	5978
1914	2827	3937	4547

Add:

6.	7.	8.	9.	10.
$8246	$2769	$4486	$28.38	$50.35
3983	7478	6615	29.15	25.97
3762	9192	7307	85.72	64.78
6219	5581	9590	72.40	99.24
6768	4803	6925	39.17	91.39

Add by columns and by lines:

	11.	12.	13.	14.	15.
16.	62384	2784	37039	7006	67809
17.	70572	4751	804	73072	96075
18.	39283	57650	2906	8002	36075
19.	50061	3807	924	7054	59763
20.	30216	45285	5407	3391	90076

21. In adding long columns, business men often use the following method:

1. (*a*)		(*b*)
527		527
841		841
576		576
439		439
765	*or*	765
28		28
22		24
29		31
3148		

In (*b*) a second addition is avoided by adding the 2 tens of the first partial sum with the tens, and the 2 hundreds of the second partial sum with the hundreds. For the sum read the last line and the last figures of the other partial sums, as shown by heavy-faced type.

Add the following:

2.	3.	4.	5.	6.
77535	49413	177	97890	46925
62914	72694	32416	345	81470
4728	47	87324	34251	14703
2872	74062	4715	26076	58
63389	7618	982	8907	57147
7604	385	5639	73529	3692
5881	92764	70048	982	81470
3007	8270	39571	8006	369
98376	54936	50860	93745	60487

7. Add three million, five hundred twenty-four thousand, one hundred two; three hundred thirty-five thousand, six hundred fifty-three; two hundred two thousand, six hundred seven.

8. In one year a man pays $275 for rent, $625 more for other expenses than for rent, and has $500 of his income left. What is his income?

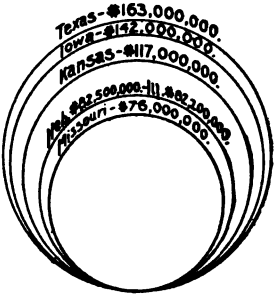
9. A farmer sold 250 bushels of apples, 190 bushels of peaches, 67 bushels of pears, and 17 bushels of plums. How many bushels of orchard fruits did he sell?

10. The area of the United States is 3,602,990 square miles; of the Philippine Islands, 143,000 square miles; of Porto Rico, 3600 square miles; and of other insular possessions, 7294 square miles. What is the total area?

11. The value of the wheat crop of Minnesota in a certain year was $50,000,000; of Ohio, $32,900,000; of North Dakota, $31,700,000; of Indiana, $22,200,000; of South Dakota, $21,000,000. What was the total value of the wheat production of these five states?

12. The apple crop of New York in a recent year was 24,110,000 bushels; of Pennsylvania, 24,060,000 bushels; of Ohio, 20,620,000 bushels; of Virginia, 9,840,000 bushels; of Illinois, 9,180,000 bushels. What was the combined crop of these five states?

13. The value of the herds in six leading cattle states in 1900 was as follows: Texas, $163,000,000; Iowa, $142,000,000; Kansas, $117,000,000; Nebraska, $82,500,000; Illinois, $82,200,000; Missouri, $76,000,000. What was the value of the cattle in the six states? Explain the diagram.



LEADING CATTLE STATES

14. In a certain year the sales of leading flowers were: roses, $6,000,000; carnations, $4,000,000; violets, $750,000; chrysanthemums, $500,000. What were the total sales?

15. For practice copy and time yourself in adding the following numbers. Repeat this from day to day, and note the progress made. Add by columns and by lines:

4	8	9	6	5	6	3	2	8
8	6	1	3	5	6	8	9	2
7	6	2	8	4	1	7	8	3
6	6	2	6	8	5	1	6	6
4	8	9	3	2	8	4	5	5
1	8	6	7	3	6	8	3	6
4	5	2	6	9	2	2	1	9
7	6	5	8	8	6	6	6	8
3	7	4	1	4	6	7	8	4

SUBTRACTION

22. 1. If Ruth has \$8 and spends \$3, how much has she left?

2. Ten marbles are how many more than 4 marbles?

3. What number of cents added to 16 cents makes 25 cents?

4. If you pay a debt of 40 cents, how much change do you get out of \$1?

23. The number that is left when one number is taken from another is called the **Remainder** or **Difference.**

24. The process of finding the difference between two numbers is called **Subtraction.**

25. The number from which another is subtracted is called the **Minuend.**

26. The number which is subtracted is called the **Subtrahend.**

702	Minuend
527	Subtrahend
175	Remainder or Difference

1. Can 3 feet be taken from 5 pounds? Why not?

2. What is the remainder when \$3 is taken from \$7? How does \$7 compare with the sum of \$3 and this remainder?

3. Subtract one number from another and see what the difference added to the smaller number equals. Observe that the minuend equals the sum of the subtrahend and the difference.

27. 1. Subtract by 2's from 20 to 0. From 21 to 1.
2. Subtract by 3's from 33 to 0. From 31 to 1.
3. Subtract by 4's from 60 to 0. From 45 to 1.
4. Subtract by 8's from 88 to 0. From 57 to 1.

28. Give differences as quickly as possible:

1. $57 - 20 =$	6. $67 - 40 =$	11. $950 - 300 =$
2. $65 - 50 =$	7. $46 - 33 =$	12. $624 - 504 =$
3. $85 - 54 =$	8. $79 - 26 =$	13. $391 - 250 =$
4. $98 - 61 =$	9. $99 - 56 =$	14. $396 - 175 =$
5. $69 - 48 =$	10. $125 - 70 =$	15. $349 - 127 =$

29. In the following name only *sums* and *differences;* thus, in the first example, say 6, 10, 5, 11, 4:

1. $6 + 4 - 5 + 6 - 7 =$	6. $6 + 9 - 4 - 7 + 12 =$
2. $5 - 3 + 8 - 5 - 4 =$	7. $13 - 8 + 6 - 7 + 9 =$
3. $4 + 5 - 6 + 8 - 7 =$	8. $4 + 9 - 8 + 5 - 7 =$
4. $7 - 7 + 6 + 7 - 8 =$	9. $3 + 8 - 9 + 6 - 4 =$
5. $3 + 8 - 6 + 9 - 6 =$	10. $2 + 11 - 7 - 5 + 9 =$

30. If you owe a merchant 18 cents and give him a quarter, he says: "18 and 2 are 20, and 5 more make 25," and hands you back 2 cents and a nickel.

In like manner call the change for:

1. \$1, when you owe 70 cents.
2. \$1, when you owe 37 cents.
3. \$2, when you owe \$1.20.
4. \$5, when you owe \$2.25.
5. \$10, when you owe \$3.75.
6. \$20, when you owe \$11.50.

31. 1. The minuend is 82, the subtrahend 30. What is the difference?

2. The minuend is 75, the difference 23. What is the subtrahend?

3. The subtrahend is 45, the difference 13. What is the minuend?

4. To what number must 50 be added to make the sum 90?

5. From what number must 36 be subtracted to make the difference 64?

ORAL EXERCISES

32. 1. Subtract by 5's from 39 to 4. From 42 to 2.

2. Subtract by 6's from 36 to 6. From 35 to 5.

3. Subtract by 9's from 45 to 0. From 55 to 1.

4. Count by 7's from 2 to 58, and then back from 58 to 2.

5. If you buy a dozen eggs for 17 cents, how much change do you get out of a half dollar?

6. Joy has 27 plants, of which 9 are geraniums and the others pinks. How many pinks has she?

7. Tom will be 20 years old 5 years hence. How old was he 6 years ago?

8. An agent purchased books for $15 and sold them for $38. How much did he gain?

9. Willie earns $45 a month and spends $15. How much does he save monthly?

10. A farmer having 38 sheep sold 18 and killed 11. How many had he left?

11. I owe my grocer $50, and pay him $30. What balance is still due?

12. Robert had a $20 bill and a $5 bill. He bought a hat for $3 and a coat for $10. How much had he left?

13. A boy wishing to ride 70 miles on his wheel rode 20 miles the first day and 30 miles the second. How many miles had he yet to ride?

14. Tom has $50 in bank and $20 in cash, but owes $30. What is his net capital?

15. A piano has 88 keys. Thirty-six of them are black. How many are white?

16. At the depot a man is loading a freight car which can haul 40,000 lb. He has put in 12,000 lb. How much more will be needed to load the car?

WRITTEN EXERCISES

33. **1.** Subtract 486 from 743.

$$\begin{array}{r} 743 \\ 486 \\ \hline 257 \end{array}$$

Since we cannot take 6 ones from 3 ones, we add 1 of the 4 tens to the 3 ones, making 13 ones. Then 6 ones from 13 ones leave 7 ones.

Having taken 1 ten from the 4 tens, 3 tens remain, from which we cannot take 8 tens. Hence, we take 1 of the 7 hundreds, or 10 tens, and add it to the 3 tens, making 13 tens. Then 8 tens from 13 tens leave 5 tens. From the 6 hundreds which remain we now take 4 hundreds.

2. From 453 take 296.

$$\begin{array}{l} 453 = 400 + 50 + 3 = 400 + 40 + 13 = 300 + 140 + 13 = 453 \\ 296 = 200 + 90 + 6 = 200 + 90 + \ \ 6 = 200 + \ 90 + \ 6 = 296 \\ \hline 157 \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad 100 + \ 50 + \ 7 = 157 \end{array}$$

The pupil should explain each step in the foregoing process.

Subtract and prove by adding the remainder and subtrahend:

3.	4.	5.	6.	7.	8.	9.
717	924	609	507	912	593	600
477	359	68	246	299	449	364

10.	11.	12.	13.	14.
8236	4228	9352	5014	7200
5753	2849	3178	4376	2904

15.	16.	17.	18.	19.
$4520	$8245	$7320	$5000	$7007
2750	3755	3575	2540	5128

20.	21.	22.	23.	24.
6003	5004	9005	5006	35216
3345	3780	4710	2607	26098

25.	26.	27.	28.	29.
47003	36210	63015	28001	30250
29017	22658	24035	6608	14785

30.	31.	32.	33.
770625	701550	703005	700000
381528	305865	60505	523260

Subtract as indicated and prove:

34. 6250 − 3795.

35. 15,008 − 6036.

36. 47,200 − 27,508.

37. 95,308 − 5006.

38. $17,500 − $2351.

39. $81,463 − $29,658.

40. $10,000 − $6789.

41. 2,963,055 − 673,695.

42. 3,123,654 – 2,745,784.
43. 7,312,894 – 3,004,045.
44. 2,000,001 – 107,003.
45. $800,800 – $60,080.
46. $75,000 – $29,127.
47. $8,076,535 – $7,067,840.

48. Subtract five thousand eighty-one from nine thousand four.

49. In a school of 715 children 285 are boys. How many girls are there?

50. The remainder is 1786, and the subtrahend 2467. What is the minuend?

51. The difference between two numbers is 796, and the larger number is 1275. What is the smaller number?

52. How many years has it been since the discovery of America by Columbus in 1492?

53. Independence was declared in 1776. How many years was that after the discovery of America?

54. The Pilgrims landed at Plymouth in 1620. How many years was that before George Washington was inaugurated President in 1789?

55. The area of England is 50,839 square miles, and that of Virginia 42,450 square miles. How much larger is England than Virginia?

56. In 1900 there were 560,892 people living in Boston, and 508,957 living in Baltimore. How much greater was the population of Boston than that of Baltimore?

57. Mr. Smith bought a farm for $7500, built a house on it for $2650 and a barn for $1100. He then sold the property for $18,000. How much did he gain?

58. If a man's income is $1500 a year, and he spends $125 for clothing, $275 for groceries, $300 for rent, and $127 for traveling expenses, how much has he left each year?

59. If the moon is 238,850 miles from the earth, and the sun 92,790,000 miles, how much farther is it to the sun than to the moon?

60. At a certain election there were 87,650 votes cast for A, B, and C. A got 58,716, B 20,580, and C the remainder. How many people voted for C?

61. Mount Everest is 29,002 ft. high, and is 14,855 ft. higher than Pike's Peak. What is the height of the latter?

62. The number of pounds of grapes grown in California in 1900 was 721,000,000. In the same year New York produced 248,000,000 lb.; Ohio, 79,200,000 lb.; Pennsylvania, 47,100,000 lb.; Michigan, 41,500,000 lb. How many more pounds did California produce than the four other states named?

63. In 1890 the street car lines carried 2,023,010,202 passengers; in 1902 they carried 4,774,211,904. Find the increase in the number carried.

64. The United States Post Office does the largest single business in the world. In the year ending June 30, 1905, it received from all sources $152,826,585, and paid out $167,399,169. Find the deficit for that year.

65. Subtract 572,891 from 5,728,914. From the remainder subtract the same subtrahend. Continue the successive subtractions until the remainder 4 is found.

Make similar problems, and apply this exercise in successive subtractions.

MULTIPLICATION

34. 1. Suppose you have a roll of $5 bills and wish to know how much money you have. How can you find out?

2. Laying out the bills one by one, you may count by 5's, thus: 5, 10, 15, 20. That is, $5+$5+$5+$5=$20. This is addition.

3. Or you may count the bills — 1, 2, 3, 4 — four bills, each $5. How many *times* is $5 repeated to make your $20? Here $4 \times \$5 = \$20$. This is multiplication. Is this process founded upon addition?

4. How much is $6+6+6+6+6$? 5 times 6? We see that five 6's are 30 by first finding the sum of five 6's.

35. The process of taking one number as many times as there are units in another is called **Multiplication.**

36. The number multiplied by another — the number taken so many times — is called the **Multiplicand.**

37. The number that tells how many times the multiplicand is taken is called the **Multiplier.**

38. The result obtained by multiplying is called the **Product.**

125	Multiplicand
4	Multiplier
500	Product

39. A number that expresses measured quantity — how much of anything — is called a **Concrete Number.**

Thus, 4 inches, 2 bushels, 6 quarts, 5 horses, are concrete numbers.

40. A number used without reference to any particular object or measure is called an **Abstract Number.**

Thus, 4, 7, 9, are abstract numbers.

1. When we multiply \$3 by 2, is the multiplicand concrete or abstract? Is the multiplier concrete or abstract? Is the product like the multiplicand or like the multiplier?

2. Can we multiply \$5 by \$3? 7 by \$3? \$5 by 2 feet? By what can we multiply \$5?

41. From the preceding it is seen that —

(1) **The multiplier is always an abstract number.**

(2) **The product is like the multiplicand.**

The expression $5 \times \$3$ is read 5 *times* \$3, and $\$3 \times 5$ is read \$3 *multiplied by* 5. 5×3 may be read 5 *times* 3, or 5 *multiplied by* 3, at will.

42. The multiplicand and the multiplier are called **Factors** (makers) of the product.

Thus, in $2 \times 5 = 10$, 2 and 5 are factors of 10.

1. Of what numbers are 5 and 7 the factors? 3 and 11?

2. One factor of 27 is 9. What is the other factor?

3. What is the product of the factors 6 and 8? 11 and 12?

4. What is the product of the factors 3 and 7? 2 and 11?

5. What is the product of the factors 6 and 5? 2, 3, and 5?

6. Of what numbers are 7 and 11 the factors? 7 and 7?

7. Name all the sets of numbers which multiplied together will make 60.

Multiplication Table

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

The basal facts of all multiplication, which are expressed by the figures included within the heavier lines, were primarily ascertained by addition.

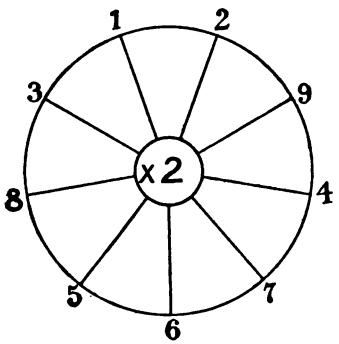
Point out the two ways in which this table may be used.

43. Announce products at sight:

1.	3×7	5×9	3×11	7×4	10×8
2.	5×7	3×10	12×3	7×9	9×9
3.	7×11	5×8	8×6	8×7	11×11
4.	9×12	7×12	11×9	8×12	6×9
5.	5×12	11×10	8×11	10×12	11×12

44. Give products at sight:

1. $12 \times 7 =$	6. $40 \times 2 =$	11. $24 \times 2 =$
2. $20 \times 3 =$	7. $30 \times 5 =$	12. $32 \times 4 =$
3. $42 \times 3 =$	8. $81 \times 3 =$	13. $110 \times 5 =$
4. $71 \times 4 =$	9. $40 \times 8 =$	14. $120 \times 3 =$
5. $31 \times 5 =$	10. $33 \times 3 =$	15. $144 \times 2 =$



For further practice in rapid work place this diagram on the blackboard, and then, using the number at the center as the multiplier, indicate the multiplicands with the pointer. Have the pupils give the results rapidly. Vary the exercise by substituting each of the other digits for the 2 in the center.

ORAL EXERCISES

45. 1. State the factors of 21; of 35; of 99.

2. Compare 4×5 with 5×4. $4 \times \$5$ with $5 \times \$4$.

3. Find the cost of 8 yards of silk at \$3 a yard.

Since 1 yard costs \$3, 8 yards cost 8 times \$3, or \$24.

4. At \$3 a day, how much can I earn in 9 days?

5. At \$4 each, how much must be paid for 11 sheep?

Do you multiply \$4 by 11 sheep?

6. At 9 cents each, what is the cost of 7 bats?

7. There are 8 quarts in a peck. How many quarts in 5 pecks?

8. A man bought 6 bars of glycerine soap at 10 cents each. How much change did he get out of a dollar bill?

State the cost of:

9. 12 dozen eggs at 20¢ a dozen.

10. 15 pounds of sugar at 6¢ a pound.

11. 5 pounds of grapes at 7¢ a pound.

12. 4 pounds of cheese at 15¢ a pound.

13. 5 pounds of tea at 90¢ a pound.

14. If an automobile goes 15 miles an hour, how far will it go in 3 hours, at the same rate?

15. If a kit of tools costs $5, how much will it cost to supply a class of 20 boys with these tools?

16. What is the cost of 75 stamped envelopes (3 packages) at 53¢ a package?

17. At 25¢ a hundred pounds, what will be the freight on 700 pounds of books?

18. At 3¢ a mile, what is the fare between two cities that are 125 miles apart?

19. On a piano there are 12 keys in each octave. How many keys on a 7-octave piano?

20. A boarding school accommodates 3 boys in each of its 150 rooms. What is its entire capacity

WRITTEN EXERCISES

46. 1. Multiply 635 by 4.

(*a*)	(*b*)	(*c*)
$635 = 600 + 30 + 5$	635	635
4	4	4
$20 = 4 \times 5$	20	2540
$120 = 4 \times 30$	12	
$2400 = 4 \times 600$	24	
2540	2540	

Why may we omit one cipher in the second *partial product*, and two ciphers in the third, as in (*b*)?

Explain how the result in (*c*) is obtained without setting down the partial products.

Find the products:

2.	3.	4.	5.	6.
\$1450 × 3	\$4348 × 4	\$3176 × 5	\$8267 × 7	\$44639 × 5
7.	**8.**	**9.**	**10.**	**11.**
26314 × 7	35283 × 6	47489 × 7	56458 × 6	67926 × 8
12.	**13.**	**14.**	**15.**	**16.**
18273 × 8	26742 × 9	129275 × 7	287064 × 5	469078 × 9

47. Examine carefully:

10 times 5 = 50.	100 × 7 = 700.
10 times 8 = 80.	100 × 9 = 900.
10 times 10 = 100.	100 × 20 = 2000.
10 times 15 = 150.	100 × 50 = 5000.

Annexing a 0 to an integer multiplies it by 10; annexing two 0's multiplies it by 100.

Announce results at sight:

1.	10 × 9 =	10 × 25 =	100 × 60 =
2.	10 × 12 =	10 × 35 =	100 × 66 =
3.	10 × 13 =	10 × 38 =	100 × 70 =
4.	10 × 18 =	10 × 40 =	100 × 75 =
5.	10 × 20 =	10 × 46 =	100 × 87 =

Teachers are urged to give pupils much practice in work that involves the equation form of expression.

6. Multiply 73 by 30.

$$\begin{array}{r} 73 \\ 30 \\ \hline 2190 \end{array} \quad \text{or} \quad \begin{array}{r} 73 \\ 30 \\ \hline 2190 \end{array}$$

30 is 10 times 3. Hence we multiply by 3 and annex a 0.

Multiply:

7.	34×20.	**11.**	49×30.	**15.**	65×80.
8.	46×30.	**12.**	38×60.	**16.**	73×70.
9.	56×40.	**13.**	47×50.	**17.**	86×90.
10.	122×30.	**14.**	134×40.	**18.**	356×50.

19. Multiply 325 by 15.

$$\begin{array}{r} 325 \\ 15 \\ \hline 1625 = 5 \times 325 \\ 3250 = 10 \times 325 \\ \hline 4875 = 15 \times 325 \end{array}$$

Are 5 times $325 + 10$ times 325 equal to 15 times 325?

20. Multiply 448 by 264.

$$\begin{array}{r} 448 \\ 264 \\ \hline 1792 = 4 \times 448 \\ 26880 = 60 \times 448 \\ 89600 = 200 \times 448 \\ \hline 118272 = 264 \times 448 \end{array} \quad \textit{or} \quad \begin{array}{r} 448 \\ 264 \\ \hline 1792 \\ 2688* \\ 896 \\ \hline 118272 \end{array}$$

* In multiplying by tens it is not customary to put the 0 in one's place. $60 \times 448 = 26880$ ones, or 2688 tens.

When we multiply by tens and omit the 0, in what place does the right-hand figure of the partial product stand? In what place when we multiply by hundreds and omit the two ciphers?

Observe that the ones, tens, and hundreds of the multiplicand are multiplied by the ones, tens, and hundreds of the multiplier, in order, and that the first figure of each product is put directly under the figure of the multiplier used in obtaining it, and that the products are then added.

21. Multiply 525 by 203.

(*a*)	(*b*)
525	525
203	203
1575	1575
000	1050
1050	106575
106575	

If either factor is 0, what is the product? Multiply 203 by 525 and compare the products.

Multiply:

22.	23.	24.	25.	26.	27.
262	364	218	564	417	764
23	27	35	36	35	37

28.	29.	30.	31.	32.	33.
623	517	743	327	586	639
46	65	58	76	65	58

34.	35.	36.	37.	38.
3247	7350	4585	5287	8465
64	75	56	78	67

39.	40.	41.	42.	43.
225	861	348	790	835
145	335	634	246	268

44.	45.	46.	47.	48.
1317	3465	5263	7375	9728
224	225	317	368	438

Find the products:

49. 4430 × 33.	**58.** $2107 × 43.	**67.** 12,425 × 45.
50. 7625 × 37.	**59.** $3206 × 75.	**68.** 33,507 × 86.
51. 5275 × 66.	**60.** $5004 × 68.	**69.** 65,310 × 93.
52. 6760 × 42.	**61.** $6760 × 74.	**70.** 28,672 × 47.
53. 7586 × 26.	**62.** $7090 × 85.	**71.** 80,836 × 75.
54. 7275 × 67.	**63.** $5805 × 39.	**72.** 50,076 × 98.
55. 8406 × 91.	**64.** $9273 × 47.	**73.** 72,035 × 87.
56. 1496 × 237.	**65.** $738 × 324.	**74.** 4728 × 385.
57. 2738 × 346.	**66.** $857 × 932.	**75.** 6387 × 624.

76. 27,583 × 4574.

77. 48,065 × 3016.

78. 72,738 × 6537.

79. 82,065 × 4962.

80. 28,256 × 4652.

81. 65,865 × 1975.

48. **1.** If a man earns $ 37 a month, how much does he earn in 8 months?

$37 in one month
8
$296 in eight months

In 8 months he earns 8 times $37, or $296.

2. A grocer bought 16 barrels of sugar each weighing 344 pounds. How many pounds did he buy?

3. If a man takes 120 steps a minute, how many steps will he take in an hour? In 8 hours?

4. A man deposited $ 18 a week in a bank. How much did he deposit in 13 weeks?

5. A vessel sails 275 miles a day. How far does it sail in 9 days?

6. Mr. Roe pays $ 325 a year for rent. How much does he pay in 7 years?

7. The distance from New York to Washington is 228 miles. How many miles does a man travel who makes 2 round trips between the two cities?

8. A man sold 36 bales of cotton, which was one tenth of his crop. What was his crop?

9. In an orchard there are 24 rows and 24 trees in a row. How many trees are there in the orchard? Draw a diagram.

10. How many lines are there on this page of your book? How many on 272 such pages?

11. There are 144 pens in a box. How many pens are there in 12 dozen boxes?

12. If a bushel of wheat weighs 60 pounds, what will 750 bushels weigh?

13. An army used 48,512 pounds of meat each week. How much was used in 23 weeks?

14. If it costs $375 a year for a boy's expenses at a preparatory school, how much are the expenses of 300 boys, at the same rate?

15. There are 1760 yd. in a mile. If steel rails weigh 90 lb. to the yard, what is the weight of the rails in two miles of single-track railway?

16. Two trains left St. Louis at the same time, one going west at the rate of 46 miles an hour, the other east at the rate of 38 miles an hour. How far apart were they in 14 hours?

17. A lady bought 3 yd. of velvet at $1.65 a yard and 28 yd. of muslin at 8 cents a yard. How much change did she get out of a ten-dollar bill?

18. If a dealer pays $28 for a suit of clothes, what will a dozen suits cost, at the same rate?

19. In a certain year there were 25,434,460 of our people living in cities. If this was one third of our total population, what was the total?

20. The freight rate on baled cotton between two of our principal cities is 15 cents a 100 pounds. What will be the freight on a shipment of 25,400 lb.?

21. A hotel uses a barrel of flour (196 lb.) each day. How many pounds supply it from Jan. 20 to Feb. 25?

Find the cost of:

22. 245 dozen eggs at 18¢ a dozen.

23. 384 pounds of sugar at 6¢ a pound.

24. 24 pounds of butter at 18¢ a pound.

25. 25 tons of clover hay at $14 a ton.

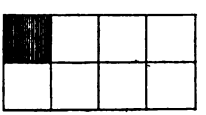
26. 345 bushels of wheat at 90¢ a bushel.

27. 5 barrels of molasses, 46 gal. each, at 35¢ a gallon.

28. 4 rolls of carpet, 50 yd. each, at 75¢ a yard.

29. 3 kegs of vinegar, 10 gal. each, at 18¢ a gallon.

49. **1.** If a rectangle 4 feet long and 2 feet wide is divided as in the figure, what will each of the small squares represent? What is the area of one of the strips? (4 × 1 sq. ft.) Then what is the area of the two strips, or of the rectangle?



2. How many square feet in the surface of a table 8 feet long and 4 feet wide? Draw a diagram.

3. What is the area of a rectangular meadow 60 rods long and 35 rods wide?

4. How many square feet of carpet will cover a room 15 feet square?

5. A rectangle is 19 feet long and 17 feet wide. What is its area?

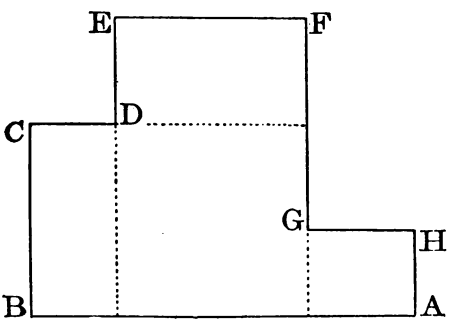
6. How many square yards of linoleum will cover a hallway 9 yards long and 3 yards wide?

7. How many square miles in a tract of land 150 miles wide and twice as long?

8. What is the floor space in a three-story building 200 ft. long and 120 ft. wide?

9. How many square yards of oilcloth would be needed to cover the kitchen floor at your home? What would it cost at 35 cents a square yard?

10. A greenhouse is 70 feet long. On one side of the slanting roof the glass surface has a width of 12 feet. On the other it is 8 feet wide. Find how many square feet of glass are needed to cover the house.



11. In this figure let $AB = 16$ ft., $BC = 8$ ft., $CD = 4$ ft., $DE = 5$ ft., $EF = 7$ ft., $FG = 9$ ft., $GH = 5$ ft., and $AH = 4$ ft. Divide the figure into rectangles and find its area.

DIVISION

50. 1. What number multiplied by 4 gives $20 as a product?

$4 \times (\$—) = \$20.$

2. If 4 times $5 is $20, then how many times is $5 contained in $20? $\$20 \div \$5 =$ ——.

3. How many times is $2 contained in $10? In $16? In $17, with what remainder?

4. If a man earns $15 in 3 days, how much does he earn a day?

$3 \times \$5 = \$15$; then $\$15 \div 3 = \$—$.

5. If the product of two numbers is 35, and one of the factors is 7, what is the other? If one of the factors is 5, what is the other?

51. The process of finding either factor when the other factor and their product are given is called **Division.**

52. The given product is called the **Dividend,** the given factor is called the **Divisor,** and the factor to be found is called the **Quotient.**

53. Division is seen to be the inverse of multiplication. In the latter, multiplicand and multiplier are given, while the product is required. In the former, dividend corresponds to product, divisor to one factor, and quotient to the other.

$3 \times 5 = 15. \quad 15 \div 3 = 5. \quad 15 \div 5 = 3.$

Point out the factors in these examples; the product; the dividends; the divisors; the quotients. Observe that

to find a quotient we need recall only how many of the given divisors equal the dividend.

54. Division includes —

(*a*) Finding one of the equal parts of a number; as, $\frac{1}{2}$ of $\$10 = \$5$. $\$10 \div 2 = \$5$.

(*b*) Finding how many times one number contains another of the same kind; as, $\$20 \div \$5 = 4$; $15 \text{ ft.} \div 5 \text{ ft.} = 3$.

1. What is one of the 3 equal parts of $\$21$?

$\frac{1}{3}$ of $\$21 = \$7$, *or* $\$21 \div 3 = \$7$.

In this case the dividend is concrete and the divisor abstract. The quotient is concrete, since it must correspond to the multiplicand.

2. How many times is $\$3$ contained in $\$21$?

$\$21 \div \$3 = 7$.

In this case the dividend and divisor are concrete. The quotient is *abstract*, since it must correspond to the multiplier.

3. When 60 ft. is the dividend and 12 the divisor, what is the quotient?

4. When $50¢$ is the dividend and $10¢$ the divisor, what is the quotient?

5. Find by trial whether the product of the quotient and divisor is equal to the dividend.

6. When $\$48$ is the dividend and $\$6$ the quotient, what is the divisor?

7. When $\$9$ is the divisor and 3 the quotient, what is the dividend?

8. When 9 is the divisor and 72 the dividend, what is the quotient?

9. If you have a dozen apples and wish to give me $\frac{1}{4}$ of them, you must give me one of the 4 equal parts of 12 apples. That is, you must divide 12 apples by 4.

10. What is $\frac{1}{3}$ of 45 feet? $\frac{1}{5}$? $\frac{1}{9}$?

55. 1. When books are \$2 each, how many can you buy for \$7? How much money will you have left?

2. How many 3's in 6? How many in 10? How many ones are left over?

3. When you take 5 cents from 18 cents three times, how many cents remain? Is the remainder a part of the 18 cents? Is it greater or less than the 5 cents?

56. The number remaining when the division is not exact is called the **Remainder.** It is a part of the dividend, and always less than the divisor.

The product of divisor and quotient, plus the remainder, if any, is equal to the dividend. Verify this by trial.

57. Show that 5 tens and 6 ones are equal to 4 tens and 16 ones.

Complete:

$24 = 2 \text{ tens} + 4 \text{ ones} = 1 \text{ ten} + 14 \text{ ones}.$

$75 = 7 \text{ tens} + 5 \text{ ones} = 6 \text{ tens} + \text{—} \text{ ones}.$

$89 = 8 \text{ tens} + 9 \text{ ones} = 7 \text{ tens} + \text{—} \text{ ones}.$

$94 = 9 \text{ tens} + 4 \text{ ones} = 8 \text{ tens} + \text{—} \text{ ones}.$

Divide 54 by 2, or find $\frac{1}{2}$ of 54.

$$54 = 4 \text{ tens} + 14 \text{ ones}.$$

$$\frac{2)4 \text{ tens} + 14 \text{ ones}}{2 \text{ tens} + 7 \text{ ones}} = \frac{2)54}{27}.$$

58. Name quotients at sight:

1. $24 \div 3$.	**4.** $49 \div 7$.	**7.** $40 \div 8$.	**10.** $60 \div 5$.
2. $35 \div 5$.	**5.** $45 \div 5$.	**8.** $56 \div 7$.	**11.** $32 \div 4$.
3. $42 \div 6$.	**6.** $54 \div 9$.	**9.** $36 \div 4$.	**12.** $63 \div 7$.

13. $54 \div 6$.	**19.** $24 \div 4$.	**25.** $88 \div 8$.	**31.** $108 \div 9$.
14. $72 \div 9$.	**20.** $36 \div 6$.	**26.** $64 \div 8$.	**32.** $110 \div 10$.
15. $81 \div 9$.	**21.** $84 \div 7$.	**27.** $72 \div 8$.	**33.** $96 \div 12$.
16. $48 \div 6$.	**22.** $48 \div 8$.	**28.** $28 \div 4$.	**34.** $121 \div 11$.
17. $56 \div 8$.	**23.** $27 \div 3$.	**29.** $55 \div 5$.	**35.** $84 \div 12$.
18. $45 \div 9$.	**24.** $63 \div 9$.	**30.** $96 \div 8$.	**36.** $42 \div 7$.

Name quotients as quickly as possible:

37.	**38.**	**39.**	**40.**	**41.**	**42.**
$3\overline{)366}$	$5\overline{)555}$	$6\overline{)186}$	$5\overline{)255}$	$4\overline{)288}$	$6\overline{)366}$
43.	**44.**	**45.**	**46.**	**47.**	**48.**
$4\overline{)388}$	$5\overline{)450}$	$7\overline{)497}$	$6\overline{)786}$	$8\overline{)728}$	$8\overline{)848}$
49.	**50.**	**51.**	**52.**	**53.**	**54.**
$6\overline{)618}$	$7\overline{)721}$	$5\overline{)350}$	$9\overline{)459}$	$8\overline{)816}$	$9\overline{)927}$

55.	**56.**	**57.**	**58.**	**59.**
$4\overline{)4088}$	$5\overline{)2050}$	$3\overline{)6129}$	$6\overline{)1806}$	$7\overline{)14021}$

Name quotients and remainders:

60.	**61.**	**62.**	**63.**	**64.**
$6\overline{)46}$	$4\overline{)45}$	$9\overline{)66}$	$8\overline{)68}$	$4\overline{)49}$
65.	**66.**	**67.**	**68.**	**69.**
$7\overline{)86}$	$6\overline{)58}$	$4\overline{)62}$	$6\overline{)49}$	$5\overline{)54}$
70.	**71.**	**72.**	**73.**	**74.**
$7\overline{)99}$	$9\overline{)86}$	$8\overline{)98}$	$11\overline{)107}$	$12\overline{)110}$

What is —

75. $\frac{1}{2}$ of 68?
76. $\frac{1}{3}$ of 54?
77. $\frac{1}{3}$ of 99?
78. $\frac{1}{5}$ of 85?
79. $\frac{1}{7}$ of 84?
80. $\frac{1}{4}$ of 56?
81. $\frac{1}{5}$ of 95?
82. $\frac{1}{4}$ of 64?
83. $\frac{1}{3}$ of 78?
84. $\frac{1}{3}$ of 51?
85. $\frac{1}{8}$ of 80?
86. $\frac{1}{6}$ of 78?

ORAL EXERCISES

59. 1. Mr. B divided \$9 equally among 3 boys. How much did each boy receive?

Since 3 boys received \$9, each boy received $\frac{1}{3}$ of \$9, or \$3. \$9÷3=\$3.

2. When chestnuts are \$3 a bushel, how many bushels can be bought for \$24?

At \$3 a bushel as many bushels can be bought for \$24 as \$3 is contained times in \$24. There are 8 times \$3 in \$24. Hence 8 bushels can be bought.

It will be observed that in the division the quotient 8 is not 8 bushels, but an abstract number, which is interpreted in the conclusion.

3. There are 4 pecks in a bushel. How many bushels are there in 36 pecks?

4. If 4 bags of the same size contain 32 bushels of beans, how many bushels are in each bag?

5. A jeweler paid \$84 for 7 watches. What was the average price paid?

6. A gardener tied 54 radishes in bunches of 6 each. How many bunches did he have?

7. If 10 yards of cloth make a dress, how many dresses can be made from 90 yards?

8. A butcher invested $84 in hogs, paying $12 for each. How many did he buy?

9. There are 12 rows of the same number of trees in an orchard containing 132 trees. How many trees are there in a row?

10. At $12 each, how many guns can be bought for $96?

11. How many times can you fill a 3-gallon bucket from a barrel that holds 42 gallons?

12. My milk bill for one week was 72 cents. If I paid 8 cents a quart, how many quarts did I get?

13. Seven hunters together killed 63 rabbits. What was each man's share, if the game was divided equally?

14. In 4 years the President receives salary to the amount of $200,000. What is his yearly salary?

15. If a man saves $12 a month, in how many months will he save $720?

16. If 2 dozen cans of tomatoes fill a case, how many cases must be shipped to fill an order for 70 dozen cans?

17. An author can compose at the average rate of 5 pages a day. How many days will be required to write a book of 350 pages?

18. An express train averages 40 miles an hour. How long will it take to run from Richmond to Raleigh, a distance of 160 miles?

19. A certain printed page has in 8 successive lines 7, 8, 5, 6, 8, 9, 11, 10 words. What is the average number of words to a line?

WRITTEN EXERCISES

60. **1.** Divide 876 by 6.

876 = 6 hundreds + 27 tens + 6 ones

= 6 hundreds + 24 tens + 36 ones

$\frac{6)6 \text{ hundreds} + 24 \text{ tens} + 36 \text{ ones}}{1 \text{ hundred} + 4 \text{ tens} + 6 \text{ ones}} = \frac{6)876}{146}$

Divide:

2.	3.	4.	5.	6.	7.
5)725	4)648	3)528	6)852	5)625	4)732
8.	**9.**	**10.**	**11.**	**12.**	**13.**
6)756	3)537	8)976	7)854	8)904	6)942
14.	**15.**	**16.**	**17.**	**18.**	
4)7524	3)5475	2)4756	5)6620	6)7806	
19.	**20.**	**21.**	**22.**	**23.**	
4)8256	3)9267	7)3675	5)8705	6)9348	

24. Divide 8556 by 12.

(*a*)

12)8556
713

(*b*)

713 quotient
12)8556
84
15
12
36
36

85 hundreds ÷ 12 = 7 hundreds, and 1 hundred remaining. 1 hundred equals 10 tens, and 10 tens plus 5 tens equal 15 tens.

15 tens ÷ 12 = 1 ten, and 3 tens remaining. 3 tens equal 30 ones, and 30 ones plus 6 ones equal 36 ones.

36 ones ÷ 12 = 3 ones.

The process as shown in (*a*) is called short division; as shown in (*b*), long division. Wherein do they differ? In what ways are they alike? Where is the first quotient figure written in short division? In long division?

25. Divide 3456 by 48.

$$\begin{array}{r} 72 \\ 48\overline{)3456} \\ 336 \\ \hline 96 \\ 96 \\ \hline \end{array}$$

As 48 is greater than 34, it is necessary to take the number represented by three figures of the dividend for the first partial dividend. 345 tens ÷ 48 = 7 tens, and a remainder. 48 × 7 tens = 336 tens. The remainder is 9 tens, and the new dividend is 9 tens and 6 ones, or 96 ones. 96 ones ÷ 48 = 2 ones. 48 × 2 ones = 96 ones.

Divide, and check results by finding the product of divisor and quotient:

26. 2670 ÷ 6.	**36.** 5654 ÷ 11.	**46.** 6625 ÷ 53.
27. 5412 ÷ 6.	**37.** 3465 ÷ 11.	**47.** 5148 ÷ 26.
28. 7315 ÷ 5.	**38.** 4830 ÷ 21.	**48.** 5592 ÷ 24.
29. 2506 ÷ 7.	**39.** 7602 ÷ 21.	**49.** 14,624 ÷ 32.
30. 3536 ÷ 8.	**40.** 3534 ÷ 31.	**50.** 14,706 ÷ 43.
31. 9216 ÷ 8.	**41.** 8190 ÷ 35.	**51.** 25,960 ÷ 55.
32. 6456 ÷ 12.	**42.** 9963 ÷ 41.	**52.** 66,528 ÷ 28.
33. 4944 ÷ 12.	**43.** 7488 ÷ 24.	**53.** 18,690 ÷ 35.
34. 7956 ÷ 12.	**44.** 6908 ÷ 22.	**54.** 12,382 ÷ 41.
35. 5076 ÷ 12.	**45.** 8211 ÷ 23.	**55.** 29,210 ÷ 46.

56. Divide 869 by 24.

$$\begin{array}{r} 36\frac{5}{24} \\ 24\overline{)869} \\ 72 \\ \hline 149 \\ 144 \\ \hline 5 \end{array}$$

869 ÷ 24 = 36, with a remainder 5 still to be divided. The quotient is written $36\frac{5}{24}$.

$36 \times 24 + 5 = 869$. Does this check the work (Art. 56)?

Find the quotients:

57. 62,296 ÷ 52.
58. 37,015 ÷ 55.
59. 11,914 ÷ 23.
60. 24,050 ÷ 37.
61. 14,964 ÷ 43.
62. 22,212 ÷ 36.
63. 24,472 ÷ 23.
64. 32,088 ÷ 56.
65. 88,020 ÷ 27.
66. 36,225 ÷ 69.
67. 43,364 ÷ 74.
68. 36,876 ÷ 84.
69. 52,776 ÷ 72.
70. 75,548 ÷ 68.
71. 64,108 ÷ 94.
72. 6,859,356 ÷ 148.
73. 4,710,612 ÷ 158.
74. 8,311,857 ÷ 129.
75. 36,913,698 ÷ 237.
76. 40,740,544 ÷ 368.
77. 18,148,366 ÷ 547.
78. 46,297,160 ÷ 637.

61. Examine carefully:

$50 \div 10 = 5.$ $300 \div 100 = 3.$

$80 \div 10 = 8.$ $700 \div 100 = 7.$

$120 \div 10 = 12.$ $1500 \div 100 = 15.$

Cutting off a 0 from an integer divides it by 10; cutting off two 0's divides it by 100.

Announce results at sight:

1. 270 ÷ 10.
2. 800 ÷ 10.
3. 950 ÷ 10.
4. 370 ÷ 10.
5. 320 ÷ 10.
6. 1500 ÷ 10.
7. 1200 ÷ 100.
8. 2700 ÷ 100.
9. 8000 ÷ 1000.
10. Divide 3600 by 300.

$3|00\overline{)}\,36|00$
12

Cutting off two 0's from the right of the dividend, or dividing by 100, gives 36. This is still to be divided by 3, since $300 = 3 \times 100$. $36 \div 3 = 12$.

The arrangement of the work is shown at the left.

Find the quotients:

11. $680 \div 20$.	**14.** $2640 \div 40$.	**17.** $31{,}500 \div 500$.
12. $3400 \div 20$.	**15.** $4590 \div 30$.	**18.** $56{,}000 \div 700$.
13. $6300 \div 90$.	**16.** $9780 \div 60$.	**19.** $64{,}800 \div 800$.

62. **1.** A crate holds 360 eggs. How many dozen eggs are there in the crate?

2. A horse dealer paid \$2928 for 24 horses. What was the average price?

3. If there are 70,000 lb. of coal in a car, how many tons are in it? (2000 lb. = 1 ton.)

4. An eighteen-acre field produced 720 bushels of oats. How many bushels to the acre did it yield?

5. During the month of September a man rode his bicycle 1440 miles. If he rode the same distance each day, what was his daily rate?

6. One day the fares on a certain street car amounted to 2340 cents. Each passenger paid 5 cents. How many persons rode on the car that day?

7. Mr. Roe divided \$187,500 equally among his 3 children. A daughter divided her share equally among her 5 children. How much did each grandchild receive?

8. A drover bought some cattle for \$7168. If the average price was \$56 a head, how many did he buy?

9. A owes B \$1170. How many months will it take to pay off the debt, if he pays \$65 a month?

10. A grocer bought 35 barrels of molasses for \$525, and sold them for \$735. How much did he gain on each barrel?

11. By selling 125 bales of cotton for $5750, a broker gained $1000. How much did he pay a bale for the cotton?

12. The dividend is 19,378, the quotient 67, and the remainder 15. What is the divisor?

13. A single-track street railway is 5 miles long. How many rails does it contain, if each rail is 60 feet long? (5280 ft. = 1 mi.)

14. A train of 12 cars contained 463,680 lb. of salt in barrels. What was the average number of barrels of salt to the car, if each barrel weighed 280 lb.?

15. A man pays $312 a year for the rent of his home. What is the monthly rate?

16. A dealer pays $36 a dozen for hats. At this rate, what does he pay for 75 hats?

17. A farmer sold 28 tons of hay at $15 a ton, and with the proceeds bought cattle at $21 a head. How many cattle did he buy?

18. A certain manufacturing plant requires a 50 horse-power engine. How many mills of this capacity could be supplied with power by the Niagara Falls Power Company with its 100,000 horse-power plant?

19. At 443 miles a day, how long would it take a steamer to go from New York to Liverpool, if the route covers a distance of 3544 miles?

20. The proposed Panama canal is 46 miles in length, and the estimated cost by one corps of engineers is $184,000,000. Find the average cost per mile on the basis of this estimate.

21. A man travels 448 miles in 14 hours. What is his average rate an hour?

22. If a man picks 147 lb. of seed cotton a day, how long will it take him to pick a bale of 490 lb. of lint cotton, allowing 3 lb. of seed cotton to 1 lb. of lint?

23. A man paid $4960 for horses and cattle, buying the same number of each. If he paid $115 a head for the horses and $45 a head for the cattle, how many of each did he buy?

24. In a certain factory the weekly pay roll contained the names of 901 employees. The total amount of the pay roll was $13,515. What was the average amount paid weekly to each employee?

25. The freight on a car of coal from the mines to Roanoke, Va., is $36.80, and the rate is $1.60 a ton of 2000 lb. How many pounds in the car?

26. An author is paid a royalty on the proceeds of the sales of tickets to his dramas. If he receives $1 out of every $50, what is his royalty on sales of tickets amounting to $1500?

27. In a stable there is available 1296 square feet for stalls. If each stall occupies 72 square feet, how many horses can be kept in the stable?

28. A manuscript accepted by a publisher contains 77,500 words. If each type page has 31 lines, averaging 10 words each, how many pages will the manuscript make?

29. If a printer can set 1550 words a day, how many days will he need to put in type the entire manuscript mentioned in Example 28?

REVIEW WORK

ORAL EXERCISES

63. 1. What name do you give to 5 tens? To 10 tens? To 10 hundreds? To 1000 thousands?

2. In the number 536, what place is occupied by the 6? By the 3? By the 5?

3. In the number 10,519,138, what name is given to the period 138? To the period 519? To the period 10? Read the entire number.

4. Could we write such numbers as 805 without the aid of 0? What use has the zero?

5. If you pay a bill of 35 cents, how much change do you get out of $1?

6. The subtrahend is 9, the minuend 27. What is the difference?

7. The difference is 10, the minuend 33. What is the subtrahend? 33 minus what number equals 10?

8. The subtrahend is 17, the remainder 19. What is the minuend? 17 plus 19 equals what number?

9. State the products obtained by multiplying each of the following numbers by 7: 4, 7, 9, 6, 8, 11.

10. Compare 3×5 with 5×3. Compare $3 \times \$5$ with $5 \times \$3$. What conclusion do you arrive at as to the order of the factors?

11. Since $5 = 10 \div 2$, to multiply an integer by 5 is the same as to annex 0 to the integer and divide by what number? Make an example and try.

12. Since $25 = 100 \div 4$, to multiply an integer by 25 we may first annex two 0's to the integer, and then divide by what number? Make a test.

13. State the quotients obtained by dividing each of the following numbers by 7: 56, 63, 280, 630, 497, 644.

14. Compare $80 \div 20$ and $8 \div 2$. Compare $900 \div 300$ and $9 \div 3$. What do you find out about dropping the same number of zeros from both dividend and divisor?

15. Explain the two cases of division shown in the following: $\$20 \div \$5$; $\$20 \div 5$.

State rapidly the sums, noting the time required for the entire exercise:

16.	$35 + 11$.	**21.**	$70 + 33$.	**26.**	$78 + 54$.
17.	$19 + 71$.	**22.**	$65 + 42$.	**27.**	$45 + 87$.
18.	$88 + 12$.	**23.**	$73 + 28$.	**28.**	$59 + 69$.
19.	$140 + 80$.	**24.**	$145 + 63$.	**29.**	$330 + 84$.
20.	$115 + 76$.	**25.**	$250 + 75$.	**30.**	$440 + 120$.

State rapidly the differences, timing yourself as above:

31.	$53 - 22$.	**36.**	$72 - 22$.	**41.**	$97 - 58$.
32.	$84 - 71$.	**37.**	$93 - 48$.	**42.**	$72 - 37$.
33.	$61 - 37$.	**38.**	$82 - 23$.	**43.**	$91 - 68$.
34.	$151 - 40$.	**39.**	$136 - 66$.	**44.**	$165 - 73$.
35.	$125 - 80$.	**40.**	$223 - 33$.	**45.**	$382 - 102$.

46. Fill the blanks with the proper numbers.

Minuend	24	48		64	72	
Subtrahend	18		18	12		15
Remainder		16	32		60	40

State rapidly the products:

47. 63×2.
48. 80×9.
49. 92×7.
50. 64×5.
51. 81×10.
52. 190×2.
53. 930×3.
54. 810×7.
55. 420×5.
56. 320×4.
57. 900×11.
58. 310×12.
59. 520×6.
60. 812×8.
61. 610×7.

State rapidly the quotients:

62. $110 \div 10$.
63. $123 \div 3$.
64. $192 \div 4$.
65. $625 \div 5$.
66. $728 \div 7$.
67. $400 \div 8$.
68. $570 \div 6$.
69. $840 \div 12$.
70. $561 \div 11$.
71. $315 \div 7$.
72. $420 \div 20$.
73. $330 \div 30$.
74. $640 \div 20$.
75. $360 \div 60$.
76. $2808 \div 9$.

77. Fill the following blanks:

$7\overline{)49}$ $\quad 6\overline{)}\ 9$ $\quad \overline{)55}\ 5$ $\quad 6\overline{)}\ 8$ $\quad \overline{)72}\ 8$ $\quad 7\overline{)84}$

State the cost of the following articles:

78. 8 lb. coffee at 25¢ a pound.
79. 3 yd. cloth at $1.50 a yard.
80. 4 lb. cheese at 15¢ a pound.
81. 5 packages oatmeal at 12¢ a package.
82. A hat at $1.75, and a cap at 25¢.

State the price of each piece, pound, or pair if —

83. 10 cakes of soap cost 80¢.

84. 7 lb. of crackers cost 56¢.

85. 8 lb. of mixed nuts cost $1.20.

86. 3 lb. of tea cost $2.70.

87. 5 pairs of skates cost $6.25.

88. How many typewriters will $4000 buy at $100 each?

89. If 6 books cost $12, how much will 1 book cost at the same rate? How much will 2 books cost?

90. If 6 pencils cost 30¢, what do 3 pencils cost at the same rate?

Since 6 pencils cost 30¢, 1 pencil costs $\frac{1}{6}$ of 30¢, or 5¢; and 3 pencils cost 3×5¢, or 15¢.

91. If 6 cords of wood cost $24, what will 1 cord cost at the same rate? What will 9 cords cost?

92. If 5 spellers cost 80 cents, what will 9 spellers cost?

93. If $\frac{1}{2}$ of a pound of candy costs 12 cents, how much must I pay for 2 pounds?

94. If 6 men can do a piece of work in 8 days, how many days will it take 4 men to do it?

If it takes 6 men 8 days to do the work, it will take 1 man 6×8 days, or 48 days; if it takes 1 man 48 days, it will take 4 men $\frac{1}{4}$ of 48 days, or 12 days.

95. If 5 men can dig a ditch in 9 days, how many men will be required to dig it in 3 days?

96. At 15 cents a dozen, what do 72 bananas cost?

97. How many pears at the rate of 3 for 5 cents can I buy for 30 cents?

98. How many pounds of sugar at 6 cents a pound must be given for 3 lb. of coffee at 20 cents a pound?

99. If you divide 48 marbles equally among 6 boys, how many marbles do 2 boys get?

100. If 9 hats cost $27, what do 5 hats cost at the same rate?

101. What number is represented by 8 ones of the first period and 5 tens of the second period?

102. How much do 13 cakes of soap cost, if 9 cakes cost 72 cents?

103. A man bought 10 sheep for $40, and sold 6 of them at the same rate. How much did he get for them?

104. How many weeks will it take A to earn $99, if he earns $18 in two weeks?

105. If 4 lb. of cheese cost 64 cents, what do 10 lb. cost?

106. If 7 men can build a fence in 9 days, how long will it take 3 men?

107. If a load of hay feeds 7 cows a week, how long will it feed 14 cows?

108. If I buy 2 quarts of milk a day at 5 cents a quart, what is my weekly milk bill?

109. If a car has 8 wheels, how many cars will 56 wheels supply?

110. A boy bought 9 lead pencils at 4¢ each, and exchanged them for tablets at 6¢ each. How many tablets did he get?

111. John is 7 years old, and his father is 4 times as old and 7 years more. How old is his father?

112. How many square feet in the floor of a room 20 feet long and 9 feet wide?

113. Jane's father gave her $4 every 4 months. How many dollars did he give her in 1 year?

114. If a cow gives 10 qt. of milk a day, how many gallons does she give in 6 days?

115. A man pays $10 a month for house rent and $7 a month for office rent. How much rent does he pay in 1 year?

116. A man bought 6 bushels of apples for $3, and exchanged them for oranges at 30 cents a dozen. How many dozen did he receive?

WRITTEN EXERCISES

64. **1.** Add 2498, 48, 6735, 9476.

2. Add 87, 975, 45, 108, 7358, 5763.

3. Add 3987, 4876, 9054, 7083, 8407, 2930.

Find the values:

4. $357 + 6048 + 867 + 15{,}008 + 3715$.

5. $5984 + 876 + 93 + 4672 + 8456 + 8769$.

6. $63{,}251 - 679$.

7. $72{,}175 - 896$.

8. $84{,}037 - 55{,}069$.

9. $90{,}524 - 8765$.

10. $54{,}321 - 23{,}456$.

11. $756{,}328 - 667{,}439$.

12. $431{,}562 - 87{,}968$.

13. $936{,}061 - 7076$.

14. $538{,}013 - 199{,}815$.

15. $832{,}415 - 643{,}749$.

16. 27,583 × 574.
17. 38,065 × 4018.
18. 72,736 × 8539.
19. 72,069 × 964.
20. 38,254 × 9651.
21. 25,864 × 3972.
22. 98,357 × 14,098.
23. 37,528 × 26,573.
24. 376,859,376 ÷ 48.
25. 238,311,908 ÷ 29.
26. 618,875,225 ÷ 745.
27. 204,963,416 ÷ 406.
28. 584,093,472 ÷ 356.
29. 230,884,080 ÷ 5684.
30. 941,108,532 ÷ 3819.
31. 810,882,072 ÷ 9009.

32. A man gave $22 for a buggy, and paid $9 for repairing it. For how much must he sell it to gain $13?

33. The earth moves in its orbit 19 miles a second. How many miles does it move in a minute? In an hour?

34. B and C start from the same place, and travel in opposite directions. B travels 7 miles an hour, and C 5 miles an hour. How far apart are they in 19 hours?

35. A clock strikes 156 times a day. How many times does it strike in the month of December?

36. Sound travels 1132 feet a second. How far away is a cannon, the report of which is heard in 17 seconds?

37. If you purchase a horse for $200, and pay $56 in cash, how long must you work at the rate of $12 a week to pay the remainder?

38. If your pulse beats 4320 times in an hour, how many times does it beat in two days?

39. Gold was discovered in California in 1848. How many years ago was the discovery made?

40. A farmer sold 120 bushels of wheat at $1 a bushel. How many calves at $6 each can he buy with the money?

41. A boy takes 19 subscriptions to the "Saturday Evening Post" at $1.25 each. How much money does he receive for them?

42. A man owed $5964. He made three payments, one of $1375, and two of $964 each. How much did he still owe?

43. An electric car line is 11 miles long. If a car makes 5 round trips daily, how many miles does it run during the month of June?

44. If 9 tons of coal are worth $45, what are 29 tons worth at the same rate?

45. America was discovered in 1492; the settlement of Jamestown was made in 1607. How many years was it from the discovery of America to the settlement at Jamestown?

46. A man paid $180 for a horse, which was 6 times as much as he paid for a wagon. How much did he pay for both?

47. The multiplicand is 99,621; the multiplier 286. What is the product?

48. If the divisor is 55 and the dividend 10,285, what is the quotient?

49. A baker has 240 loaves of bread. He sells 4 of them at 7¢ each, and the remainder at 6¢ each. How much does he get for his bread?

50. A farmer clipped 1254 lb. of wool from his sheep. If each fleece weighed 6 lb., how many sheep had he?

51. Divide 4536 by 21; also by 3, and the resulting quotient by 7. Compare results. What do you find?

52. A teacher earns $1125 in 9 months. How many months will it take her to earn $750?

53. In 100 years the population of the United States increased from 3,929,250 to 62,622,250. What was the average yearly increase?

54. A train runs 350 miles in 11 hours. Twenty stops of 3 minutes each are made. Find the train's average rate an hour while running.

55. Find the cost of 250 tierces of lard, each weighing 340 pounds, at 7¢ a pound.

56. A produce buyer paid $135 for 3 car loads of melons. At that rate, how much do 10 car loads cost?

57. A farmer bought 160 acres of land at $40 an acre. After spending $600 for improvements, he sold it at a gain of $520. At what price an acre did he sell?

58. If a train runs 36 miles an hour, in how many hours can it run to a place 144 miles distant and return, allowing 1 hour for time lost at the terminus?

59. A farmer owing $790, gave in part payment 250 bushels of wheat at $1 a bushel, and paid the remainder with flour at $6 a barrel. How many barrels of flour were required?

60. I have an income of $1800, and my expenses during the year are as follows: rent $300, food $400, service $150, fuel $75, light $20, repairing and replenishing $40, clothing $200, insurance $100, emergency $35, church and charity $50, literature and travel $130. What is my yearly saving?

61. The contractor's bid for building a dwelling house was as follows: excavation $40, foundation and masonry $115.20, brick work and plastering $400, cement floor $35, lumber $800, mill work $175.50, hardware $60, painting $110, plumbing and tinning $260, doors and sash $160, heating $150, labor $680. What was the total bid?

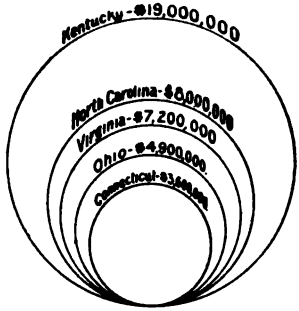
RELATED PROBLEMS

65. The picture gives a good idea of the ordinary farm as laid out into fields for various purposes. The problems which follow show the immense value of our farms as sources of national wealth.

1. In a recent year the yield in bushels of certain leading agricultural products was as follows: corn, 2,030,000,000; oats, 811,000,000; wheat, 632,000,000; potatoes, 273,000,000; apples, 176,000,000; barley, 85,000,000; rye, 27,000,000; buckwheat, 12,000,000. What was the total number of bushels produced? What was the value of the wheat at 75¢ a bushel? If the yield of wheat averaged 16 bu. an acre, how many acres were in wheat?

2. The value of the corn crop in the five greatest corn-producing states in 1900 was as follows: Illinois, $115,000,000; Iowa, $97,000,000; Missouri, $61,000,000; Kansas, $58,000,000; Indiana, $52,000,000. What was the total value for the five states? If 4 bu. of corn sold for $1, how many bushels did each state produce?

3. The value of the tobacco crop in five of the greatest tobacco-producing states in the same year was as follows: Kentucky, $19,000,000; North Carolina, $8,000,000; Virginia, $7,200,000; Ohio, $4,900,000; Connecticut, $3,600,000. What was the total value of the tobacco crop in these states? Virginia's annual product was how many times the value of the crop produced by Connecticut?



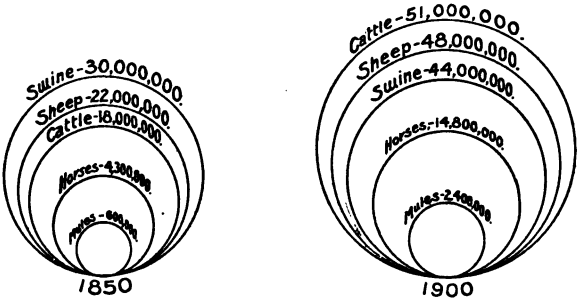
LEADING TOBACCO STATES

4. The value of the cotton produced in five leading cotton states in a recent year was as follows: Texas, $154,989,000; Mississippi, $85,957,000; Georgia, $76,024,000; Alabama, $57,000,000; South Carolina, $47,219,000. Find the total value of the cotton crop in the five states. If the cotton is valued at $45 a bale, of 500 pounds each, how many bales did Texas produce?

5. In a certain year the value of the vegetable crop in the United States was as follows: Irish potatoes, $98,000,000; sweet potatoes, $20,000,000; onions, $6,600,000; sugar beets, $3,300,000; miscellaneous vegetables, $114,000,000. Find how much the total value of the vegetable crop is exceeded by the value of the cotton crop in the five states named in Example 4.

6. In a recent year the apple crop in five leading states was as follows: New York, 24,110,000 bu.; Pennsylvania, 24,060,000 bu.; Ohio, 20,620,000 bu.; Virginia, 9,840,000 bu.; Illinois, 9,180,000 bu. What was the total value at 35¢ a bushel? What was the value of Virginia's crop?

7. In the year 1850 the number of farm animals in the United States was as follows: swine, 30,000,000; sheep, 22,000,000; cattle, 18,000,000; horses, 4,300,000; mules,



600,000. In 1900 the number was as follows: cattle, 51,000,000; sheep, 48,000,000; swine, 44,000,000; horses, 14,800,000; mules, 2,400,000. Find the average yearly increase of each kind in the 50 years.

8. In a certain year this country had 69,336,000 head of cattle. What was their total value, if the average value was $24 a head?

9. In a recent year this country produced 287,400,000 lb. of wool. If 3 lb. could be bought for $1, what was the total value of the wool produced?

10. In a recent year the value of the horses in five leading states was as follows: Iowa, $77,000,000; Illinois, $69,000,000; Ohio, $50,000,000; New York, $48,000,000; Kansas, $44,000,000. If the total value of the horses in the United States was $1,050,000,000, what was the value of the horses in all the other states? If the average value of horses was $50 a head, how many horses had Iowa that year?

FACTORS AND MULTIPLES

66. 1. What two numbers multiplied together make 6? Then what are the factors (*makers*) of 6?

2. Is each factor of 6 an exact divisor of 6?

3. Name two factors of 21. Of 35. Of 33.

67. The integers which multiplied together produce a number are called **Factors** of that number.

Thus, 5 and 6, or 2, 3, and 5, are the factors of 30.

1. What are the exact divisors (factors) of 10? Of 14? Of 16?

2. Name a factor that is common to both 10 and 24. To both 25 and 35.

3. Name the greatest common factor of 12 and 18.

We have need for factors in changing fractions to smaller denominators.

68. A factor that is common to two or more numbers is called a common factor, or **Common Divisor.**

Thus, 3 is a common divisor of 9 and 21.

69. The greatest factor common to two or more numbers is called their **Greatest Common Divisor.**

Thus, 8 is the greatest common divisor of 16, 24, and 32. Why? Is 4 also a common divisor?

70. A number whose only exact divisors are itself and one is called a **Prime Number.**

A number that has other exact divisors is called a **Composite Number.**

Thus, 1, 2, 3, 5, etc., are prime numbers, and 4, 6, 8, 9, etc., are composite numbers.

1. Make a list of all the prime numbers to 50.

2. Make a list of all the numbers from 1 to 50 that are exactly divisible by 2. Make a list of those not exactly divisible by 2.

71. A number that is exactly divisible by 2 is called an **Even Number.** All other numbers are called **Odd Numbers.**

72. The exact divisors, or factors, of a number are found by trial. The following tests are very helpful in finding factors:

Any number is exactly divisible —

(1) By 2, when the right-hand digit is 0, 2, 4, 6, or 8.

(2) By 3, when the sum of its digits is divisible by 3.

(3) By 4, when the number represented by the two right-hand digits is divisible by 4.

(4) By 5, when the right-hand digit is 0 or 5.

(5) By 6, when it is divisible by both 2 and 3.

(6) By 8, when the number represented by the three right-hand digits is divisible by 8.

(7) By 9, when the sum of its digits is divisible by 9.

Find some divisors of the following by inspection:

1. 125.	4. 2406.	7. 9070.	10. 3654.
2. 240.	5. 5300.	8. 5200.	11. 4435.
3. 360.	6. 1904.	9. 3357.	12. 9525.

73. 1. What prime numbers multiplied together make 6? 10? 22?

2. What prime numbers (two or more) will exactly divide 12, or what are the *prime factors* of 12?

74. Prime numbers used as factors are called **Prime Factors.**

Thus, 2, 3, 5, are the prime factors of 30.

75. Two numbers that have no common factor are said to be *prime to each other.* May one or both of them be composite?

1. Since 3 is a factor of 6, it is a factor of two 6's, or 12. Is it a factor of any number of 6's?

2. Twelve is a factor of 36. Are all the factors of 12 also factors of 36? Make several tests.

76. An exact divisor of a factor of a number is a factor of the number itself.

77. 1. What are the prime factors of 462?

2	462
3	231
7	77
	11

Since the prime number 2 is a divisor of 462, it is one of the factors, and 231 is another. Since 3 is an exact divisor of 231, it is a factor of 462 (Art. 76), as is 77 also. Likewise 7 and 11 being exact divisors of 77 are also factors of 462. Hence the prime factors are 2, 3, 7, 11.

2. Find the greatest common divisor of 462 and 330.

(*a*)		(*b*)	
2	462	2	330
3	231	3	165
7	77	5	55
	11		11

Finding the prime factors of the two numbers, as in the last example, we have

$$462 = 2 \times 3 \times 7 \times 11,$$

$$330 = 2 \times 3 \times 11 \times 5,$$

in which 2, 3, and 11 are the only common factors. Therefore $2 \times 3 \times 11$, or 66 is the greatest common divisor.

Find the prime factors of:

3. 42.	9. 315.	15. 1754.
4. 72.	10. 428.	16. 1365.
5. 120.	11. 1826.	17. 4585.
6. 144.	12. 1056.	18. 2716.
7. 308.	13. 2310.	19. 2002.
8. 225.	14. 4806.	20. 1576.

Find the greatest common divisor of:

21. 64 and 72.	24. 210 and 525.
22. 49 and 63.	25. 125, 175, and 275.
23. 1440 and 1728.	26. 132, 154, and 198.

78. The product of two or more integers is called a **Multiple** of those numbers. It follows that every number is a multiple of any of its factors.

1. Does 6 exactly contain both 2 and 3?
2. What number contains 5 and 3 without a remainder?
3. Name a multiple that is *common* to 5 and 7. To 4 and 8. To 2, 3, and 5.

79. The least number that is exactly divisible by each of two or more numbers is called their **Least Common Multiple** (L. C. M.).

We have need to find the L. C. M. of the denominators when we wish to change two or more fractions to fractions having the least common denominator.

1. What are the prime factors of 6? Of 8? What is their L. C. M.?
2. What are the prime factors of 30? How do they compare with those of 6 and 10?

3. What factor is common to 6 and 10? Does it occur also in the factors of 30?

4. Since 30 contains both 6 and 10, it contains all their prime factors.

5. The factors of 6 are 2 and 3, and those of 15 are 3 and 5. Which of these factors must be multiplied together to produce the L. C. M. of 6 and 15?

80. The least common multiple of two or more numbers contains all the prime factors of those numbers, and no others.

WRITTEN EXERCISES

81. 1. Find the least common multiple of 25, 30, and 42.

$25 = 5 \times 5.$

$30 = 5 \times 2 \times 3.$

$42 = 2 \times 3 \times 7.$

$5 \times 5 \times 2 \times 3 \times 7 = 1050.$

The least common multiple must contain all the prime factors of 25, 30, and 42; that is, 2, 3, 5, and 7. Each of these must be contained as often as it occurs in any one set of factors. The only factor that occurs twice in one number is 5. Hence the factors of the L. C. M. are 2, 3, 5, 5, 7. Their product is 1050, the L. C. M.

When several numbers have no common factor, their product is their L. C. M.

Find the L. C. M. of the following:

2. 6, 24, 30.

3. 12, 27, 32.

4. 22, 55, 77.

5. 24, 30, 60.

6. 32, 60, 80.

7. 3, 5, 9, 8, 12.

8. 7, 6, 3, 4, 5.

9. 2, 7, 8, 10, 14.

10. 4, 6, 8, 16, 24.

11. 3, 5, 7, 11, 13.

UNITED STATES MONEY

AND

DECIMALS

82. 1. Name the different pieces of money in the picture. Tell something you can buy with each piece. Will the silver piece buy more than the pile of copper money?

2. How many cents make one dime? Then a cent is what part of a dime?

3. Add 1¢, 2¢, 3¢, and 4¢, and express the sum in two ways.

4. Can you subtract two cents from a dime?

5. By what must 2¢ be multiplied to make a dime?

6. How many piles of 5¢ each can be made of the cents in the picture?

What operations are used in the last four exercises?

7. Tell what you know of United States coins of less value than a dime.

83. 1. Name the coins in the picture. Tell what you can buy with one of each kind. Will the pile of small coins buy more than the large coin?

2. How many dimes make one dollar? Then a dime is what part of a dollar?

3. Add 4 dimes, 3 dimes, and 3 dimes; express the sum in two ways.

4. If a pocketknife costs 6 dimes, how much change would be returned to you if you offered a dollar in payment?

5. John is paid 2 dimes a day for cutting wood in the afternoon of each school day. How much will he earn in a week?

6. An uncle gives his three nieces and two nephews a dollar so that each child has the same amount of money. How much does each receive?

What operations are used in the last four exercises?

7. Tell all you know of United States coins greater than a dime and less than a dollar.

8. One tenth of a cent is called a mill. Then how many mills make 1 cent?

9. How many mills in 10 cents? In 100 cents, or $1? Then 1 mill is what part of a dollar?

84. Table of United States money:

10 mills = 1 cent.

10 cents = 1 dime.

10 dimes = 1 dollar.

The dollar is our monetary unit. Values are expressed in dollars as whole numbers, dimes as tenths, and cents as hundredths. Such expressions are called **Decimals,** because the scale is by *tens.*

The mill is not coined, but we sometimes use the name for the value.

85. Five dollars, two dimes, seven cents = $5.27.

Write decimally:

1. One dollar, two dimes, one cent.
2. Four dollars, six dimes, two cents.
3. Sixteen dollars, four dimes, five cents.
4. Eighty-five dollars, seven dimes, four cents.

Read in dollars, dimes, and cents:

5. $14.28.	8. $17.98.	11. $182.06.
6. $16.39.	9. $16.47.	12. $ 45.21.
7. $27.82.	10. $87.60.	13. $147.20.

14. Tell the kinds of money used for the simplest way of counting out the amounts given in Exercises 5–13. For example, $14.28 may be made up of one ten-dollar bill, two two-dollar bills, a quarter, and three coppers.

15. A twenty-dollar gold piece is offered in payment of each of the amounts in Exercises 5, 6, 8, and 9. How much change must be given in each case?

16. Tell all you can about gold coins. About paper money.

86. 1. A cent is a tenth of a dime. How many tenths of a dime make one dime?

How many tenths of a dollar make one dollar?

How many tenths make one whole?

2. Read the table from left to right, calling each number by the name above it.

Read the sum which is printed in the row at the bottom, using the word "and" in place of the point.

Replace 1 by 5 in each column, add, and read the result.

It is not necessary to read "no tens," "no ones," etc.; simply read those numbers which have some value.

hundreds	tens	ones	point	tenths	hundredths
1	0	0	.		
	1	0	.		
		1	.		
			.	1	
			.	0	1
1	1	1	.	1	1

3. Copy the numbers in the table and read them. You have already been directed to omit the word "and" between orders and periods when reading integers. In reading decimals, use it only between the *ones* and *tenths*. Thus, 558.28 should be read five hundred fifty-eight *and* twenty-eight hundredths.

4. If we use paper ruled like this table, there is no use of a point, inasmuch as the dollars and cents are separated by a heavy vertical line instead of by points. In most accounts such a system is used.

1	2	3	57
	2	5	89
		6	87
2	4	7	84
	8	6	91
	6	7	20
5	5	8	28

5. Read the numbers and make the addition. Then read the sum as dollars and cents, and also as dollars and hundredths of a dollar.

87. The Arabic is a decimal system, in which numbers are expressed in 10's and multiples of 10.

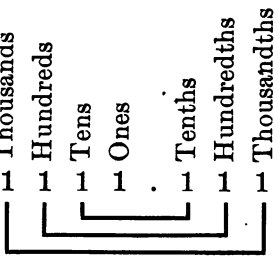
United States money is likewise a decimal system with which every one is more or less familiar, and appropriately introduces the study of decimals.

88. The point used to separate ones and tenths is called the **Decimal Point.**

89. Examine carefully the following:

The places to the right of the decimal point are called **decimal places.** Compare the names of the places to the right of the ones with those of the corresponding places on the left.

What does the first figure at the right of the decimal point express? Which figure expresses hundredths? Which expresses thousandths?



90. Read as dimes and cents:

$.04.	$.05.	$.50.	$.25.	$.07.
$.08.	$.30.	$.20.	$.15.	$.75.
$.35.	$.45.	$.95.	$.60.	$.80.
$.28.	$.55.	$.86.	$.72.	$.99.

Read as tenths and hundredths:

.03.	.45.	.05.	.5.	.7.	.25.
.23.	.13.	.65.	.15.	.35.	.21.
.42.	.44.	.85.	.75.	.95.	.77.
.36.	.27.	.76.	.83.	.66.	.94.

WRITTEN EXERCISES

91. Copy and complete:

5 hundredths of a dollar = 5 ¢ = $.05.

4 hundredths of a dollar =

7 hundredths of a dollar =

2 tenths of a dollar =

8 tenths of a dollar =

1 tenth, 5 hundredths of a dollar = $.15.

25 hundredths of a dollar =

5 tenths, 5 hundredths of a dollar =

92. Write with the decimal point:

2 hundredths.	2 tenths, 4 hundredths.
8 hundredths.	3 tenths, 3 hundredths.
5 tenths.	7 tenths, 3 hundredths.
9 tenths.	1 tenth, 7 hundredths.
46 hundredths.	10 hundredths.

3 tenths, 6 hundredths, 5 thousandths.

5 tenths, 3 hundredths, 6 thousandths.

8 tenths, 9 hundredths, 7 thousandths.

93. Copy and read:

.1.	.12.	.125.
14.5.	9.05.	8.125.
7.052.	600.075.	.675.

94. Add:

1.	2.	3.	4.	5.
25.5	16.4	75.7	72.4	$65.25
38.6	68.3	27.4	68.68	$37.43
67.5	92.3	91.2	65.78	$98.86

Subtract:

6.	7.	8.	9.	10.
75.35	96.30	92.55	89.5	$170.6
22.12	30.75	36.99	46.8	$125.8

Where do we place the decimal point in the results?

95. Write 1000 and put a point in different places.

For example, 1st, 1000; 2d, 100.0; 3d, 10.00; 4th, 1.000. Note the change in value as you pass from first to second, second to third, third to fourth.

The third is how many times the fourth? The second is how many times the third? The first is how many times the second?

Compare the following:

$1 \times 10 = 10.$ $\qquad$ $1000 \div 10 = 100.$

$10 \times 10 = 100.$ $\qquad$ $100 \div 10 = 10.$

$100 \times 10 = 1000.$ $\qquad$ $10 \div 10 = 1.$

From the preceding it is seen that —

Multiplying by ten is the same as moving the point one place to the right. Dividing by ten is the same as moving the point one place to the left.

WRITTEN EXERCISES

96. Multiply each of the following numbers by ten by moving the decimal point:

1. 43.62.	4. 65.3.	7. 42.7.
2. 2.78.	5. 78.75.	8. .643.
3. 484.2.	6. 82.06.	9. .062.

Divide each of the following numbers by ten by moving the decimal point:

10. 278.2.	14. 6.473.	18. 2.793.
11. 3.541.	15. 257.9.	19. 289.1.
12. 657.	16. 867.	20. 78.47.
13. 4847.	17. 2480.	21. 8.965.

97. Tell how many places the points have been moved, and in which direction, in the decimals compared with whole numbers composed of the same digits:

65.3 and 653.	3.74 and 374.
47.2 and 472.	43.62 and 4362.

To multiply one decimal by another, *first note how many places the point has been moved* (*i.e.*, how many times a whole number expressed by the same figures has been divided by ten) *in each case to make the given decimal. Multiply the numbers without regarding the points and divide the product by ten as many times as it has been divided in the two given decimals.*

The product of two decimals contains as many decimal places as there are places in the two given decimals.

WRITTEN EXERCISES

98. 1. Multiply 3.74 by 9.2.

$$\begin{array}{r} 374 \\ 92 \\ \hline 748 \\ 3366 \\ \hline 34.408 \end{array}$$

$3.74 = 374 \div 100$ (*i.e.*, by 10×10); $9.2 = 92 \div 10$. Hence 34408, the product of the whole numbers, should be divided by 1000 (*i.e.*, by $10 \times 10 \times 10$), and we have 34.408 as the product of the decimals. Or, applying the principle we find two decimal places in the multiplicand and one in the multiplier. Hence three decimal places must be pointed off in the product, and again we have 34.408.

Find the products:

2. $.8 \times .7$.	16. $.217 \times 3.9$.	30. $\$145.50 \times 6.5$.
3. $.16 \times 8.7$.	17. 104×4.01.	31. $\$650 \times .375$.
4. $.45 \times .16$.	18. $\$1.35 \times 1.08$.	32. 72.5×10.6.
5. $10 \times .15$.	19. 5.125×2.5.	33. 122.5×1.48.
6. $.13 \times 25$.	20. $1000 \times .001$.	34. $25.8 \times .45$.
7. $21.6 \times .26$.	21. $17.5 \times .05$.	35. $\$4.05 \times 20.2$.
8. $.488 \times .153$.	22. $.258 \times .158$.	36. 12.5×4.658.
9. $.017 \times .09$.	23. $.1 \times 1000$.	37. 2.165×8.35.
10. $.01 \times .001$.	24. $.1 \times .1$.	38. $\$27.40 \times 16.3$.
11. $\$150 \times .25$.	25. $55 \times .5$.	39. $\$89.27 \times 48.5$.
12. $\$2.30 \times .15$.	26. $55 \times .05$.	40. $68.293 \times .78$.
13. 7.75×19.6.	27. $5.5 \times .005$.	41. $\$85.67 \times 3.12$.
14. 3.5×5.3.	28. 34.6×9.25.	42. $\$84.48 \times .037$.
15. $\$175 \times .06$.	29. 325.25×2.4.	43. $276.58 \times .57$.

99. Consider the following example:

Divide \$34.56 by 24; also 34.56 by 24.

(*a*)	(*b*)
\$1.44	1.44
24)\$34.56	24)34.56
24	24
10 5	10 5
9 6	9 6
96	96
96	96

(*a*)

$\frac{1}{24}$ of \$34 = \$1, and \$10 remaining. \$10 = 100 dimes. 100 dimes + 5 dimes = 105 dimes.

$\frac{1}{24}$ of 105 dimes = 4 dimes, and 9 dimes remaining. 9 dimes = 90 cents. 90 cents + 6 cents = 96 cents.

$\frac{1}{24}$ of 96 cents = 4 cents.

(*b*)

34 ÷ 24 = 1, and 10 remaining. 10 = 100 tenths. 100 tenths + 5 tenths = 105 tenths.

105 tenths ÷ 24 = 4 tenths, and 9 tenths remaining. 9 tenths = 90 hundredths. 90 hundredths + 6 hundredths = 96 hundredths.

96 hundredths ÷ 24 = 4 hundredths.

To divide one decimal by another, *change the divisor to a whole number by moving the decimal point to the right; move the point a like number of places in the dividend, annexing ciphers if required. Divide as in whole numbers, placing the decimal point in the quotient directly over that in the dividend.*

WRITTEN EXERCISES

100. 1. Divide 34.408 by 9.2.

$$
\begin{array}{r}
3.74 \\
92\overline{)344.08} \\
276 \\
\hline
680 \\
644 \\
\hline
368 \\
368 \\
\hline
\end{array}
$$

$34.468 \div 9.2 = 344.08 \div 92.$

Have the pupils explain the other steps in the process.

2. Divide 427.5 by 2.25; also 4.275 by 225.

$$
\begin{array}{r}
(a) \\
190. \\
225\overline{)42750.} \\
225 \\
\hline
2025 \\
2025 \\
\hline
0
\end{array}
\qquad\qquad
\begin{array}{r}
(b) \\
.019 \\
225\overline{)4.275} \\
2\ 25 \\
\hline
2\ 025 \\
2\ 025 \\
\hline
\end{array}
$$

Remember that annexing ciphers to the right of the decimal point does not change the value of the number.

An integer may be considered as having a decimal point at the end.

Let the pupil now complete and learn the principle for pointing off decimals when dividing:

There are as many decimal places in the quotient as the number of places in the ——— diminished by number of places in the ———.

Find the quotients:

3. $62.5 \div 25$.
4. $62.5 \div 2.5$.
5. $16.8 \div .12$.
6. $.884 \div .34$.
7. $3.55 \div 7.1$.
8. $16.02 \div .45$.
9. $.924 \div .42$.
10. $.0375 \div .25$.
11. $921.6 \div 96$.
12. $227.151 \div 5.37$.
13. $2697.5 \div 8300$.
14. $1563.25 \div 32.5$.
15. $31.16 \div .41$.
16. $17.28 \div .024$.
17. $11.56 \div 6.8$.
18. $52.8 \div 1.76$.
19. $3.116 \div .82$.
20. $20.72 \div 2.8$.
21. $26.975 \div .83$.
22. $444.08 \div 5.6$.
23. $2880 \div 1.6$.
24. $1440 \div .32$.
25. $323.817 \div 7.41$.
26. $2165.44 \div 80.8$.
27. $146.3775 \div 21.75$.
28. $525.525 \div 8.25$.

29. At 18¢ a pound, what will 6.5 pounds of steak cost?

30. At 22¢ a pound, what will 7.5 pounds of butter cost?

31. At 12¢ a yard, what will 8.5 yards of ribbon cost?

32. At 36¢ a gallon, what will 4.5 gallons of oil cost?

33. A dealer sells 3 turkeys for $4.50. At the same rate, how many does he sell for $31.50?

34. A grocer paid $19.44 for 324 pounds of rice. What did it cost a pound?

35. If 8 pounds of tea cost $3.48, what will 6 pounds cost?

36. A man receives $12.50 a month as rent on a cottage. It costs him each month for taxes $1.25, for insurance $.90, and for repairs $2.35. How much does he save a month?

37. At 2.5¢ each, how much will 5.5 dozen oranges cost?

38. If 6 dozen pairs of gloves cost $28.80, what is the price a pair?

39. A merchant pays $66 for 2 dozen hats. At this rate what does he pay for 100 hats?

40. A druggist pays $5.25 for 150 cakes of soap. What is the price a cake?

41. I buy 7.5 pounds of coffee for $1.80. What is the price a pound?

42. A woman pays the grocer $1.80 for 24 pounds of prunes. What is the price a pound?

43. A merchant bought 6 dozen pairs of shoes for $2.25 a pair, and sold them at $3.75 a pair. How much did he gain?

44. A dealer sells 36 yd. of cloth for $50.40, thereby gaining $7.20. How much did it cost him a yard?

45. A dealer bought timothy hay at $12.50 a ton, and sold it at $14.75, gaining $18. How many tons did he buy?

46. Find the cost of peas to plant 6 A., if 2.5 bushels are used to the acre, and the seed are worth $2.30 a bushel.

47. A farmer had 16 acres planted in tobacco. The average yield was 813.5 pounds to the acre. What was the total yield? What did his crop bring at $21 per 100 lb.?

48. The number of pounds in the several parts of a dressed mutton and the cost of each per pound were as follows: leg, 30.5 pounds at 8¢; loin, 24.5 pounds at 6¢; rib, 20.25 pounds at 8¢; chuck, 26.75 pounds at 4¢. Find the total value.

RELATED PROBLEMS

101. 1. The picture represents a 90-acre field of corn that averaged 130 bu. per acre. What was the total yield?

2. If this corn was sold at an average price of \$.45 a bushel, what was the total value of the crop?

3. If it cost the owner \$2268 to plow the ground, plant, cultivate, and gather the crop, how much more did the corn bring than the cost of production?

4. A bushel of corn weighs 56 lb. How much does the corn grown in the above field weigh?

5. At 22¢ per 100 lb., what is the freight on this crop to market?

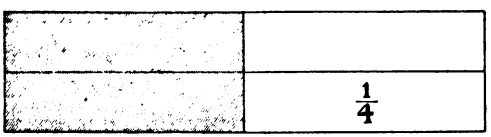
6. How many car loads does this crop make, allowing 32,760 lb. to the car?

7. A neighboring farmer had 160 acres of land in corn, and his average yield was 45 bu. to the acre. If he sold the crop at the same price, what did he receive for his corn?

COMMON FRACTIONS

102. Fraction is derived from a word which means *broken*, and so whenever the word is seen or heard, it refers to something (some whole) which has been broken or divided into equal parts.

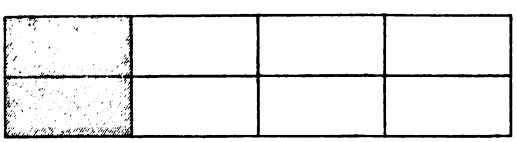
1. What part of this rectangle is dark? What part is white?



Into how many equal parts is it divided? What is the name of each part? Write it as a number and as a word. How many of these parts make the shaded portion of the figure?

2. Complete: $\frac{3}{4} = \frac{?}{?}$.

3. Apply the same questions to some object in the schoolroom which has been divided into four equal parts.



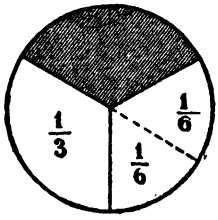
103. 1. Count the number of equal parts into which this rectangle is divided. How many eighths are shaded? What two names may be given to the shaded portion of the figure?

2. Complete: $\frac{2}{8} = \frac{}{4}$; $\frac{6}{8} = \frac{}{4}$.

3. Find the values:

$\frac{1}{8} + \frac{1}{4} =$; $\frac{3}{8} + \frac{1}{4} =$; $\frac{3}{8} + \frac{2}{4} + \frac{1}{8} =$.

104. 1. Into how many equal parts is the circle divided by the black lines? What do you think is the use of the dotted line?



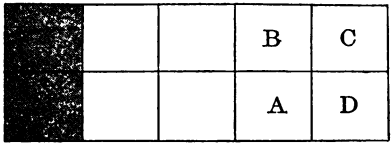
Then show how you would find one twelfth of the circle.

2. Complete: $\frac{1}{3} = \frac{}{6}$; $\frac{1}{3} = \frac{}{12}$.

3. Find the values:

$\frac{1}{6} + \frac{2}{3} =$; $\frac{1}{6} + \frac{1}{3} =$; $\frac{1}{12} + \frac{1}{6} + \frac{1}{3} =$.

105. 1. What part of this rectangle is dark? How many fifths are light? What is the name of each of the parts marked A, B, C, D? Why?



2. Add B to C, and their sum to D. Can you add B to the dark portion?

3. Complete: $\frac{1}{5} = \frac{}{10}$; $\frac{1}{2} = \frac{}{10}$.

4. A cent is one —— of a dime. A nickel is one —— of a dime, or is five —— of a dime.

A cent is one —— of a nickel.

106. 1. Draw a 10-inch square and divide it into square inches. Each small square is what part of the whole? How many *hundredths* in the upper row? Then how many hundredths make one tenth?

2. Find how many hundredths make $\frac{2}{10}$, or $\frac{1}{5}$. Find how many hundredths make $\frac{5}{10}$, or $\frac{1}{2}$. One tenth is how many times one hundredth?

3. Find how many hundredths in $\frac{1}{4}$ of the large square. In $\frac{3}{4}$ of it.

One hundredth is what part of one tenth?

4. Point out $\frac{1}{10}$ of one tenth of the large square, and tell what it is called. Point out $\frac{3}{10}$ of one tenth, and tell what it is called.

5. Complete:

$$\frac{1}{2} = \frac{2}{4} = \frac{\quad}{6} = \frac{\quad}{8} = \frac{\quad}{10} = \frac{\quad}{12} = \frac{\quad}{100}.$$

$$1 = \frac{2}{2} = \frac{\quad}{4} = \frac{\quad}{6} = \frac{\quad}{8} = \frac{\quad}{10} = \frac{\quad}{12} = \frac{\quad}{100}.$$

107. According to the number of parts into which the whole is divided, the fraction is named. You have seen in the preceding, that when a whole is broken into *two*, *four*, *eight*, or *ten* equal parts, these parts are called *halves*, *fourths*, *eighths*, *tenths*, respectively.

A fraction written as a number is composed of two figures, one above and the other below a short straight line.

For example, the fraction three fifths is written $\frac{3}{5}$.

Fractions so written are called **Common Fractions.**

The figure below the line is called the **Denominator**, or namer. It shows the number of equal parts into which some whole has been divided.

The figure above the line is called the **Numerator.** It shows the number of parts expressed.

Show by some object within easy reach that you divide a whole into five equal parts, and count out three of them to make three fifths.

108. The sum of an integer and a fraction is called a **Mixed Number.** Thus, $\$5\frac{1}{4}$ (the sum of $\$5$ and $\$\frac{1}{4}$) is a mixed number.

109. Read and write in words, naming numerator and denominator of each fraction:

1. $\frac{1}{4}$, $\frac{2}{3}$, $\frac{5}{6}$.
2. $\frac{4}{5}$, $\frac{8}{9}$, $\frac{5}{7}$.
3. $\frac{7}{9}$, $\frac{6}{7}$, $\frac{7}{11}$.
4. $\frac{12}{13}$, $\frac{9}{20}$, $\frac{7}{32}$.
5. $\frac{25}{64}$, $\frac{21}{25}$, $\frac{35}{48}$.
6. $\frac{13}{35}$, $\frac{11}{12}$, $\frac{1}{33}$.
7. $\frac{27}{80}$, $\frac{7}{24}$, $\frac{23}{48}$.
8. $2\frac{4}{5}$, $5\frac{7}{8}$, $8\frac{3}{16}$.

Write in figures:

9. Four fifths.
10. Two thirds.
11. Ten elevenths.
12. Eight twentieths.
13. Twelve twenty-thirds.
14. Two thirty-seconds.
15. Fifty-two hundredths.
16. Fifty two-hundredths.
17. Eleven eightieths.
18. Twenty-seven thirty-fifths.
19. Nineteen eighty-sevenths.
20. Twenty-seven fiftieths.

ORAL EXERCISES

110. 1. If James earns $\frac{1}{2}$ of a dollar a day, how many half dollars does he earn in 4 days? How many dollars?

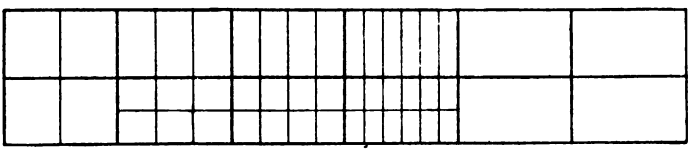
2. If $\frac{1}{2}$ of a yard of silk costs 26¢, what does $\frac{1}{4}$ of a yard cost? What is the cost of a whole yard?

3. How many envelopes at the rate of $\frac{1}{5}$¢ apiece can you buy for 5¢?

4. Marshall spent $\frac{1}{2}$ of a dollar one day and $\frac{3}{4}$ of a dollar the next. How much did he spend in the two days?

5. Each side of a square is $\frac{3}{4}$ yd. long. How many yards around it?

6. A box is $\frac{3}{4}$ yd. long and $\frac{1}{2}$ yd. wide. By how much is the length more than the width?



111. This figure is divided into twelve equal parts by the heavy lines. When each twelfth is further divided, the number of the parts is increased.

Both numerator and denominator may be multiplied by the same number without changing the value of the fraction.

For example, $\frac{1}{12} = \frac{2 \times 1}{2 \times 12} = \frac{2}{24}$.

112. 1. Multiply numerator and denominator of each of the following fractions by 2, 3, 4, 5, 6, successively:

$\frac{1}{12}$, $\frac{1}{3}$, $\frac{2}{5}$, $\frac{3}{4}$, $\frac{3}{10}$, $\frac{4}{7}$, $\frac{5}{12}$, $\frac{9}{16}$.

2. Change $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$ to 12ths.

Both numerator and denominator of $\frac{1}{2}$ must be multiplied by $12 \div 2$, or 6.

Both numerator and denominator of $\frac{2}{3}$ must be multiplied by $12 \div 3$, or 4.

By what must both numerator and denominator of $\frac{3}{4}$ be multiplied?

Change:

3. $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ to 12ths.

4. $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{1}{8}$ to 8ths.

5. $\frac{1}{2}$, $\frac{2}{5}$, and $\frac{3}{10}$ to 10ths.

6. $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{5}{6}$ to 12ths.

7. $\frac{1}{2}$, $\frac{3}{4}$, $\frac{4}{5}$, and $\frac{7}{10}$ to 20ths.

8. $\frac{1}{4}$, $\frac{3}{5}$, $\frac{5}{8}$, and $\frac{1}{10}$ to 40ths.

9. $\frac{3}{4}$, $\frac{1}{2}$, $\frac{2}{5}$, and $\frac{3}{10}$ to 100ths.

113. How many sixths in one third? How many twelfths in one third? How many tenths in one fifth?

Compare:

1. $\frac{2}{6}$ and $\frac{1}{3}$.
2. $\frac{4}{12}$ and $\frac{1}{3}$.
3. $\frac{2}{10}$ and $\frac{1}{5}$.
4. $\frac{6}{18}$ and $\frac{1}{3}$.
5. $\frac{4}{10}$ and $\frac{2}{5}$.
6. $\frac{12}{18}$ and $\frac{2}{3}$.

Both numerator and denominator may be divided by the same number without changing the value of the fraction.

For example, $\frac{4}{6}=\frac{4\div 2}{6\div 2}=\frac{2}{3}$.

114. Dividing both numerator and denominator of a fraction by the same number is called **canceling** the common factor.

115. To change to Smallest Denominators.

By Art. 114, we may cancel common factors from the numerator and denominator; when they have no factor in common remaining, the fraction is said to be expressed with its smallest denominator.

WRITTEN EXERCISES

116. 1. Change $\frac{30}{36}$ to its smallest denominator.

(*a*)

$$\frac{30}{36}=\frac{30\div 2}{36\div 2}=\frac{15}{18}.$$

$$\frac{15}{18}=\frac{15\div 3}{18\div 3}=\frac{5}{6}.$$

Divide the numerator and denominator by 2, then by 3.

Since no number will divide both numerator and denominator of $\frac{5}{6}$, the fraction is changed, or *reduced*, to its smallest denominator.

(*b*)

$$\frac{30}{36}=\frac{2\times 3\times 5}{2\times 3\times 6}=\frac{5}{6}.$$

In (*b*) the numerator and denominator are first resolved into prime factors; the common factors are then canceled.

Change to smallest denominators:

2. $\frac{45}{60}$, $\frac{30}{45}$, $\frac{45}{75}$.
3. $\frac{20}{25}$, $\frac{21}{24}$, $\frac{40}{50}$.
4. $\frac{63}{77}$, $\frac{45}{55}$, $\frac{44}{52}$.
5. $\frac{30}{45}$, $\frac{18}{60}$, $\frac{70}{95}$.
6. $\frac{36}{54}$, $\frac{28}{42}$, $\frac{49}{63}$.
7. $\frac{24}{32}$, $\frac{64}{96}$, $\frac{66}{99}$.
8. $\frac{48}{80}$, $\frac{45}{72}$, $\frac{96}{160}$.
9. $\frac{57}{93}$, $\frac{27}{57}$, $\frac{66}{78}$.
10. $\frac{28}{36}$, $\frac{54}{60}$, $\frac{56}{72}$.
11. $\frac{12}{144}$, $\frac{111}{123}$, $\frac{130}{195}$.
12. $\frac{96}{120}$, $\frac{36}{162}$, $\frac{48}{108}$.
13. $\frac{144}{156}$, $\frac{120}{192}$, $\frac{121}{143}$.

117. To change Integers and Mixed Numbers to Fractional Form, and the Reverse.

1. How many quarters (fourths) are there in a dollar? In 2 dollars? In $2\frac{1}{4}$ dollars?

2. How many halves are there in 1? In $2\frac{1}{2}$? In 4? In $4\frac{1}{2}$?

3. How many 1's are there in 4 fourths? In 8 fourths? In $\frac{16}{4}$? In $\frac{24}{4}$?

4. How many 1's are there in $\frac{15}{3}$? In $\frac{28}{4}$? In $\frac{30}{5}$?

WRITTEN EXERCISES

118. 1. Change $5\frac{2}{3}$ to thirds.

Since $1 = \frac{3}{3}$,

$5 = 5 \times \frac{3}{3} = \frac{15}{3}$;

$\frac{15}{3} + \frac{2}{3} = \frac{17}{3}$.

There are 3 thirds in 1. How many thirds in 5? Why add $\frac{2}{3}$ to $\frac{15}{3}$?

2. Change $\frac{20}{3}$ to a mixed number.

$20 \div 3 = 6\frac{2}{3}$.

There are 3 thirds in 1. How many 1's are there in 20 thirds? How many thirds are left over?

Change to fractional form:

3. $5\frac{3}{8}$.
4. $15\frac{3}{4}$.
5. $16\frac{2}{3}$.
6. $12\frac{1}{2}$.
7. $13\frac{5}{8}$.
8. $6\frac{2}{3}$.
9. $33\frac{1}{3}$.
10. $25\frac{3}{20}$.
11. $38\frac{2}{9}$.
12. $24\frac{5}{7}$.
13. $3\frac{5}{6}$.
14. $66\frac{2}{3}$.
15. $34\frac{17}{21}$.
16. $125\frac{3}{40}$.
17. $666\frac{2}{3}$.

Change to integers or mixed numbers:

18. $\frac{29}{3}$. **21.** $\frac{67}{8}$. **24.** $\frac{316}{16}$. **27.** $\frac{881}{42}$. **30.** $\frac{124}{5}$.

19. $\frac{63}{25}$. **22.** $\frac{87}{11}$. **25.** $\frac{168}{33}$. **28.** $\frac{179}{16}$. **31.** $\frac{825}{33}$.

20. $\frac{79}{5}$. **23.** $\frac{196}{7}$. **26.** $\frac{625}{25}$. **29.** $\frac{280}{8}$. **32.** $\frac{432}{26}$.

119. To change to Least Common Denominator.

1. Have $\frac{1}{2}$ and $\frac{3}{8}$ the same denominator? Express $\frac{1}{2}$ as eighths.

$$\frac{1}{2} + \frac{3}{8} = \frac{4}{8} + \frac{3}{8} =$$

2. In adding $\frac{1}{2}$ and $\frac{3}{8}$, we first change $\frac{1}{2}$ to a fraction with what denominator?

120. Fractions that have the same denominators are said to be *similar fractions* and have a *common denominator.*

121. When the common denominator is the smallest possible, the fractions are said to have the **Least Common Denominator** (L. C. D.).

WRITTEN EXERCISES

122. 1. Change $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$ to fractions having the least common denominator.

$$\frac{1}{2} = \frac{1 \times 6}{2 \times 6} = \frac{6}{12}.$$

$$\frac{2}{3} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12}.$$

$$\frac{3}{4} = \frac{3 \times 3}{4 \times 3} = \frac{9}{12}.$$

Since the required denominator must contain 2, 3, and 4, it must be their least common multiple. The L. C. M. is 12 (Art. 81).

Since $12 \div 2 = 6$, the numerator and denominator of the first fraction must be multiplied by 6. How are the other multipliers found?

Change to similar fractions with their L. C. D.:

2. $\frac{3}{4}$, $\frac{5}{8}$, $\frac{7}{9}$. **5.** $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{7}$. **8.** $\frac{7}{8}$, $\frac{3}{16}$, $\frac{5}{32}$. **11.** $\frac{3}{20}$, $\frac{3}{10}$, $\frac{4}{15}$.

3. $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$. **6.** $\frac{1}{4}$, $\frac{3}{8}$, $\frac{7}{12}$. **9.** $\frac{11}{15}$, $\frac{5}{10}$, $\frac{4}{5}$. **12.** $\frac{7}{12}$, $\frac{8}{9}$, $\frac{5}{8}$, $\frac{1}{6}$.

4. $\frac{2}{9}$, $\frac{2}{3}$, $\frac{5}{6}$. **7.** $\frac{3}{4}$, $\frac{1}{6}$, $\frac{4}{15}$. **10.** $\frac{3}{7}$, $\frac{4}{15}$, $\frac{7}{10}$. **13.** $\frac{3}{5}$, $\frac{7}{8}$, $\frac{11}{20}$, $\frac{2}{15}$.

ADDITION AND SUBTRACTION

123. 1. What is the sum of a quarter and a half dollar? The half dollar = 2 quarters. 2 quarters + 1 quarter = 3 quarters.

What is the sum of $\frac{1}{2}$ bu. and a peck? (1 pk. = $\frac{1}{4}$ bu.)

2. Add $\frac{1}{2}$ gal. and $\frac{1}{4}$ gal. $\frac{1}{2} + \frac{1}{4} = \frac{\quad}{4} + \frac{1}{4} = \frac{\quad}{4}$.

Why can you not add halves and fourths without changing the halves to fourths?

3. Use your foot rule to find the sum of $\frac{1}{4}$ in. and $\frac{1}{8}$ in. To find the sum of $\frac{1}{2}$ in. and $\frac{1}{8}$ in.

4. Complete: $\frac{1}{4} + \frac{1}{8} = \frac{\quad}{8}$. $\frac{1}{2} + \frac{1}{8} = \frac{\quad}{8}$.

What must be done with $\frac{1}{2}$ and $\frac{1}{4}$ before they can be added to $\frac{1}{8}$?

5. Find the difference between $\frac{1}{2}$ in. and $\frac{1}{4}$ in.

$\frac{1}{2}$ in. $- \frac{1}{4}$ in. $= \frac{\quad}{4}$ in. $- \frac{1}{4}$ in. $= \frac{\quad}{4}$ in.

Fractions must have the same denominator to be added or subtracted.

124. Change fractions to the same denominator, and add or subtract as indicated:

1. $\frac{2}{3} + \frac{1}{6} = \frac{\quad}{6}$.	**9.** $\frac{5}{6} + \frac{7}{12} = \frac{\quad}{12}$.	**17.** $\frac{2}{3} - \frac{1}{6} = \frac{\quad}{6}$.
2. $\frac{2}{3} + \frac{5}{6} = \frac{\quad}{6}$.	**10.** $\frac{1}{3} + \frac{2}{5} = \frac{\quad}{15}$.	**18.** $\frac{5}{6} - \frac{2}{3} = \frac{\quad}{6}$.
3. $\frac{1}{3} + \frac{1}{6} = \frac{\quad}{6}$.	**11.** $\frac{2}{3} + \frac{3}{5} = \frac{\quad}{15}$.	**19.** $\frac{1}{2} - \frac{1}{3} = \frac{\quad}{6}$.
4. $\frac{1}{2} + \frac{3}{4} = \frac{\quad}{8}$.	**12.** $\frac{2}{3} + \frac{1}{15} = \frac{\quad}{15}$.	**20.** $\frac{1}{2} - \frac{1}{5} = \frac{\quad}{10}$.
5. $\frac{1}{4} + \frac{1}{8} = \frac{\quad}{8}$.	**13.** $\frac{3}{10} + \frac{4}{5} = \frac{\quad}{20}$.	**21.** $\frac{2}{5} - \frac{1}{10} = \frac{\quad}{10}$.
6. $\frac{3}{4} + \frac{7}{8} = \frac{\quad}{8}$.	**14.** $\frac{2}{5} + \frac{3}{4} = \frac{\quad}{20}$.	**22.** $\frac{1}{6} - \frac{1}{12} = \frac{\quad}{12}$.
7. $\frac{1}{6} + \frac{1}{12} = \frac{\quad}{12}$.	**15.** $\frac{7}{10} + \frac{3}{20} = \frac{\quad}{20}$.	**23.** $\frac{7}{12} - \frac{1}{2} = \frac{\quad}{12}$.
8. $\frac{1}{6} + \frac{5}{12} = \frac{\quad}{12}$.	**16.** $\frac{1}{3} + \frac{1}{8} = \frac{\quad}{24}$.	**24.** $\frac{1}{3} - \frac{1}{12} = \frac{\quad}{12}$.

25. Add a nickel and a dime; a dime and a quarter.

Observe that the process of changing to the same kind must be done before adding.

WRITTEN EXERCISES

125. 1. Find the sum of $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{5}{6}$.

$\frac{2}{3} = \frac{8}{12}$. $\frac{3}{4} = \frac{9}{12}$. $\frac{5}{6} = \frac{10}{12}$.

$\frac{8}{12} + \frac{9}{12} + \frac{10}{12} = \frac{27}{12} = 2\frac{3}{12} = 2\frac{1}{4}$.

Changing the fractions to similar ones having their least common denominator, we have 8 twelfths, 9 twelfths, and 10 twelfths; the sum of these is $\frac{27}{12}$, or $2\frac{1}{4}$.

After fractions are added or subtracted, the results should be simplified if possible.

2. Subtract $\frac{2}{3}$ from $\frac{6}{7}$.

$\frac{2}{3} = \frac{14}{21}$. $\frac{6}{7} = \frac{18}{21}$.

$\frac{18}{21} - \frac{14}{21} = \frac{4}{21}$.

Changing to similar fractions, we have 14 twenty-firsts and 18 twenty-firsts, whose difference is easily found.

3. Subtract $4\frac{5}{7}$ from $7\frac{2}{3}$.

(a) (b)

$7\frac{2}{3} = 7\frac{14}{21} = 6\frac{35}{21}$.

$4\frac{5}{7} = 4\frac{15}{21} = 4\frac{15}{21}$.

$6\frac{35}{21} - 4\frac{15}{21} = 2\frac{20}{21}$.

The fractions are made similar, as in (a). Since $\frac{15}{21}$ is greater than $\frac{14}{21}$, we take 1, or $\frac{21}{21}$, from 7, and add it to $\frac{14}{21}$, as in (b). Then $\frac{15}{21}$ from $\frac{35}{21}$ leaves $\frac{20}{21}$, and 4 from 6 leaves 2.

In practice, the work under (b) should be done mentally. The mixed numbers may be changed to fractional form, and then subtracted.

Find the values:

4. $\frac{1}{2} + \frac{5}{6} =$

5. $\frac{2}{5} + \frac{3}{10} =$

6. $\frac{3}{5} + \frac{1}{6} =$

7. $\frac{1}{2} + \frac{4}{5} =$

8. $\frac{1}{10} + \frac{2}{3} =$

9. $\frac{1}{5} + \frac{1}{10} =$

10. $\frac{3}{4} + \frac{2}{3} =$

11. $\frac{2}{5} + \frac{1}{4} =$

12. $\frac{1}{3} + \frac{3}{4} =$

13. $\frac{3}{10} + \frac{1}{4} =$

14. $\frac{2}{5} - \frac{3}{10} =$

15. $\frac{2}{3} - \frac{2}{5} =$

16. $\frac{1}{2} - \frac{2}{7} =$

17. $\frac{3}{4} - \frac{1}{6} =$

18. $\frac{6}{7} - \frac{2}{3} =$

Add the integers and fractions separately, and then add the results:

19. $4\frac{1}{2} + \frac{1}{4} =$

20. $3\frac{1}{2} + 5\frac{7}{8} =$

21. $6\frac{2}{5} + 3\frac{1}{4} =$

22. $3\frac{2}{3} + 4\frac{5}{6} =$

23. $9\frac{1}{2} + 1\frac{7}{8} =$

24. $7\frac{1}{2} + 2\frac{5}{8} + 4\frac{5}{6} =$

25. $8\frac{2}{3} + 3\frac{5}{6} + 4\frac{7}{12} =$

26. $8\frac{1}{4} + 7\frac{3}{10} + 12\frac{5}{6} =$

27. $2\frac{3}{5} + 4\frac{7}{20} + 14\frac{3}{10} =$

28. $10\frac{3}{4} + 9\frac{7}{8} + 16\frac{5}{24} =$

Subtract:

29. $6\frac{1}{4}$ from $11\frac{1}{2}$.

30. $3\frac{2}{3}$ from $6\frac{1}{4}$.

31. $5\frac{3}{5}$ from $8\frac{1}{3}$.

32. $4\frac{1}{3}$ from $7\frac{3}{5}$.

33. $5\frac{3}{5}$ from $9\frac{4}{5}$.

34. $12\frac{1}{8}$ from $16\frac{3}{4}$.

35. $15\frac{1}{3}$ from $30\frac{3}{4}$.

36. $31\frac{1}{8}$ from $42\frac{1}{4}$.

37. $13\frac{5}{6}$ from $28\frac{5}{12}$.

38. $37\frac{1}{2}$ from $75\frac{1}{12}$.

Add or subtract as indicated:

39. $7\frac{5}{8} + 3\frac{2}{5} =$

40. $13\frac{3}{20} - 4\frac{1}{4} =$

41. $12\frac{1}{5} + 3\frac{14}{15} =$

42. $252\frac{1}{3} + 125\frac{5}{9} =$

43. $200\frac{4}{7} - 78\frac{13}{14} =$

44. $126\frac{2}{9} - 109\frac{5}{27} =$

Find the values:

45. $\frac{3}{8} + \frac{5}{12} + \frac{7}{16} =$

46. $\frac{5}{6} + \frac{3}{8} + \frac{1}{12} =$

47. $\frac{3}{5} + \frac{5}{9} + \frac{7}{12} =$

48. $\frac{6}{7} + \frac{3}{5} + \frac{9}{35} =$

49. $\frac{1}{6} + \frac{2}{3} - \frac{1}{2} =$

50. $\frac{6}{7} + \frac{1}{3} + \frac{5}{21} =$

51. $2\frac{1}{3} + 3\frac{4}{7} - 5\frac{11}{21} =$

52. $3\frac{3}{4} + 6\frac{2}{3} - 3\frac{7}{10} =$

53. One week James earned $\$3\frac{2}{5}$, Homer $\$4\frac{7}{10}$, and David $\$12\frac{3}{4}$. How much did all earn?

54. If Jack has a stick $15\frac{1}{8}$ in. long, how much must he saw off to make it $12\frac{1}{2}$ in. long?

55. An automobile goes $8\frac{1}{3}$ mi. the first hour, $5\frac{3}{8}$ mi. farther the second hour than the first, and as far the third hour as in both the others. How far does it go the third hour?

56. Frank is $16\frac{2}{3}$ years old, Martha $2\frac{1}{2}$ years older, and Fred is $1\frac{1}{4}$ years younger than Martha. How old is Fred?

57. How many rods of fence will a farmer need to fence a rectangular field whose length is $180\frac{5}{6}$ rods and whose width is $27\frac{3}{8}$ rods?

58. John bought a book for $\$\frac{4}{5}$, a knife for $\$\frac{3}{4}$, and a tennis racket for $\$1\frac{3}{5}$. How much change did he receive if he offered a five-dollar bill in payment?

59. A farmer sold $114\frac{3}{4}$ bu. of wheat, which was $45\frac{1}{2}$ bu. less than the number of bushels left on hand. How much wheat had he before the sale?

MULTIPLICATION

126. Note carefully:

3×1 man $= 3$ men.	$3 \times 1 = 3$.
3×2 boys $= 6$ boys.	$3 \times 2 = 6$.
3×3 girls $= 9$ girls.	$3 \times 3 = 9$.
3×4 cents $= ?$	$3 \times 4 = ?$
3×1 half $= 3$ halves.	3×3 fourths $= 9$ fourths.
3×2 thirds $= 6$ thirds.	3×4 fifths $= ?$

Writing the last four expressions as numbers only, we have —

$3 \times \frac{1}{2} = \frac{3}{2}$, *or* $\frac{1}{2} \times 3 = \frac{3}{2}$.	$3 \times \frac{3}{4} = \frac{9}{4}$, *or* $\frac{3}{4} \times 3 = \frac{9}{4}$.
$3 \times \frac{2}{3} = \frac{6}{3}$, *or* $\frac{2}{3} \times 3 = \frac{6}{3}$.	$3 \times \frac{4}{5} = ?$ *or* $\frac{4}{5} \times 3 = ?$

Hence, to multiply fractions and whole numbers, *multiply the numerator by the given whole number.*

The expression $\frac{1}{2} \times 3$ may be read or written $\frac{1}{2}$ of 3; that is, in such expressions the sign $\times$ and the word "of" are interchangeable.

Find the products:

1. $7 \times \frac{5}{6}$.
2. $9 \times \frac{7}{8}$.
3. $\frac{8}{9} \times 10$.
4. $\frac{10}{13} \times 4$.
5. $5 \times \frac{12}{13}$.
6. $\frac{14}{15} \times 7$.
7. $\frac{9}{10} \times 11$.
8. $9 \times \frac{15}{16}$.
9. $13 \times \frac{3}{20}$.

127. After fractions are multiplied, the pupil should simplify the results if possible.

For example, $8 \times \frac{5}{16} = \frac{40}{16}$.

Dividing both numerator and denominator by 8, $\frac{40 \div 8}{16 \div 8} = \frac{5}{2} = 2\frac{1}{2}$.

The same result is conveniently accomplished by indicating the multiplication and canceling common factors.

For example, $8 \times \frac{5}{16} = \frac{\overset{1}{\cancel{8}} \times 5}{\underset{2}{\cancel{16}}} = \frac{5}{2}$, or $2\frac{1}{2}$.

Find the products:

1. $14 \times \frac{8}{35} =$
2. $10 \times \frac{7}{15} =$
3. $8 \times \frac{5}{12} =$
4. $7 \times \frac{3}{14} =$
5. $\frac{20}{27} \times 9 =$
6. $\frac{15}{26} \times 13 =$
7. $18 \times \frac{16}{17} =$
8. $\frac{17}{25} \times 15 =$
9. $\frac{19}{28} \times 14 =$
10. $\frac{27}{64} \times 14 =$
11. $18 \times \frac{7}{9} =$
12. $3 \times \frac{16}{21} =$

128. Any whole number may be written in fractional form.

For example, $2 = \frac{2}{1}$; $5 = \frac{5}{1}$; etc.

Compare with the multiplications in Art. 126:

$\frac{1}{2} \times \frac{3}{1} = \frac{3}{2}$, *or* $\frac{3}{1} \times \frac{1}{2} = \frac{3}{2}$.

$\frac{2}{3} \times \frac{3}{1} = \frac{6}{3}$, *or* $\frac{3}{1} \times \frac{2}{3} = \frac{6}{3}$.

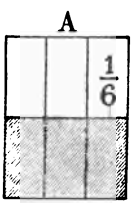
$\frac{3}{4} \times \frac{3}{1} = \frac{9}{4}$, *or* $\frac{3}{1} \times \frac{3}{4} = \frac{9}{4}$.

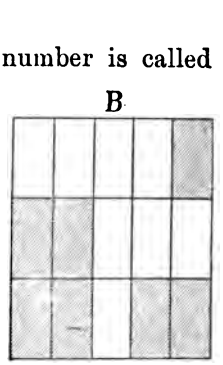
$\frac{4}{5} \times \frac{3}{1} = \frac{12}{5}$, *or* $\frac{3}{1} \times \frac{4}{5} = \frac{12}{5}$.

How do you obtain the new numerators and new denominators of the product?

129. To take a fractional part of a number is called *multiplying by a fraction.*

1. If A is divided by vertical lines into thirds, and by a horizontal line into halves, how many parts are there? What is each part called? $\frac{1}{2}$ of $\frac{1}{3} = \frac{1}{6}$.





2. If B is divided by vertical lines into fifths, and by horizontal lines into thirds, how many parts are there?

What is each part called? $\frac{1}{3}$ of $\frac{1}{5} = \frac{1}{15}$.

Point out $\frac{1}{15}$ (*i.e.*, $\frac{1}{3}$ of $\frac{1}{5}$) in the figure. Point out $\frac{2}{15}$ ($\frac{1}{3}$ of $\frac{2}{5}$). $\frac{1}{3}$ of $\frac{2}{5} = \frac{2}{15}$.

3. Since $\frac{1}{3}$ of $\frac{2}{5}$ is $\frac{2}{15}$, $\frac{2}{3}$ of $\frac{2}{5}$ is twice as much, or $\frac{2}{3} \times \frac{2}{5} = \frac{4}{15}$. Point out $\frac{4}{15}$.

4. In finding that $\frac{2}{3}$ of $\frac{2}{5}$ equals $\frac{4}{15}$, how do you obtain 4 (the numerator)? How do you obtain 15 (the denominator)?

130. To multiply one fraction by another, *take the product of the numerators for the new numerator, and of the denominators for the new denominator.*

Before multiplying, it is better to indicate the operation and cancel if possible.

For example, $\frac{2}{3} \times \frac{3}{4} = \frac{2 \times \cancel{3}}{\cancel{3} \times 4} = \frac{1}{2}$.
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad 2$

WRITTEN EXERCISES

131. Find the products:

1. $\frac{3}{4} \times \frac{8}{9}$.
2. $\frac{3}{5} \times \frac{5}{8}$.
3. $\frac{3}{4} \times \frac{5}{6}$.
4. $\frac{5}{8} \times \frac{2}{5}$.
5. $\frac{3}{4} \times \frac{5}{12}$.
6. $\frac{2}{3} \times \frac{5}{16}$.

7. $\frac{2}{3} \times \frac{3}{7}$.
8. $\frac{2}{5} \times \frac{3}{4}$.
9. $\frac{3}{5} \times \frac{2}{3}$.
10. $\frac{5}{8} \times \frac{3}{16}$.
11. $\frac{5}{6} \times \frac{3}{8}$.
12. $\frac{3}{5} \times \frac{7}{10}$.
13. $\frac{5}{6} \times \frac{9}{10}$.
14. $\frac{5}{6} \times \frac{4}{5}$.
15. $\frac{2}{5} \times \frac{7}{8}$.
16. $\frac{5}{6} \times \frac{3}{10}$.
17. $\frac{8}{15} \times \frac{5}{64}$.
18. $\frac{8}{9} \times \frac{15}{16}$.

19. Multiply $15\frac{3}{4}$ by 8.

$$\begin{array}{r} 15\frac{3}{4} \\ 8 \\ \hline 120 = 8 \times 15 \\ 6 = 8 \times \frac{3}{4} \\ \hline 126 = 8 \times 15\frac{3}{4} \end{array}$$

Explain this process.

Find the products:

20. $6 \times 7\frac{1}{3}$.
21. $8 \times 2\frac{1}{4}$.
22. $7\frac{3}{4} \times 52$.
23. $9 \times 3\frac{3}{5}$.
24. $9\frac{5}{6} \times 9$.
25. $72 \times 6\frac{2}{3}$.
26. $40 \times 5\frac{5}{8}$.
27. $3\frac{5}{6} \times 84$.
28. $65 \times 4\frac{4}{5}$.
29. $7\frac{1}{6} \times 90$.
30. $14 \times 12\frac{1}{2}$.
31. $87\frac{1}{2} \times 16$.
32. $24 \times 16\frac{2}{3}$.
33. $33\frac{1}{3} \times 3$.
34. $18 \times 62\frac{1}{2}$.

132. To multiply two mixed numbers, *change both mixed numbers to fractional form; then multiply as shown in Art.* 130.

WRITTEN EXERCISES

133. 1. Multiply $3\frac{2}{5} \times 5\frac{5}{8}$.

$$3\frac{2}{5} \times 5\frac{5}{8} = \frac{17}{\cancel{5}} \times \frac{\overset{9}{\cancel{45}}}{8} = \frac{153}{8}, \text{ or } 19\tfrac{1}{8}.$$

Multiply:

2. $7\frac{1}{8}$ by $2\frac{3}{10}$.
3. $8\frac{4}{5}$ by $6\frac{7}{8}$.
4. $7\frac{2}{5}$ by $17\frac{4}{7}$.
5. $9\frac{3}{8} \times 14\frac{2}{5}$.
6. $10\frac{5}{16} \times 11\frac{3}{5}$.
7. $11\frac{2}{24} \times 142\frac{3}{8}$.

Find the values:

8. $\frac{3}{5} \times \frac{5}{6}$.
9. $\frac{3}{4} \times \frac{4}{9}$.
10. $\frac{2}{3} \times \frac{5}{8}$.
11. $\frac{7}{12} \times \frac{3}{20}$.
12. $\frac{3}{5} \times \frac{8}{9}$.
13. $\frac{11}{60} \times \frac{15}{22}$.
14. $12\frac{3}{4} \times \frac{9}{10}$.
15. $11\frac{1}{12} \times \frac{4}{11}$.
16. $6\frac{7}{8} \times 32$.
17. $10\frac{8}{11} \times 33$.
18. $14\frac{3}{4} \times 16\frac{2}{3}$.
19. $12\frac{5}{8} \times 11\frac{2}{5}$.
20. $\frac{5}{9} \times \frac{3}{5} \times \frac{1}{5}$.
21. $\frac{4}{5} \times 11\frac{1}{12} \times \frac{5}{6}$.
22. $\frac{2}{3} \times \frac{4}{5} \times \frac{7}{12}$.
23. $\frac{3}{8} \times 7\frac{1}{2} \times 11\frac{1}{12}$.
24. $3\frac{1}{2} \times 2\frac{1}{2} \times \frac{2}{5}$.
25. $90 \times 3\frac{1}{4} \times \frac{5}{6}$.

26. What is the weight of $2\frac{3}{4}$ bushels of corn, allowing 56 pounds to a bushel?

27. A farmer bought $56\frac{3}{4}$ acres of land at $\$27\frac{1}{2}$ an acre, and sold it at $\$32\frac{1}{4}$ an acre. How much did he gain?

28. What is the cost of 6 yd. of silk at $\$1.87\frac{1}{2}$ a yard?

29. Find the cost of 9 yd. of lace at $\$1.16\frac{2}{3}$ a yard.

30. Find the cost of 2 dozen pocketknives at $66\frac{2}{3}¢$ each.

31. What must I pay for 25 cakes of soap at the rate of $5\frac{1}{2}¢$ a cake? (In results $\frac{1}{2}¢$ or over is counted as a cent; less than $\frac{1}{2}¢$ is disregarded.)

32. What must I pay for 5 handkerchiefs at $16\frac{2}{3}¢$ each?

33. At $\$2\frac{3}{4}$ a yard, how much will $2\frac{1}{2}$ yd. of cloth cost?

34. What is the price of a mileage book (1000 miles), at $2\frac{1}{2}¢$ a mile?

35. If the freight rate is $1\frac{9}{10}¢$ a ton per mile, what will it cost to ship 10 tons of fertilizer 100 miles?

36. If you live $\frac{3}{4}$ of a mile from school, and I live $\frac{1}{3}$ as far in the opposite direction, how far apart do we live?

37. A woman bought from her grocer 10 lb. coffee at $27\frac{1}{2}¢$ a pound, 20 lb. sugar at $5\frac{1}{2}¢$ a pound, and 4 lb. butter at $18\frac{3}{4}¢$ a pound. How much was the bill?

DIVISION

134. Note carefully:

$\frac{1}{3}$ of 3 men = 1 man.	$\frac{1}{3}$ of 3 = 1.
$\frac{1}{3}$ of 6 boys = 2 boys.	$\frac{1}{3}$ of 6 = 2.
$\frac{1}{3}$ of 9 girls = 3 girls.	$\frac{1}{3}$ of 9 = 3.
$\frac{1}{3}$ of 12 cents = ?	$\frac{1}{3}$ of 12 = ?
3 halves ÷ 3 = 1 half.	$\frac{3}{2} \div 3 = \frac{1}{2}$.
6 thirds ÷ 3 = 2 thirds.	$\frac{6}{3} \div 3 = \frac{2}{3}$.
9 fourths ÷ 3 = 3 fourths.	$\frac{9}{4} \div 3 = \frac{3}{4}$.
12 fifths ÷ 3 = ?	$\frac{12}{5} \div 3 = ?$

One way of dividing a fraction by a whole number is *to divide the numerator by the whole number.*

Find the quotients:

1. $\frac{75}{3} \div 25$.	**3.** $\frac{18}{9} \div 2$.	**5.** $\frac{35}{82} \div 7$.	**7.** $\frac{42}{10} \div 6$
2. $\frac{75}{64} \div 3$.	**4.** $\frac{25}{7} \div 5$.	**6.** $\frac{27}{16} \div 9$.	**8.** $\frac{49}{24} \div 7$

135. As before stated, whole numbers may be put in fractional form; that is, $25 = \frac{25}{1}$; $3 = \frac{3}{1}$; $7 = \frac{7}{1}$.

Writing the exercises of the last article by use of this form, we have:

$$\frac{3}{2} \div \frac{3}{1} = \frac{1}{2}, \quad or \quad \frac{3}{2} \times \frac{1}{3} = \frac{3}{6} = \frac{1}{2}.$$

$$\frac{6}{3} \div \frac{3}{1} = \frac{2}{3}, \quad or \quad \frac{6}{3} \times \frac{1}{3} = \frac{6}{9} = \frac{2}{3}.$$

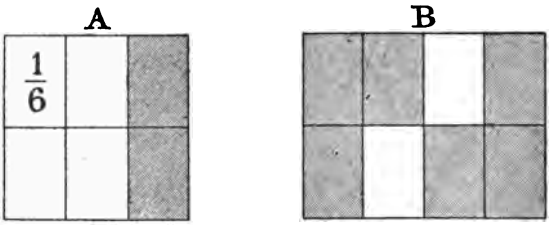
$$\frac{9}{4} \div \frac{3}{1} = \frac{3}{4}, \quad or \quad \frac{9}{4} \times \frac{1}{3} = \frac{9}{12} = \frac{3}{4}.$$

$$\frac{12}{5} \div \frac{3}{1} = —, \quad or \quad \frac{12}{5} \times \frac{1}{3} = —.$$

To divide a fraction by a fraction, *multiply the inverted divisor and the dividend.*

Shorten the work by canceling common factors when practicable.

136. The diagrams may be used to illustrate this process:



1. Tell by inspection:

How many sixths in A? $1 \div \frac{1}{6} = 6$, *i.e.*, $1 \times \frac{6}{1} = 6$.

How many sixths in $\frac{1}{3}$ of A? $\frac{1}{3} \div \frac{1}{6} = 2$, *i.e.*, $\frac{1}{3} \times \frac{6}{1} = 2$.

How many sixths in $\frac{2}{3}$ of A? $\frac{2}{3} \div \frac{1}{6} = 4$, *i.e.*, $\frac{2}{3} \times \frac{6}{1} = \frac{12}{3} = 4$.

How many $\frac{2}{6}$'s in 1? $1 \div \frac{2}{6} = 3$, *i.e.*, $1 \times \frac{6}{2} = \frac{6}{2} = 3$.

How many $\frac{2}{6}$'s in $\frac{1}{3}$? $\frac{1}{3} \div \frac{2}{6} = 1$, *i.e.*, $\frac{1}{3} \times \frac{6}{2} = \frac{6}{6} = 1$.

2. From B, tell by inspection:

How many $\frac{1}{8}$'s in 1? $1 \div \frac{1}{8} = 8$, *i.e.*, $1 \times \frac{8}{1} = 8$.

How many $\frac{2}{8}$'s in 1? $1 \div \frac{2}{8} = 4$, *i.e.*, $1 \times \frac{8}{2} = \frac{8}{2} = 4$.

How many $\frac{3}{8}$'s in 1? $1 \div \frac{3}{8} = 2\frac{2}{3}$, *i.e.*, $1 \times \frac{8}{3} = \frac{8}{3} = 2\frac{2}{3}$.

What part of $\frac{3}{8}$ in $\frac{1}{4}$? $\frac{1}{4} \div \frac{3}{8} = \frac{2}{3}$, *i.e.*, $\frac{1}{4} \times \frac{8}{3} = \frac{8}{12} = \frac{2}{3}$.

3. Instead of dividing 1 by $\frac{1}{6}$, by what number may you multiply 1 and get the same result? Instead of dividing $\frac{1}{8}$ by $\frac{1}{6}$, by what number may you multiply $\frac{1}{8}$ and get the same result?

WRITTEN EXERCISES

137. **1.** Divide 4 by $\frac{2}{3}$.

$$4 \div \frac{2}{3} = \frac{4}{1} \times \frac{3}{2} = \frac{\overset{2}{\cancel{4}} \times 3}{\underset{1}{\cancel{2}}} = 6.$$

Divide:

2. 4 by $\frac{1}{8}$.
3. 7 by $\frac{1}{4}$.
4. 5 by $\frac{1}{3}$.
5. 8 by $\frac{2}{5}$.
6. 3 by $\frac{3}{8}$.
7. 4 by $\frac{5}{12}$.
8. 5 by $\frac{8}{10}$.
9. 4 by $\frac{7}{16}$.
10. 2 by $\frac{9}{24}$.

11. Divide $\frac{2}{3}$ by $\frac{3}{5}$.

Inverted divisor $\frac{5}{3}$ = number of $\frac{3}{5}$'s in 1.
$\frac{2}{3} \times \frac{5}{3} = \frac{10}{9}$, or $1\frac{1}{9}$, number of $\frac{3}{5}$'s in $\frac{2}{3}$.

Find the quotients:

12. $\frac{1}{3} \div \frac{1}{2}$.
13. $\frac{3}{4} \div \frac{1}{2}$.
14. $\frac{1}{2} \div \frac{1}{8}$.
15. $\frac{1}{2} \div \frac{1}{12}$.
16. $\frac{1}{2} \div \frac{2}{3}$.
17. $\frac{2}{3} \div \frac{1}{4}$.
18. $\frac{3}{8} \div \frac{3}{4}$.
19. $\frac{5}{12} \div \frac{2}{3}$.
20. $\frac{5}{12} \div \frac{1}{2}$.
21. $\frac{5}{6} \div \frac{2}{3}$.
22. $\frac{3}{20} \div \frac{3}{10}$.
23. $\frac{7}{12} \div \frac{7}{8}$.
24. $\frac{9}{24} \div \frac{3}{4}$.
25. $\frac{13}{24} \div \frac{4}{13}$.
26. $\frac{17}{48} \div \frac{11}{12}$.
27. $18 \div \frac{9}{32}$.
28. $3\frac{2}{3} \div \frac{5}{8}$.
29. $4\frac{5}{6} \div \frac{2}{3}$.
30. $3\frac{9}{10} \div 13$.
31. $\frac{3}{4} \div 5\frac{1}{2}$.
32. $2\frac{1}{24} \div 2\frac{1}{3}$.

33. If a mechanic earns $\$2\frac{3}{4}$ a day, how long will it take him to earn \$33?

34. If $3\frac{1}{2}$ doz. hats cost $\$73\frac{1}{2}$, how much does each cost?

35. A drover paid $\$4\frac{3}{4}$ a head for sheep. How many did he buy for \$285?

36. If $6\frac{4}{5}$ tons of cotton seed cost $\$81\frac{3}{5}$, what is the cost a ton?

37. If a motor car runs $78\frac{3}{4}$ miles in 5 hours, what is its average rate an hour?

38. A dealer bought 24 bu. of peaches for \$15, and sold them for $\$17\frac{2}{5}$. How much did he gain a bushel?

39. A man paid out $\$49\frac{1}{2}$ for oats at $\$\frac{2}{5}$ a bushel. How many bushels did he buy?

REVIEW WORK

ORAL EXERCISES

138. 1. In measuring a line with a foot rule you express the measure in what? What is the *unit of measure?*

2. If you use a half bushel in measuring wheat, what is the *measuring unit?* How many such units in a bushel? The measuring unit is what part of a bushel?

3. In the fraction $\frac{3}{4}$ what does the 4 show? What does the 3 show?

4. State as integers: $\frac{8}{4}$, $\frac{15}{3}$, $\frac{25}{5}$, $\frac{81}{9}$.

5. State as mixed numbers: $\frac{9}{4}$, $\frac{15}{7}$, $\frac{20}{3}$, $\frac{38}{9}$.

6. What is the sum of $\frac{3}{4}$ and $\frac{1}{8}$? In what common unit do we express these fractions before adding them?

7. A dairyman sold $\frac{1}{2}$ gal. of milk to one customer, and $\frac{3}{4}$ gal. to another. How much did he sell to both?

8. James traveled $\frac{1}{3}$ of a mile in one direction and returned along the same route $\frac{1}{5}$ of a mile. How far was he from the starting point?

9. How much will $2\frac{1}{2}$ lb. of sugar cost at 6¢ a pound?

10. How many strips of carpet $\frac{3}{4}$ of a yard wide will be required to cover a room 6 yd. wide?

11. If $\frac{1}{8}$ lb. of tea cost 8¢, how many pounds can you buy for $1.28?

12. I bought $\frac{3}{5}$ of a bushel of apples, and gave away $\frac{1}{4}$ of them. What part of a bushel had I left?

13. At $\frac{1}{5}$ of a dollar a peck, how many pecks of sweet potatoes can be bought for \$1? For \$5?

14. If a boy earns $\$\frac{3}{5}$ a day, in how many days will he earn \$3?

15. At $\frac{1}{8}$ of a dollar a pound, how many pounds of coffee can I buy for \$1? For \$2?

16. James sold 2 bushels of chestnuts at the rate of 25¢ for $\frac{1}{8}$ of a bushel. How much did he get for them?

17. If $\frac{1}{2}$ lb. of bacon costs 5¢, what will 5 lb. cost at the same rate?

18. Ralph lives $1\frac{5}{8}$ mi. south of the schoolhouse, and Keller lives $1\frac{1}{4}$ mi. north of it. How far apart do the boys live?

19. From a piece of ribbon $\frac{3}{4}$ yd. long, $\frac{3}{12}$ of a yard were cut off. What was the length of the remainder?

20. If I own a lot containing $1\frac{2}{3}$ acres, and sell $\frac{1}{5}$ of it, what part of an acre do I sell?

21. How many coats can be cut from 35 yards of cloth, allowing $3\frac{1}{2}$ yards to a coat?

WRITTEN EXERCISES

139. Find the sums:

1. $23\frac{1}{2} + 39\frac{1}{3}$.	**4.** $\frac{2}{36} + 3\frac{2}{9}$.	**7.** $79\frac{8}{9} + 125\frac{2}{3}$.
2. $37\frac{2}{3} + 43\frac{8}{9}$.	**5.** $23\frac{1}{3} + 16\frac{2}{3}$.	**8.** $95\frac{2}{5} + 129\frac{2}{3}$.
3. $29\frac{4}{5} + 36\frac{7}{10}$.	**6.** $412\frac{1}{3} + 17\frac{3}{5}$.	**9.** $48\frac{2}{3} + 26\frac{5}{18}$.

Find the differences:

10. $89\frac{4}{5} - 12\frac{1}{2}$.	**13.** $23\frac{5}{6} - 16\frac{2}{3}$.	**16.** $52\frac{3}{4} - 17\frac{2}{3}$.
11. $121\frac{2}{5} - 68\frac{3}{10}$.	**14.** $412\frac{1}{3} - 17\frac{3}{5}$.	**17.** $49\frac{1}{16} - 12\frac{7}{8}$.
12 $28\frac{9}{10} - 2\frac{17}{20}$.	**15.** $47 - 22\frac{8}{9}$.	**18.** $53\frac{8}{9} - 16\frac{2}{3}$.

Find the products:

19. $35 \times 14\frac{2}{5}$.
20. $42 \times 2\frac{2}{3}$.
21. $48 \times 3\frac{9}{12}$.
22. $45 \times 17\frac{2}{9}$.
23. $54 \times 20\frac{5}{6}$.
24. $28 \times 13\frac{2}{7}$.
25. $27 \times 151\frac{2}{3}$.
26. $64 \times 27\frac{3}{4}$.
27. $6\frac{1}{3} \times 9\frac{1}{5}$.
28. $16\frac{2}{5} \times 17\frac{2}{9}$.
29. $37\frac{1}{3} \times 16\frac{2}{3}$.
30. $48\frac{1}{4} \times 162\frac{3}{5}$.
31. $28\frac{5}{6} \times 131\frac{2}{3}$.
32. $271\frac{5}{6} \times 23\frac{5}{8}$.
33. $251\frac{2}{3} \times 17\frac{1}{3}$.

Find the quotients:

34. $\frac{7}{8} \div \frac{4}{5}$.
35. $\frac{51}{256} \div \frac{11}{32}$.
36. $\frac{5}{64} \div \frac{8}{15}$.
37. $\frac{13}{24} \div \frac{11}{12}$.
38. $\frac{48}{30} \div 1\frac{2}{5}$.
39. $\frac{29}{25} \div \frac{9}{7}$.
40. $\frac{7}{9} \div \frac{35}{6}$.
41. $\frac{47}{64} \div \frac{7}{8}$.
42. $12\frac{4}{5} \div 3\frac{1}{2}$.
43. $12\frac{5}{16} \div 2\frac{7}{8}$.
44. $57\frac{2}{5} \div 3\frac{2}{9}$.
45. $412\frac{3}{4} \div 45\frac{5}{6}$.

46. A lady bought $3\frac{3}{4}$ lb. of mixed nuts, and divided them equally among 4 girls. How much did each get?

47. A rectangular box is $\frac{1}{3}$ ft. long and $\frac{1}{6}$ ft. wide. What is the distance around it (its perimeter)?

48. How many pieces of velvet, each $\frac{1}{2}$ yd. long, can be cut from a piece $9\frac{1}{2}$ yd. long?

49. Each of the three joints of my fishing rod is $3\frac{5}{8}$ ft. long. How long is the rod?

50. An automobile ran $68\frac{3}{4}$ mi. in $4\frac{7}{12}$ hours. What was the hourly rate?

51. A newsdealer pays $3\frac{3}{10}$¢ each for copies of the "Saturday Evening Post." How many does he get for $\$6\frac{3}{5}$?

52. A carpenter has a board $15\frac{5}{12}$ ft. long. Into how many pieces each $3\frac{1}{12}$ ft. long can he saw it?

53. How much is saved by buying a barrel of sugar (350 lb.) for $\$17\frac{1}{2}$, instead of buying the same quantity at $5\frac{1}{2}$¢ a pound?

54. An electric locomotive can run $5\frac{1}{5}$ miles an hour faster than a steam locomotive which can make $57\frac{2}{5}$ miles an hour. How much farther will the former run than the latter in 5 hours?

55. A drover bought 15 head of cattle averaging 900 lb. each, and 15 head averaging 1000 lb. each, at $3\frac{1}{2}¢$ a pound. How much did he pay for them?

56. When cotton is quoted at $10\frac{5}{8}¢$ a pound, what is the cost of 2 bales weighing 500 lb. each?

57. How many square inches in the bottom of a box 16 in. long and $11\frac{3}{4}$ in. wide?

58. I paid \$2.08 for $6\frac{1}{2}$ lb. of coffee. What did I pay a pound?

59. How many loaves of bread can a baker make from a 48-pound sack of flour, if he uses $\frac{3}{5}$ of a pound to the loaf?

60. A boy in a factory earns \$7 a week. He spends $\frac{1}{4}$ of his salary for board, $\frac{1}{3}$ of what he has left for clothes, and $\$2\frac{1}{2}$ for other expenses. If he saves the remainder, how much does he save in a year (52 weeks)?

RELATED PROBLEMS

140. 1. A merchant pays \$32 for 48 yards of dress goods, and sells it at $87\frac{1}{2}¢$ a yard. How much does he gain?

2. A merchant bought a 16-yard silk dress pattern at \$2.50 a yard, being allowed $\frac{1}{5}$ off for cash. He sold it to a customer at \$2.60 a yard. What was his gain?

3. A dressmaker bought 6 pieces of cheviot, $24\frac{1}{4}$ yd. in each piece, at $87\frac{1}{2}¢$ a yard. She sold it at an average price of \$1 a yard. What did she gain?

4. The ruffles on a dress are $2\frac{1}{2}$ in. wide. Allowing $\frac{1}{8}$ in. along each edge to turn under, how wide must a piece of cloth be to make a ruffle?

5. A girl who works in the dressmaking department of a store is paid at the rate of $22\frac{1}{2}¢$ for 3 hours' work. How much does she earn in an 8-hour day?

6. A merchant sold 8 yd. of sateen at $25¢$ a yard, 11 yd. of lining at $12\frac{1}{2}¢$ a yard, $6\frac{1}{2}$ yd. of lawn at $33\frac{1}{3}¢$ a yard, and 8 yd. of suiting at $87\frac{1}{2}¢$ a yard. If the customer gave a \$20-bill in payment, how much change was returned?

7. I purchased the following materials for a raincoat: $6\frac{3}{4}$ yd. of 54-in. cravenette at \$1.75 a yard, buttons and sewing cotton $50¢$. I paid \$5 for making it. What did it cost me?

8. A merchant sold the following materials for a bath-robe: $8\frac{1}{8}$ yd. of eiderdown at $50¢$ a yard, $\frac{3}{4}$ yd. taffeta silk at $75¢$ a yard, 1 silk girdle $50¢$. What was the amount of the bill?

9. Find the cost of the following materials for a dressing sack: $3\frac{1}{2}$ yd. of 27-in. flannelette at $15¢$ a yard, $2\frac{1}{2}$ yd. of ribbon at $20¢$ a yard, $\frac{1}{2}$ yd. of trimming at $15¢$ a yard, 6 buttons $3¢$.

10. Find the cost of the materials for an everyday frock, as follows:

$7\frac{7}{8}$ yd. of cashmere at $75¢$ a yard	\$
$1\frac{3}{4}$ yd. of waist lining at $12\frac{1}{2}¢$ a yard . . .	
$\frac{5}{8}$ yd. of embroidery at $75¢$ a yard	
Findings	.30
	\$

RELATIONS, RATIO

141. 1. Draw a line 12 inches long and another 36 inches. Compare their lengths, using a foot rule and yardstick.

2. The 36-inch line contains the 12-inch line how many times? Then the 12-inch line is what part of the 36-inch line?

$$\frac{12 \text{ in.}}{36 \text{ in.}} = \frac{1}{3}.$$

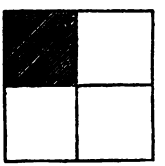
The relation of 12 in. to 36 in., expressed by dividing the former by the latter, is called their **Ratio.**

3. The 36-inch line is how many times as long as the 12-inch line? Then what is the ratio of 36 in. to 12 in.?

$$\frac{36 \text{ in.}}{12 \text{ in.}} =$$

4. Measure your foot rule with a 3-inch unit. What is the ratio of 3 in. to the foot rule? Of the foot rule to 3 inches?

5. In this diagram, what is the ratio of the shaded part to the whole square? Of the whole to the shaded part?



6. What is the ratio of a dime to a dollar? Of a dollar to a dime?

7. Two is what part of 10? Then what is the ratio of 2 to 10? $\frac{2}{10} =$

What is the ratio of 10 to 2? $\frac{10}{2} =$

8. What is the ratio of $\frac{3}{6}$ to $\frac{2}{6}$? Of $\frac{1}{2}$ to $\frac{1}{6}$? Of $\frac{1}{8}$ to $\frac{1}{4}$? Of 4 to $\frac{1}{4}$? Show by drawing and measuring lines.

9. By what operation do you find the ratio of one number to another?

The ratio of one number to another is found by —— the first number by the second.

ORAL EXERCISES

142. **1.** If one apple costs 2¢, what will 5 apples cost?

1 apple	2 cents
5 apples	10 cents

What is the ratio of 1 apple to 5 apples? Of 2 cents to 10 cents? Are these ratios equal or unequal?

2. If 4 lemons cost 10 cents, what will 12 lemons cost?

4 lemons	10 cents
12 lemons	— cents

What is the ratio between 4 lemons and 12 lemons? What must be the ratio between 10 cents and the cost of 12 lemons?

3. If a dozen eggs cost 18 cents, what will 4 eggs cost? (Arrange as in Examples 1 and 2, and fill the blanks by inspection. Note that only numbers of the same kind can be compared in a ratio.)

4. If 3 lb. of rice cost 22 cents, what will 12 lb. cost?

5. If 3 lb. of rice cost 22 cents, how many pounds can be bought for 88 cents?

3 lb. of rice	22 cents
— lb. of rice	88 cents

6. What will 16 lb. of sugar cost, if 4 lb. cost 26 cents?

7. If 6 gal. of oil cost 90 cents, how many gallons can be bought for 30 cents?

8. A lady paid 36 cents for 3 yd. of ribbon. At that rate, how much would she pay for 12 yd.?

9. How many steps does a boy take in running 100 ft., if he takes 3 steps every 10 ft.?

10. A man paid 60¢ for 2 caps. At that rate, what would 6 caps cost?

11. If 15 lb. of pork bring 90¢, how much will 10 lb. bring?

12. If 3 pairs of gloves cost $4.50, what will 6 pairs cost at the same rate?

13. If $5\frac{1}{2}$ lb. of butter cost $1.10, what will 11 lb. cost at the same rate?

14. What is the ratio of a rectangle 3 in. by 5 in. to one 6 in. by 10 in.?

143. Equal Parts of 100.

The *equal parts* of 100 are those numbers that are quotients of 100 divided by integers.

The ratio of —

50 to 100 is $\frac{1}{2}$.	Hence, $50 = \frac{1}{2}$ of 100.
$33\frac{1}{3}$ to 100 is $\frac{1}{3}$.	Hence, $33\frac{1}{3} = \frac{1}{3}$ of 100.
25 to 100 is $\frac{1}{4}$.	Hence, $25 = \frac{1}{4}$ of 100.
20 to 100 is $\frac{1}{5}$.	Hence, $20 = \frac{1}{5}$ of 100.
$16\frac{2}{3}$ to 100 is $\frac{1}{6}$.	Hence, $16\frac{2}{3} = \frac{1}{6}$ of 100.
$12\frac{1}{2}$ to 100 is $\frac{1}{8}$.	Hence, $12\frac{1}{2} = \frac{1}{8}$ of 100.
10 to 100 is $\frac{1}{10}$.	Hence, $10 = \frac{1}{10}$ of 100.
$8\frac{1}{3}$ to 100 is $\frac{1}{12}$.	Hence, $8\frac{1}{3} = \frac{1}{12}$ of 100.

The numbers in the first column are the equal parts of 100. From them may be derived other parts of 100. For example, $66\frac{2}{3} = \frac{2}{3}$ of 100.

In solving problems, these equal parts may be used to great advantage in shortening computations.

SOLUTION OF PROBLEMS

144. You have now learned to add, subtract, multiply, and divide whole numbers and fractions. In order to apply this knowledge successfully to the solution of problems, experience and good common sense are necessary. The illustrative solutions and the examples given below are intended to serve as models, and to give needed practice in solving problems.

The following general directions will be found helpful:

1. **Learn what the statement of the problem says.** A good test of your understanding the statement is your ability to close the book and tell your teacher what you are trying to do.

2. **State what is given.**

3. **State what is to be found.**

4. **Plan an outline of the work which you think will yield the required result.**

5. **After making the computations, test the result.**

145. 1. A dealer pays \$24 a dozen for hats. At this rate, what does he pay for 25 hats?

GIVEN: 12 hats cost \$24, and from this we find that
1 hat costs $\frac{1}{12}$ of \$24, or \$2.

TO FIND: Cost of 25 hats.
1 hat costs \$2.
25 hats cost $25 \times \$2$, or \$50.

2. A dealer pays \$48 for 8 heaters. At this rate, what does he pay for 5 heaters?

3. A man pays $64 for 10 barrels of flour. At that rate, what do 18 barrels cost?

4. If I pay $240 a year rent for a residence, what is the rent for 5 months at the same rate?

5. A drover paid $900 for 20 head of cattle, and sold them at $50 a head. How much did he gain?

GIVEN: $900 paid for 20 head of cattle.

TO FIND: Gain after selling at $50 a head.

20 cattle at $50 = 20 × $50 = $1000.

$1000 − $900 = $100, gain.

6. A fruit buyer bought 60 barrels of apples at $1.20 a barrel, and was compelled to sell them at $1 a barrel. How much did he lose?

$1.20 − $1.00 = $.20, loss on 1 barrel.

60 × $.20 = $12, the loss on 60 barrels.

Have the pupil explain the steps.

7. If I buy 60 sheep for $240, and sell them at $5.20 a head, what do I gain?

8. A farmer paid $8000 for a farm of 160 acres, and sold it at $60 an acre. What did he gain?

9. A grocer bought 100 bu. of potatoes at 70 ct. a bushel, and sold them at 84 ct. a bushel. What did he gain? (Solve in two ways.)

10. I am selling 100 pairs of shoes that cost $350 at $2.75 a pair. What do I lose on each pair?

11. At $12\frac{1}{2}$¢ a yard, what will 18 yd. of gingham cost? At $16\frac{2}{3}$¢? At $33\frac{1}{3}$¢?

$12\frac{1}{2}$¢ = $\frac{1}{8}$; $16\frac{2}{3}$¢ = $\frac{1}{6}$; $33\frac{1}{3}$¢ = $\frac{1}{3}$.

Hence, 18 yd. at $12\frac{1}{2}$¢ cost 18 times $\frac{1}{8}$, or $2.25.

18 yd. at $16\frac{2}{3}$¢ cost 18 times $\frac{1}{6}$, or $3.00.

18 yd. at $33\frac{1}{3}$¢ cost 18 times $\frac{1}{3}$, or $6.00.

How is the work shortened in this solution?

12. At $12\frac{1}{2}¢$ a pound, what will 16 lb. of cheese cost?

13. At $16\frac{2}{3}¢$ a pound, what will 24 lb. of butter cost?

14. At $33\frac{1}{3}¢$ a yard, what will 48 yd. of cloth cost?

15. Ten is $\frac{2}{3}$ of what number?

Since $\frac{2}{3}$ of the number = 10,

$\frac{1}{3}$ of the number = $\frac{1}{2}$ of 10, or 5.

Therefore $\frac{3}{3}$ of the number = 3 × 5, or 15.

Since 10 is $\frac{2}{3}$ of some number, 1 third of that number is $\frac{1}{2}$ of 10, or 5; and since 5 is $\frac{1}{3}$ of the number, 3 thirds, or the number, equals 3 times 5, or 15. Hence, 10 is $\frac{2}{3}$ of 15.

16. If $\frac{5}{8}$ of a yard of velvet cost \$1.50, what is the price a yard?

∵ $\frac{5}{8}$ yd. cost \$1.50,

$\frac{1}{8}$ yd. costs $\frac{1}{5}$ of \$1.50, or \$0.30.

∴ $\frac{8}{8}$ yd. cost 8 × \$0.30, or \$2.40.

In written work we shall find it convenient to use the sign ∵ for the word "since," and the sign ∴ for the word "therefore."

17. If $\frac{2}{3}$ of the price of a bicycle is \$40, what amount must you pay for it?

∵ $\frac{2}{3}$ of the cost = \$40,

$\frac{1}{3}$ of the cost = $\frac{1}{2}$ of \$40, or \$20.

∴ $\frac{3}{3}$ of the cost = 3 × \$20, or \$60.

18. Twelve is $\frac{3}{4}$ of what number?

19. Twenty is $\frac{2}{5}$ of what number?

20. Thirty-two locomotives are $\frac{4}{5}$ of the number owned by a certain railroad. How many locomotives does it own?

21. A lady spent \$24, which was $\frac{3}{4}$ of her money. How much had she at first?

22. James rode 30 miles in one day, which was $\frac{3}{5}$ of the distance Marshall rode. How many miles did Marshall ride?

23. In a school $\frac{1}{20}$ of the pupils are absent and 570 are present. How many are absent?

$\frac{19}{20}$ of the pupils $-$ $\frac{1}{20}$ of the pupils $=$ $\frac{18}{20}$ of them.

$\frac{18}{20}$ of the pupils $=$ 570 pupils.

$\frac{1}{20}$ of the pupils $=$ $\frac{1}{19}$ of 570 pupils, or 30 pupils.

24. If electricity passes through 7200 miles of wire in $\frac{2}{3}$ of a second, through how many miles does it pass in a second?

25. If a horse trots $\frac{5}{6}$ of a mile in $1\frac{2}{3}$ minutes, in what time can he trot a mile?

26. One half of a pole was painted red, $\frac{1}{3}$ white, and the remaining $7\frac{1}{2}$ feet blue. What was the length of the pole?

27. A man spent $\frac{1}{2}$ of his money for a cow, $\frac{2}{3}$ of the remainder for feed, and had \$15 left. How much had he at first?

He had $\frac{1}{2}$ of his money left after buying the cow.

$\frac{2}{3}$ of $\frac{1}{2}$ $=$ $\frac{1}{3}$ of his money, or cost of feed.

$\frac{1}{2}$ of his money $-$ $\frac{1}{3}$ of it $=$ $\frac{1}{6}$ of it, part left.

$\frac{1}{6}$ of his money $=$ \$15,

and $\frac{6}{6}$ of his money $= 6 \times$ \$15, or \$90.

28. A college spent $\frac{3}{4}$ of its income in salaries, $\frac{1}{5}$ of the remainder for insurance and repairs, and the remaining \$8000 for new buildings. What was its income?

29. In a primary school $\frac{2}{7}$ of the pupils are in the third grade, $\frac{3}{5}$ of the remainder in the second, and the remaining 100 in the first. How many pupils are there in the school?

30. A man, his wife, son, and daughter weigh respectively 165 lb., 120 lb., 145 lb., and 106 lb. What is the average weight?

165 lb. + 120 lb. + 145 lb. + 106 lb. = 536 lb., total weight.

$\frac{1}{4}$ of 536 lb. = 134 lb., average weight.

31. The attendance of pupils at a certain school for a week was as follows: Monday 45, Tuesday 48, Wednesday 43, Thursday 46, Friday 43. What was the average daily attendance?

32. The *per capita* indebtedness of France is $151.60; of Germany, $10.51; of Great Britain and Ireland, $96.28; of Russia, $31.35; of Spain, $108.84; of Austria-Hungary, $24.32; and of the United States, $11.38. What is the average *per capita* indebtedness of these countries?

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33. Find the cost of a dozen spoons, if $\frac{3}{5}$ of the cost is $9.

34. Six is $\frac{2}{5}$ of what number?

35. Thirty is $\frac{3}{7}$ of what number?

36. Twenty-four is $\frac{7}{8}$ of what number?

37. If $\frac{5}{6}$ of a number is $12\frac{1}{2}$, what is the number?

38. If $\frac{3}{4}$ lb. of almonds cost 15¢, what is the cost of 1 pound? Of 5 pounds?

39. If $\frac{5}{8}$ yd. of silk cost 80¢, what will 1 yd. cost?

40. If half a dozen lemons cost 15¢, what will 4 dozen cost?

41. If $\frac{5}{6}$ lb. of butter cost $.25, what is the price a pound?

42. If $\frac{3}{5}$ lb. of cheese cost 9¢, what will 4 lb. cost?

43. What is the cost of $2\frac{3}{4}$ lb. of mixed candy at $.20 a pound?

44. What is the cost of 12 bunches of celery at $12\frac{1}{2}$¢ a bunch?

45. What will $12\frac{1}{2}$ dozen eggs cost at 20¢ a dozen?

46. A grocer sells 25 lb. of coffee for $4.50. At this rate, what does he get for 6 pounds?

47. A dealer bought 20 tons of coal at $3.25 a ton, and sold it at $4 a ton. If drayage cost him $.50 a ton, what did he gain?

48. At $66\frac{2}{3}$¢ a yard, what will 9 yd. of cheviot cost?

49. If 10 boxes of oranges cost $22.50, how many boxes can be bought for $33.75?

50. How many pounds of sugar at $6\frac{2}{3}$¢ a pound can be bought for a dollar?

51. A farmer sold $\frac{1}{5}$ of his corn at one time, $\frac{2}{3}$ of it at another, and then had 80 bu. left. How much had he at first?

52. A boy had $\frac{1}{2}$ lb. of caramels, and gave away $\frac{1}{4}$ of them. What part of a pound had he left?

53. If $\frac{1}{4}$ lb. of dried beef cost $.05, how many pounds can be bought for a dollar?

54. If a dozen pencils cost 40¢, how many pencils can be bought for $1.40?

55. If $\frac{4}{5}$ of a cord of wood cost $2.80, what will $5\frac{1}{2}$ cords cost?

56. If $144 is $\frac{8}{5}$ of my annual rent, what do I pay a month at this rate?

57. At $\frac{1}{4}$ of a dollar a bushel, how many bushels of apples can be bought for $16.50?

58. If $5\frac{1}{4}$ lb. of nuts are divided equally among 7 boys, what is the share of each?

59. If a furnace consumes $\frac{4}{5}$ of a ton of coal in a day, how much will it consume in $8\frac{4}{7}$ weeks, at the same rate?

60. In a hennery there are 80 hens. In a year (52 weeks) they laid 12,480 eggs. How many eggs did each average a year? A week? At 20¢ a dozen, what did these eggs bring?

61. When butter was worth 18¢ a pound, I sold $11.34 worth in 7 days. How many pounds did I average a day?

62. A grocer sells a sack of coffee weighing 125 lb. for $18.75. What is the price a pound?

63. A dealer buys 12 dozen 3-pound cans of tomatoes for $13.80, and sells them at 12¢ a can. How much does he gain?

64. How many rosettes, containing $1\frac{5}{8}$ yd. each, can be made from 26 yd. of ribbon?

65. At $\frac{3}{4}$ of a cent a square foot, what will it cost to slate a blackboard 20 ft. long and 4 ft. wide?

66. How many miles an hour is an automobile moving when traveling a mile every 4 minutes?

67. If $\frac{3}{20}$ of the population of Richmond, Va., in 1900, was 12,750, what was the population?

68. A dealer bought 500 bu. of wheat at 90¢ a bushel. He sold 300 bu. at 98¢ a bushel, and the rest at 85¢ a bushel. How much did he gain?

69. A man who earns $200 in 8 weeks, spends $7 a week for board and $2 a week for other purposes. If he deposits the remainder in a savings bank, what is his average weekly deposit? How much would he deposit in 52 weeks at the same rate?

70. The width of a stream at several points is as follows: $42\frac{1}{8}$ feet, $37\frac{1}{2}$ feet, $35\frac{5}{8}$ feet, $43\frac{1}{2}$ feet, 52 feet, and $46\frac{5}{8}$ feet. What is the average width?

71. An electric light plant runs from 8 P.M. to 4 A.M. in one month; from 7 P.M. to 5 A.M. in another; from 6 P.M. to 5.30 A.M. in another; and from 5.30 P.M. to 6 A.M. in another. What is the average monthly run?

72. The pressure of water in a city reservoir raises water from a hose nipple in Ward A to a height of 60 ft., in Ward B to a height of 50 ft., in Ward C to a height of 45 ft., and in Ward D to a height of 35 ft. What is the average height to which the pressure of the reservoir will raise a stream?

73. Find the cost of the following list of groceries, using equal parts of a dollar:

12 lb. of butter at 20¢ a pound,
6 dozen eggs at $16\frac{2}{3}$¢ a dozen,
5 lb. of coffee at 25¢ a pound,
12 qt. of cranberries at $12\frac{1}{2}$¢ a quart,
6 lb. of baking powder at $33\frac{1}{3}$¢ a pound,
3 lb. of tea at $66\frac{2}{3}$¢ a pound,
9 lb. of rice at $8\frac{1}{3}$¢ a pound,
4 lb. of macaroni at 10¢ a pound.

74. The following is a statement of the expenses, exclusive of transportation charges, of a two weeks' camping trip of four girls:

Drayage	$0.50	6 loaves of bread	$0.30
5 lb. bacon	.75	10 lb. sugar	.60
5 lb. oatmeal	.25	$2\frac{1}{2}$ lb. cheese	.50
4 boxes crackers	.60	3 lb. butter	.60
$2\frac{1}{2}$ lb. ginger snaps	.25	Fruit	1.03
Vegetables	1.20	3 gal. oil	.54
Rent of tent and cots	2.00	Fishing tackle	.40

What is the average weekly expense of each?

RELATED PROBLEMS

146. **1.** The picture represents beef cattle in a pasture. In April the farmer bought 25 three-year-old steers averaging 1000 lb. each, at $3\frac{1}{2}¢$ a pound gross. He grazed them 6 months, during which time they took on an average of 400 lb. each. If the expense of salting and attention was $2.50 a month for the lot, what did he gain by selling them on foot to a dealer at $5¢$ a pound?

2. Instead of buying cattle as above, the farmer could have rented this pasture for grazing for the same length of time at $3 per acre. If it takes 3 acres to the steer, how much rent would he have received for the pasture required for the cattle mentioned in Example 1?

3. How much more did he gain by dealing in cattle than he would have received by renting his pasture?

4. If the dealer who bought the cattle in Example 1 paid $2.50 a head for freight and other expenses, how much did he gain by selling them at $6\frac{1}{2}¢$ a pound, allowing 50 lb. to the steer for loss in weight?

5. A farmer bought in the fall 50 steers averaging 1000 lb. each at $3\frac{1}{2}¢$ a pound, and 50 heifers averaging 900 lb. each at $2\frac{1}{2}¢$ a pound. After feeding and grazing them 11 months at an average cost of $20 a head, he found that his steers averaged 1500 lb. each and his heifers 1300 lb. each. If he sold his steers at $5\frac{3}{4}¢$ a pound, and his heifers at $4\frac{3}{4}¢$ a pound, what were his profits?

6. A dealer bought for export 100 steers averaging 1600 lb. each, at $6¢$ a pound. The freight and other expenses from the pasture to Liverpool amounted to $15 a head. If the loss in weight averaged 80 lb. per head, how much did he gain by selling them at $8\frac{1}{2}¢$ a pound?

7. A grazer bought 30 head of cattle, averaging 1100 lb. each, at $37.50 a head, and after grazing them 6 months, sold them at $5\frac{1}{4}¢$ a pound. If they gained 350 lb. each, how much more did he receive for them than they cost him?

8. A car load of cattle contained 20 head, averaging 1350 lb. each. If .6 of the gross weight is salable meat, what will this car load of cattle bring when butchered, cut, and sold at an average price of $18¢$ a pound?

9. In a recent year the number of cattle, other than milch cows, in the United States, was 43,629,428. The average farm value was $16.32 per head. What was the total value?

10. In a recent year the number of milch cows in this country was 17,570,000, and their estimated value was $481,945,100. What was their average value per head?

11. In a year when the number of cattle exported was 402,176, and their estimated value at the ports was $29,861,568, what was the average price per head?

COMPOUND NUMBERS

147. In the United States the English system of Weights and Measures is the one in general use. In Washington are preserved standards of capacity, weight, and length; and each state government fixes these by law as its own.

This system of which we shall treat in the succeeding pages is not a complete system in the theoretical sense, but it is the product of the experience of ages.

The Metric system, also called the French system, is more scientific, yet for most practical needs the English is convenient and sufficient.

We shall treat in order:

(1) Measures of Extension;
(2) Measures of Capacity;
(3) Measures of Weight;
(4) Measures of Time.

Every school should be supplied with the following: foot rule and yardstick; pint, quart, and gallon measures; and 1-ounce, 2-ounce, 4-ounce, 8-ounce, and 1-pound weights.

MEASURES OF EXTENSION

148. Extension may have three dimensions, called length, breadth, and thickness or height.

A line has but one dimension — length.

A surface has two dimensions — length and breadth.

A solid has three dimensions — length, breadth, and thickness.

149. Measures of Extension are used in measuring and computing lengths, surfaces, and solids.

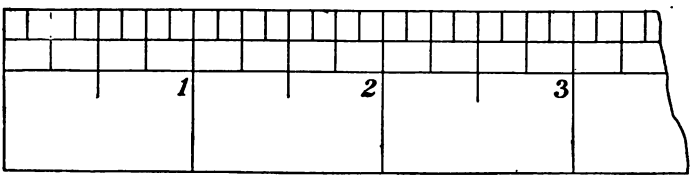
Linear Measure

150. Linear Measure is used in measuring lengths.

TABLE OF LENGTH

12 inches (in.)	= **1 foot** (ft.).
3 feet	= **1 yard** (yd.).
5½ yards or **16½ feet**	= **1 rod** (rd.).
320 rods	= **1 mile** (mi.).

mi.	rd.	yd.	ft.	in.
1	= 320	= 1760	= 5280	= 63,360.



151. The cut shows a section of a foot rule. What part of a foot is 3 inches? Apply this measure to a line four times. We have four 3-inches. How many of the 3-inch lengths will measure your desk? How many 1-foot lengths? Is it necessary to measure a second time to answer the last question?

You have used two ways to express the same length. A change in this way of expressing a length is called **Reduction.** If any measure is expressed in more than one unit, we have a **Compound Number.** For example, "My desk is 3 ft. 2 in. long."

152. Complete :

1. 1 rd. 2 yd. = $7\frac{1}{2}$ yd.
2. 2 ft. 3 in. = —— in.
3. 1 yd. 2 ft. = —— ft.
4. 1 ft. 4 in. = —— in.
5. 1 rd. 2 ft. = —— ft.
6. 1 mi. 20 rd. = —— rd.
7. 4 yd. 2 ft. = —— ft.
8. 2 yd. 8 ft. = —— ft.
9. 3 ft. 6 in. = —— ft.
10. 2 yd. 1 ft. = —— yd.

ORAL EXERCISES

153. 1. If Henry walks 20 rd. a minute, how far will he walk in 16 minutes?

2. If a boy walks a mile in 20 minutes, how many rods does he walk in a minute?

3. What is the perimeter of a field 20 rd. square?

4. If you are to buy enough carpet to make a strip down a large hall 150 feet long, how many yards must you order?

5. A fishing line is sold in lengths of 25 yd. each. How many such lengths would be needed to supply a party of 20 with lines each 5 yd. long?

WRITTEN EXERCISES

154. 1. Reduce 5 yd. 2 ft. 6 in. to inches.

(*a*)

∵ 1 yd. = 3 ft., 5 yd. = 5 × 3 ft., or 15 ft.

15 ft. + 2 ft. = 17 ft.

∵ 1 ft. = 12 in., 17 ft. = 17 × 12 in., or 204 in.

204 in. + 6 in. = 210 in.

The work is conveniently performed as in (*b*).

(*b*)

5
3
15
2
17 (ft.)
12
204
6
210 (in.)

Reduce :

2. 12 ft. 9 in. to inches.
3. 5 yd. 2 ft. to inches.
4. 2 mi. 20 rd. to rods.
5. 7 yd. 8 in. to inches.
6. $\frac{1}{2}$ mi. to feet.
7. 80 rd. to inches.

8. Reduce 210 in. to yards, feet, and inches.

(*a*)

$\because$ 12 in. = 1 ft., 1 in. = $\frac{1}{12}$ ft.,
and 210 in. = 210 × $\frac{1}{12}$ ft. = 17 ft., and 6 in. remaining.
$\because$ 3 ft. = 1 yd., 1 ft. = $\frac{1}{3}$ yd.,
and 17 ft. = 17 × $\frac{1}{3}$ yd. = 5 yd., and 2 ft. remaining.
$\therefore$ 210 in. = 5 yd. 2 ft. 6 in.

(*b*)

$$\begin{array}{r|l} 12 & 210 \\ \hline 3 & 17 + 6 \text{ (in.)} \\ & 5 + 2 \text{ (ft.)} \end{array}$$

The actual work is best performed as in (*b*).

Reduce :

9. 5280 yd. to miles.
10. 10,560 ft. to miles.
11. 800 rd. to miles.
12. 550 ft. to rods and feet.
13. 816 in. to yards and feet.
14. 385 in. to yards, feet, and inches.

Reduce :

15. $\frac{1}{2}$ rd. to inches.
16. 2640 ft. to rods.
17. $6\frac{1}{2}$ yd. to inches.
18. $8\frac{8}{15}$ mi. to feet.
19. 25 mi. to rods.
20. 250 in. to feet and inches.
21. 2 mi. 40 rd. to feet.
22. 288 in. to yards.
23. 3 rd. 1 yd. 2 ft. to inches.
24. $\frac{3}{4}$ ft. to the fraction of a yard.
25. What part of a yard is 1 ft. 4 in.?

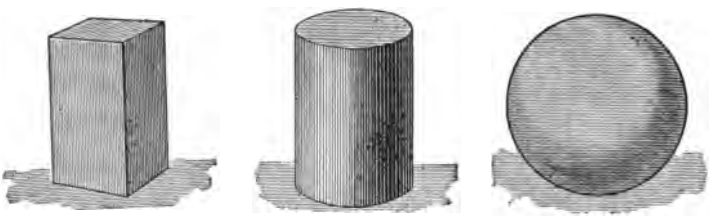
26. A tennis court has four lines each 72 ft. long, two 36 ft. long, two 27 ft. long, and two 18 ft. long. How many yards of tape will mark the court?

27. At 60¢ a rod, how much must be paid for building a fence 10 rd. 11 ft. long?

28. How many 4-inch bandage strips can be torn lengthwise from cloth that is $\frac{2}{3}$ yd. wide?

Surface Measure

155. Boundaries of solids are called **Surfaces.**



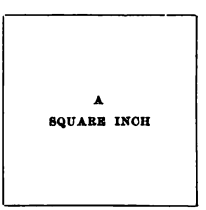
Examine the figures and tell about their surfaces. Which are flat surfaces? Any figure which can be made on a flat surface is called a **plane figure.**

156. A four-sided plane figure with square corners and all sides equal is called a **Square.** If its sides are not all equal the figure is a **Rectangle.** A square is a special form of rectangle.

157. The *corners* of a rectangle are **right angles.** The corners of any plane figure are **angles.**

158. When each side of a square is *one inch* long, the figure is called a *square inch.*

What is a square foot? A square yard? A *unit* of *square measure?*



159. The **Area** of a plane figure or surface is the number of units of square measure it contains.

160. Square Measure is used in computing the areas of surfaces.

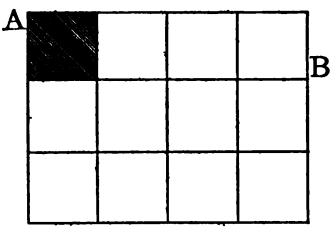
TABLE OF SQUARE MEASURE

144	**square inches** (sq. in.)	= **1 square foot** (sq. ft.).
9	**square feet**	= **1 square yard** (sq. yd.).
30¼	**square yards**	= **1 square rod** (sq. rd.).
160	**square rods**	= **1 acre** (A.).
640	**acres**	= **1 square mile** (sq. mi.).

sq. mi. A. sq. rd. sq. yd. sq. ft. sq. in.

1 = 640 = 102,400 = 3,097,600 = 27,878,400 = 4,014,489,600.

161. The figure represents a rectangle whose length is 4 in. and width 3 in. It is divided into 3 strips, each representing 1 in. in width, and each strip into 4 equal parts, each part representing 1 sq. in.



Then the area of the strip AB is 4×1 sq. in., or 4 sq. in.; and the total area of the rectangle is 3×4 sq. in., or 12 sq. in.

162. The area of a rectangle is expressed by the product of the numbers that measure its length and breadth.

Both dimensions must be expressed in like units.

ORAL EXERCISES

163. 1. A room is 12 ft. long and 10 ft. wide. How many square feet does the floor contain?

2. A table is 8 ft. long and 3½ ft. wide. How many square feet are there in its surface?

3. How many square yards of oilcloth will cover a floor 15 feet square?

4. How many square yards of linoleum are needed to cover a hall 30 ft. long by 9 ft. wide? What will it cost at $1 a square yard?

5. A city lot is 8 rd. long and 5 rd. wide. How many square rods in it? What part of an acre?

6. In 2 acres how many square rods? In 5 acres?

7. A rug is 9 ft. by 12 ft. How many square feet in it? How many square yards?

8. How do you find one factor when the other factor and the product are given?

9. The area of a rectangle 8 ft. long is 40 sq. ft. What is its width?

10. What is the area of a board 12 ft. long and $1\frac{1}{2}$ ft. wide?

11. If the perimeter of a square is 72 inches, how long is each side?

12. What is a square mile? How many rods long is one side of it?

WRITTEN EXERCISES

164. Reduce:

1. 1 sq. ft. 10 sq. in. to square inches.
2. 2 sq. yd. 6 sq. ft. to square inches.
3. 7 sq. yd. 97 sq. in. to square inches.
4. 12 sq. rd. 5 sq. yd. to square feet.
5. 5 A. 40 sq. rd. to square rods.
6. 3 sq. yd. 8 sq. in. to square inches.
7. 1 sq. mi. 160 A. to acres.

8. 900 sq. in. to square feet and square inches.

9. 1200 sq. ft. to square yards and square feet.

10. 1500 sq. in. to square yards, square feet, and square inches.

11. 2000 sq. rd. to acres and square rods.

12. 1000 A. to square miles and acres.

13. $\frac{3}{8}$ sq. yd. to square inches.

14. What part of an acre is $\frac{4}{5}$ of a square rod?

15. A floor is 18 ft. long and 16 ft. wide. How many square feet does it contain?

16. A town lot is 18 ft. by 121 ft. What is its area in square rods?

17. How many square yards in a piece of carpet 16 ft. long and $\frac{3}{4}$ yd. wide?

18. A field is 40 rods long and 25 rods wide. What will it cost at $40 an acre?

19. How many square yards in an acre? In 10 acres?

20. Farmers often step off a square 70 steps × 70 steps for an acre. Can you explain this approximation?

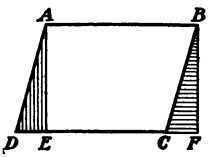
21. A farmer raised oats on a rectangular field 60 rd. long by 40 rd. wide. The total yield was 690 bu. What was the average yield per acre?

PARALLELOGRAMS

165. How does the figure $ABCD$ on the next page differ from a rectangle? Which of its sides are equal? Which are parallel? Are its angles equal?

166. A four-sided figure whose opposite sides are parallel is called a **Parallelogram.** A rectangle is a special form of parallelogram.

167. The side of a figure on which it is supposed to stand is called the **Base**; as, *CD*. The **Altitude** is the perpendicular distance between the base and the opposite side; as, *AE*. If we cut off the end *DAE*, and place it at the other end, on *CBF*, what kind of figure do we have? Will the area be changed?



1. How is the area of the rectangle *ABFE* found? Then how may the area of the parallelogram *ABCD* be found?

It follows that —

The area of a parallelogram is the product of the numbers that measure its base and altitude.

168. Find the areas of the following parallelograms:

1. Base 12 ft., alt. 4 ft.
2. Base 40 ft., alt. 18 ft.
3. Base 64 ft., alt. 27 ft.
4. Base 24 ft., alt. 8 in.
5. Base 42 rd., alt. 5 rd.
6. Base 120 in., alt. 25 ft.

7. A board in the form of a parallelogram is 6 ft. 6 in. long on each side and 1 ft. 4 in. wide. Find the area of the board.

6 ft. 6 in. $= 6\frac{1}{2}$ ft. $= \frac{13}{2}$ ft.

1 ft. 4 in. $= 1\frac{1}{3}$ ft. $= \frac{4}{3}$ ft.

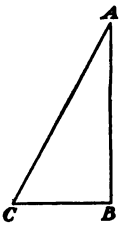
$$\frac{13}{2} \times \frac{4}{3} = \frac{26}{3} = 8\frac{2}{3},$$ number of square feet in area.

The answer $8\frac{2}{3}$ sq. ft. may be expressed as 8 sq. ft. 96 sq. in. The tendency, however, is to express results in one denomination because they can be more easily comprehended.

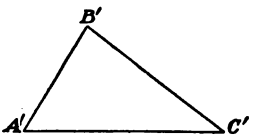
8. A parallelogram is 5 yd. 2 ft. in length, and 1 yd. 6 in. in width (altitude). Find its area.

TRIANGLES

169. A plane figure bounded by three straight lines is called a **Triangle.** Examine the three corners of the triangles, ABC, $A'B'C'$.

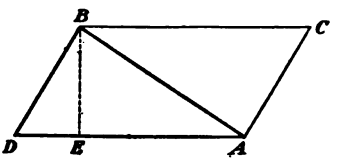


ABC has a square corner at B, and is therefore called a **right triangle.**



Is triangle $A'B'C'$ a right triangle?

170. 1. Into how many triangles does the diagonal AB divide the parallelogram $BCAD$?



Cut out a figure and by fitting one triangle upon the other see that ABD is equal to ABC.

2. Then the area of each triangle is what part of the area of the parallelogram? Has the triangle ABD the same altitude as the parallelogram? Has it the same base?

3. Since the area of the parallelogram is expressed by $AD \times BE$, how is one half its area, or the area of the triangle ABD, expressed?

The area of a triangle is expressed by one half the product of the numbers that measure its base and altitude.

171. Find the areas of the following triangles:

1. Base 16 ft., alt. 9 ft.
2. Base 19 ft., alt. 14 ft.
3. Base 45 rd., alt. 165 yd.
4. Base 18 ft., alt. 8 ft. 6 in.

5. The base of a triangular piece of slate is 24 inches and the altitude 32 inches. Find the area. (Draw a figure, using $\frac{1}{8}$ in. to represent an inch.)

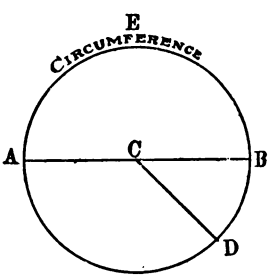
6. How many triangles, base 4 ft. and altitude 3 ft., are equal in area to a parallelogram whose base is 40 ft., and whose altitude is 30 ft.?

7. A triangle whose altitude is 6 ft. has an area of 96 sq. ft. What is its base?

CIRCLES

172. 1. Is the portion of the page inclosed by *ADBE* a plane figure?

2. What kind of line bounds the figure? Is any point in the line nearer the center than any other point in that line?



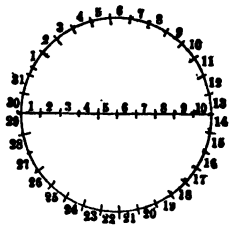
173. A plane figure whose bounding line is everywhere equally distant from a point within, called the center, is a **Circle.**

1. The curved line that bounds a circle is called the **Circumference.**

2. A straight line passing through the center and ended each way by the circumference is called a **Diameter**; as, AB.

3. A line from the center to the circumference is called a **Radius**. It is *half* a diameter; as, CD or CA.

174. By drawing a circle, say 1 inch in diameter, dividing the diameter into 10 equal parts, and then laying off on the circumference distances each equal to one of the divisions of the diameter, it will be found that the circumference is a little more than 3 times the diameter.



To test this, draw a circle of any convenient radius on cardboard; cut it out and stretch a string around it; then measure the string and the diameter.

175. In this book the circumference is taken as $3\frac{1}{7}$ times the diameter. For practical purposes, this value is sufficiently accurate. More exactly, the ratio of the circumference to the diameter is 3.1416.

The circumference of any circle is $3\frac{1}{7}$ (more exactly, 3.1416) **times the diameter.**

176. What is the circumference of —

1. A circle whose diameter is 7 in.? 14 in.? 21 in.?

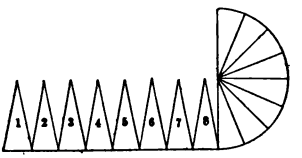
2. A circle whose radius is 8 ft.? 28 ft.? $3\frac{1}{2}$ ft.?

What is the diameter of —

3. A circle whose circumference is $3\frac{1}{7}$ in.? 44 in.?
(To find the diameter divide the circumference by $3\frac{1}{7}$.)

4. A circle whose circumference is 66 ft.? 15.4 ft.?

177. Draw a circle on a piece of leather, and cut it out. Then cut from center to circumference along one radius and along other radii almost to the circumference. Straighten out as in the figure. The resulting figures 1, 2, 3, etc., may be considered triangles. If they were exact triangles, the altitude of each would be the radius, and the sum of the bases the circumference.



Hence, **to find the area of a circle, multiply the circumference by half the radius.**

178. 1. Find the area of a circle whose radius is 7 inches.

$\frac{22}{7} \times 14$ in. $= 44$ in., circumference.

$\frac{1}{2}$ of 7 in. $= 3\frac{1}{2}$ in.

$44 \times 3\frac{1}{2} = 154$, number of square inches.

What is the area of:

2. A circle whose radius is 77 inches?

3. A circle whose radius is 91 feet?

4. A circle whose radius is $\frac{7}{11}$ of a yard?

5. At $64 an acre, what is the value of the land in a circular park whose diameter is 70 rods?

6. The distance from the center to the edge of a circular reservoir is 210 ft. How many square feet of water surface has the reservoir?

7. A cowboy uses a rope 10.5 feet long to tether his pony. Over how many square feet of ground can it graze?

Solid Measure

179. On page 124 there are three solids. Any object in the room is a solid, because it occupies space.

180. A solid with six square faces is called a **Cube.**

1. How many edges has a cube? Point to them.

2. Are the faces equal? Name an object that has the form of a cube.

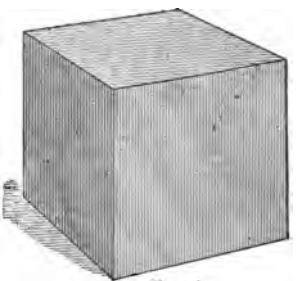


FIG. 1.

181. Figure 1 represents in actual size a cubic inch. Each face is a square inch. A solid 1 inch long, 1 inch wide, and 1 inch thick is called a **cubic inch.**

What are the dimensions of a cubic foot?

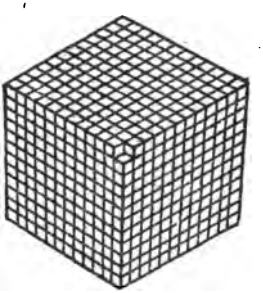


FIG. 2.

182. Figure 2 represents in diminished size a cubic foot. If we cut off a layer an inch thick and divide it into inch cubes, how many of them shall we have? How many such pieces can be cut from a cubic foot? Then how many cubic inches in a cubic foot?

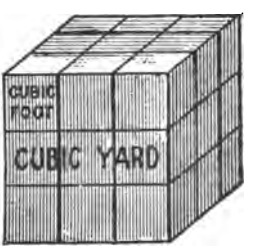


FIG. 3.

183. Figure 3 represents in diminished size a cubic yard. If we cut it into cubes as indicated, what will each little cube represent? How many are there in one layer? In three layers? Then how many cubic feet in a cubic yard?

184. The number of cubic units in a solid is called its **Volume.**

185. Cubic Measure is used in computing the volumes of solids.

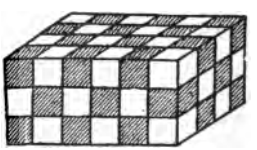
TABLE OF CUBIC MEASURE

1728 cubic inches (cu. in.) = **1 cubic foot** (cu. ft.).
27 cubic feet = **1 cubic yard** (cu. yd.).

cu. yd.	cu. ft.	cu. in.
1	= 27	= 46,656.

186. A solid bounded by six rectangular faces is called a **Rectangular Solid.**

187. The figure represents a rectangular solid, 6 in. long, 4 in. wide, and 3 in. thick. It is divided into three layers, each containing 4 rows of the little cubes. Each row of a layer contains 6 little cubes, or *cubic inches*. Hence the 4 rows of a layer contain 24 cubic inches. Since there are 24 cu. in. in each layer, in the 3 layers there are 3×24 cu. in., or 72 cu. in.



188. The volume of a rectangular solid is the product of the numbers that measure its length, breadth, and thickness.

ORAL EXERCISES

189. 1. How many cubic inches in a brick 8 in. long, 4 in. wide, and $2\frac{1}{2}$ in. thick?

2. How many cubic feet in a wall 20 ft. long, 5 ft. high, and 2 ft. thick?

3. A bin is 5 ft. square at the bottom and 7 ft. deep. How many cubic feet of grain will it hold?

4. How many cubic feet in a pile of wood 8 ft. long, 4 ft. wide, and 4 ft. high?

5. A cubic foot of water weighs 1000 oz. What is the weight of a cubic yard of water?

6. An ice box holds 24 cu. ft. of ice. It is 4 ft. long and 2 ft. deep. How wide is it?

WRITTEN EXERCISES

190. Reduce:

1. 2 cu. ft. to cu. in.
2. 3 cu. yd. to cu. in.
3. 5 cu. yd. to cu. ft.
4. 51 cu. ft. to cu. in.
5. 2 cu. yd. 10 cu. ft. to cu. in.
6. 135 cu. ft to cu. yd.
7. What part of a cubic yard is $\frac{3}{5}$ of a cubic foot?
8. Reduce $\frac{2}{3}$ cu. yd. to cubic feet.
9. Reduce .75 cu. ft. to cubic inches.
10. A block of granite is 3 ft. square and 2 ft. thick. What does it weigh, allowing 165 lb. to a cubic foot?
11. How many cubic feet of water fell during a 2-inch rainfall on a lot 50 ft. by 150 ft.?
12. In building a railroad the engineer finds that he must make a fill whose length is 60 ft. and average depth 17 ft. If the average width of the fill is 18 ft., how many cubic yards of earth must be used?

$$\frac{\overset{20}{\cancel{60}} \times 17 \times \overset{2}{\cancel{18}}}{\underset{3}{\cancel{27}}} = 680, \text{ number of cubic yards.}$$ Explain the process.

13. A mountain gulch has a dam across it that holds a body of water of 12 ft. average depth, 35 ft. average width, and 150 ft. long. How many cubic feet of water does it hold?

MEASURES OF CAPACITY

191. Liquid Measure is used in measuring liquids. The principal unit is the gallon, which contains 231 cubic inches.

TABLE OF LIQUID MEASURE

4 gills (gi.) = **1 pint** (pt.).
2 pints = **1 quart** (qt.).
4 quarts = **1 gallon** (gal.).

gal. qt. pt. gi.
1 = 4 = 8 = 32.

In estimating the capacity of tanks and cisterns, 31½ gal. are considered a barrel; but in commerce barrels vary in size.

192. Dry Measure is used in measuring grain, fruit, vegetables, etc. The principal unit is the bushel, which contains 2150.42 cubic inches.

TABLE OF DRY MEASURE

2 pints (pt.) = **1 quart** (qt.).
8 quarts = **1 peck** (pk.).
4 pecks = **1 bushel** (bu.).

bu. pk. qt. pt.
1 = 4 = 32 = 64.

A dry quart contains 67.2 cubic inches, while a liquid quart contains only 57.75 cubic inches.

ORAL EXERCISES

193. 1. How many quarts in 10 pt.? In 24 pt.?

2. How many gallons in 16 qt.? In 24 qt.? In 26 qt.?

3. What do 4 qt. of milk cost at the rate of 1¢ a gill?

4. At the rate of 32 cents a gallon, how much will a quart of vinegar cost?

5. What will a barrel of apples cost at 50¢ a bushel, if 3 bushels equal a barrel?

6. How much would a man gain by retailing this barrel of apples at 10 cents a half peck?

7. How many pecks in 32 quarts? In 40 quarts?

8. In 5 pecks how many pints? In 15 pecks?

9. How many pecks in 10 bushels? In $7\frac{1}{2}$ bushels?

10. At a stand a man sold half a bushel of peanuts at 5 cents a pint. How much did he get for them?

WRITTEN EXERCISES

194. Reduce:

1. 5 gal. 3 qt. to pints.
2. $\frac{5}{8}$ gal. to quarts.
3. 2 bu. 2 qt. to quarts.
4. $\frac{3}{16}$ bu. to quarts.
5. 16 gal. 1 qt. 1 pt. to pints.
6. $\frac{3}{4}$ gal. to pints.
7. 13 bu. 6 pk. to pecks.
8. .75 pk. to quarts.
9. 1235 quarts to gallons and quarts.
10. 1200 quarts to bushels and pecks.
11. 231 pecks to bushels and pecks.
12. 1645 quarts to pecks and quarts.
13. 303 pints to pecks, quarts, and pints.
14. What part of a gallon is 2 qt. 1 pt.?
15. What part of a gallon is $\frac{2}{3}$ of a quart?
16. If onion sets sell for $1 a half bushel, what is a quart worth?
17. A farmer sold his apple crop at $1.50 a barrel. How many bushels must be gathered to bring him $6000, allowing 3 bushels to the barrel?

MEASURES OF WEIGHT

195. Avoirdupois Weight is used in weighing all coarse and heavy articles, such as cattle, horses, coal, grain, and groceries. It is the common commercial weight.

TABLE OF WEIGHT

16 ounces (oz.) = **1 pound** (lb.).
100 pounds = **1 hundredweight** (cwt.).
20 hundredweight = **1 ton** (T.).

T.	cwt.	lb.	oz.
1	= 20	= 2000	= 32,000.

Troy Weight is used in weighing gold, silver, and jewels. It is not of enough general importance to be taught at this stage, but the table is given for reference.

24 grains = 1 pennyweight.
20 pennyweights = 1 ounce.
12 ounces = 1 pound.

The avoirdupois pound contains 7000 grains; the Troy pound, 5760 grains. In weighing diamonds the unit generally used is the *carat*, which equals 3.2 grains. The word *carat* is also used to indicate the purity of gold, meaning $\frac{1}{24}$ of the mass. Thus, 18 carat gold contains 18 parts gold and 6 parts alloy, or is $\frac{3}{4}$ pure.

In mixing and retailing medicines, druggists use *Apothecaries' Weight*, as follows:

20 grains = 1 scruple.
3 scruples = 1 dram.
8 drams = 1 ounce.
12 ounces = 1 pound.

196. There is an increasing tendency to let the avoirdupois weights take the place of dry measures. For this reason each state has made certain weights of produce the legal equivalent of a bushel. Some of these weights are given on the following page:

LEGAL WEIGHTS OF A BUSHEL

	N. Y.	N. J.	Penn.	Md.	Va.	W. Va.	N. C.	S. C.	Ga.	Fla.	Ala.	Miss.	La.	Tex.	Ark.	Tenn.	Ky.	Mass.	Mo.	Ohio
Barley	48	48	48	—	48	48	48	48	47	48	47	48	—	48	48	48	47	48	48	48
Buckwheat	48	50	48	—	52	52	50	56	52	48	48	—	—	42	52	50	56	48	52	50
Corn (shelled)	58	56	56	—	56	56	54	56	56	56	56	56	56	56	56	56	56	56	56	56
Oats	32	30	32	26	30	32	30	32	32	32	32	32	32	32	32	32	32	32	32	32
Potatoes	60	60	60	56	56	60	—	60	60	60	60	60	—	60	60	60	60	60	60	60
Rye	56	56	56	56	56	56	56	56	56	56	56	56	—	56	56	56	56	56	56	56
Clover seed	60	64	62	—	60	60	64	60	60	60	60	60	—	60	60	60	60	—	60	60
Timothy seed	44	45	—	—	45	45	—	—	45	45	45	45	—	45	60	45	45	45	45	45
Wheat	60	60	60	—	60	60	60	60	60	60	60	60	—	60	60	60	60	60	60	60

A barrel of flour weighs 196 lb.

ORAL EXERCISES

197. **1.** In 6000 lb. how many tons? In 2500 lb.?

2. Find the cost of 1 T. 10 cwt. of clover hay at $12 a ton.

3. What will 2 lb. of nutmegs cost at 3 cents an ounce?

4. At $.16 a pound, what will 3 lb. 4 oz. of cheese cost?

5. What part of a pound is 12 ounces?

6. How many pounds in a car of coal weighing 30 tons?

7. At $25 a ton, what is cottonseed meal worth by the hundredweight?

8. At $22 a ton what is the cost of 3000 pounds of fertilizer?

9. If a barrel of salt weighs 280 pounds, how many hundredweight in 100 barrels?

10. If a bushel of potatoes weighs 60 pounds, what is the weight of 10 barrels each containing 3 bushels?

WRITTEN EXERCISES

198. Reduce:

1. 2 T. 4 cwt. to pounds.
2. 3 cwt. 50 lb. to ounces.
3. 14 cwt. 75 lb. to pounds.
4. 89 cwt. to ounces.
5. $7\frac{3}{8}$ cwt. to ounces.
6. 62.5 lb. to ounces.
7. 480 oz. to pounds.
8. 610 oz. to lb. and oz.
9. 5025 lb. to cwt. and lb.
10. 1000 oz. to pounds.
11. 75,376 oz. to tons, hundredweight, and pounds.
12. What is the cost of 560 lb. of potatoes at 75¢ a bushel? (56 lb. = 1 bu.)
13. When bran is worth $18 a ton, how much must be paid for 2250 lb.?
14. A man bought 1800 lb. of wheat at 88¢ a bushel. How much did the wheat cost?
15. If 10 persons eat $\frac{1}{2}$ lb. of butter at each of the three meals, how many pounds will 100 persons consume in 30 days? What will it cost at 25¢ a pound?

MEASURES OF TIME

199. The time required for the earth to rotate once on its axis is called a *day*, which is the principal unit of time. Other divisions of time are shown in the following:

TABLE OF TIME

60 seconds (sec.)	= **1 minute** (min.).
60 minutes	= **1 hour** (hr.).
24 hours	= **1 day** (da.).
7 days	= **1 week** (wk.).
365 days or **12 months**	= **1 year** (yr.).

da. hr. min. sec.

1 = 24 = 1440 = 86,400.

1. A period of 100 years is called a *century*.

2. In order that the pupil may learn the number of days in each month, it will be well for him to memorize the following:

Thirty days hath September,
April, June, and November,
All the rest have thirty-one,
Except February alone,
To which we twenty-eight assign,
Till leap year gives it twenty-nine.

3. A leap year of 366 days occurs in every year whose date number is divisible by 4, except when this number ends in two ciphers; then it must be divisible by 400. For example, 1600 was a leap year, 1900 was not, 2000 will be.

ORAL EXERCISES

200. 1. How many work days in November, if its first day is Wednesday and one Thursday a holiday?

2. How many school days in the same month?

3. If Jan. 1, 1905, was Sunday, what day of the week was Dec. 31, 1905?

4. What day of the week was Jan. 1, 1906?

5. How many days from Jan. 15 to Feb. 15? From March 15 to April 15? From April 15 to May 15?

6. Was 1800 a leap year? 1804? 1836?

7. How many hours does a man work in a week, if he works from 9 A.M. until 6 P.M. and takes an hour a day for dinner?

8. A man spends 15 minutes in each round trip between his home and his office. If he goes home to dinner every day, how many hours does he consume in this way in six weeks, excluding Sundays?

9. A laborer gets $1 a day working from 7 A.M. until 6 P.M., and is allowed an hour's rest for dinner. How much is due him at the end of a week, if he stops at noon Saturday?

WRITTEN EXERCISES

201. 1. The siege of Port Arthur began May 25, 1904, and ended Jan. 2, 1905. How many days did the siege continue?

2. Jan. 1, 1905, was Sunday. Make a calendar for the first three months of that year.

3. How many days did a vacation last, if one session ended June 3 and the next began Sept. 10? How many weeks?

4. The "Richmond Times-Dispatch" began in 1905 to issue a Monday morning paper, thus making it a daily. How many additional issues were printed during the three months ending Dec. 31? (Dec. 31 was Sunday.)

5. A dynamo makes 975 revolutions in one minute. How many will it make in a six-hour run?

6. A passenger train starts from A, and runs at the rate of 30 miles an hour. How many stops of 3 minutes each are made if the train reaches B at the end of the third hour, the distance from A to B being 75 miles?

7. Calculate the number of school days from Sept. 15 (Monday) to the Christmas holidays, which began on Dec. 20. (Count Sept. 15, and allow one day for Thanksgiving.)

8. A mechanic works from 7.30 A.M. until noon, and from 1 P.M. to 5.30 P.M., at 25¢ an hour. How much does he earn in 4 weeks if he stops work at noon on Saturdays?

ADDITION

202. Compound numbers may be added, subtracted, multiplied, and divided. Some practice is necessary to make these operations understood.

203. **1.** Find the sum of 3 gal. 2 qt. 1 pt., 7 gal. 3 qt., and 8 gal. 1 qt. 1 pt.

(*a*)

gal.	qt.	pt.
3	2	1
7	3	
8	1	1
19	3	0

The sum of the pints is 2 pt., which equal 1 qt. The sum of the quarts is 6 qt., which with the 1 qt. equal 7 qt., or 1 gal. 3 qt. The sum of the gallons is 18 gal., which with the 1 gal. equal 19 gal.

(*b*)

	gal.	qt.	pt.
	3	2	1
	7	3	
	8	1	1
1st row,	18	6	2
2d row,	1	1	
Sum,	19	3	0

In adding large numbers involving several denominations, we may work as in (*b*). Adding each column separately, we get the 1st row, 18 gal. 6 qt. 2 pt.

Now starting at the right, we reduce pints to quarts, and set down in the 2d row the 1 qt. carried over to that denomination. Adding again and reducing, we repeat the process for the several columns, showing the sum in the last row.

2. Find the sum of 5 yd. 2 ft. 6 in., 2 yd. 1 ft. 8 in., and 4 yd. 7 in.

3. Find the sum of 3 bu. 1 pk., 9 bu. 3 pk., and 12 bu. 2 pk.

4. Find the sum of 2 mi. 4 rd., 3 mi. 70 rd., 6 mi. 50 rd., and 1 mi. 215 rd.

5. Add 70 sq. yd. 4 sq. ft. and 89 sq. yd. 20 sq. ft.

6. Find the sum of 2 gal. 3 qt., 4 gal. 2 qt., and 3 gal.

7. Add 15 lb. 12 oz. and 20 lb. 7 oz.

8. Add 5 yr. 6 mo. 9 da. to 8 yr. 7 mo. 27 da.

9. Add 22 hr. 25 min. 2 sec. to 20 hr. 35 min. 57 sec.

SUBTRACTION

204. **1.** Subtract 5 yr. 6 mo. 9 da. from 8 yr. 2 mo. 16 da.

	yr.	mo.	da.
	8	2	16
	5	6	9
Changing to —			
	7	14	16
Subtracting,	5	6	9
	2	8	7

Since we cannot subtract 6 mo. from 2 mo., we add 1 of the 8 yr. to 2 mo., giving the minuend as shown in the second form. The subtraction is then readily performed.

In practice the change in the form of the minuend should be made mentally, and the work completed without rewriting the numbers.

2. Subtract 2 mi. 155 rd. from 5 mi. 150 rd.

3. Subtract 20 da. 2 hr. from 30 da.

4. Subtract 20 A. 20 sq. rd. from 50 A.

5. Subtract 5 bu. 2 pk. from 18 bu. 1 pk.

6. Subtract 60 lb. 14 oz. from 325 lb. 8 oz.

7. Subtract 30 ft. 9 in. from 56 ft. 2 in.

8. From a barrel containing 52 gal. of molasses, a dealer sold 20 gal. 2 qt. How much remained?

9. Find the time from June 16, 1905, to September 4, 1907.

Write June 16, 1905, as 1905 yr. 6 mo. 16 da.

10. How long was it from May 10, 1900, to July 2, 1906?

MULTIPLICATION

205. **1.** Multiply 6 gal. 3 qt. 1 pt. by 5.

gal.	qt.	pt.
6	3	1
		5
34	1	1

5×1 pt. $= 5$ pt., or 2 qt. 1 pt.

5×3 qt. $= 15$ qt.; 15 qt. $+ 2$ qt. $= 17$ qt., or 4 gal. 1 qt.

5×6 gal. $= 30$ gal.; 30 gal. $+ 4$ gal. $= 34$ gal.

In practice the work is conveniently arranged as shown at the left.

Multiply:

2. 3 yd. 2 ft. 2 in. by 7.
4. 6 bu. 3 pk. by 6.
3. 17 hr. 4 min. by 15.
5. 1 gal. 1 pt. by 8.
6. 16 sq. yd. 20 sq. ft. by 9.
7. 2 cu. yd. 15 cu. ft. by 150.
8. 27 rd. 4 ft. 7 in. by 8.
9. A boy gathered 5 bags of walnuts, each containing 2 bu. 1 pk. How many did he gather?
10. If the average weight of 12 boys is 90 lb. 6 oz., what is their total weight?

DIVISION

206. 1. Divide 34 gal. 1 qt. 1 pt. into 5 equal parts.

gal.	qt.	pt.
5)34	1	1
6	3	1

$\frac{1}{5}$ of 34 gal. = 6 gal., and 4 gal. (16 qt.) remaining. 16 qt. + 1 qt. = 17 qt.

$\frac{1}{5}$ of 17 qt. = 3 qt., and 2 qt. (4 pt.) remaining. 4 pt. + 1 pt. = 5 pt.

$\frac{1}{5}$ of 5 pt. = 1 pt.

In practice the work is arranged as shown at the left.

Divide:

2. 11 T. 12 cwt. into 8 equal parts.
3. 5 yd. 2 ft. 6 in. by 6.
4. 27 da. 4 hr. by 5.
5. How many pieces 2 ft. 4 in. long can be sawed from a log 16 ft. 4 in long?

Before dividing, reduce both numbers to the same denomination.

6. The distance around a square room is 65 ft. 4 in. What is the length of one side?
7. The total weight of the members of a class was 1278 lb. 12 oz. If their average weight was 85 lb. 4 oz., how many members were in the class?

REVIEW WORK

WRITTEN EXERCISES

207. 1. Reduce 280 rd. to inches.

2. Reduce 300 pt. to gallons.

3. Reduce 267 pt. to bushels, quarts, and pints.

4. Reduce 24 sq. rd. 6 sq. ft. to square feet.

5. Reduce 25 cu. yd. 15 cu. ft. to cubic inches.

6. Reduce 6425 lb. to tons and pounds.

7. Reduce 6600 sec. to hours and minutes.

8. Reduce $\frac{5}{8}$ in. to the fraction of a yard.

9. Multiply 8 lb. 10 oz. by 15.

10. Divide 76 yd. 1 in. by 17.

11. A butcher cuts 30 lb. of beef into roasts weighing 7 lb. 8 oz. each. How many roasts does he make?

12. A hall 20 ft. by 30 ft. is paved with tiles 4 in. by 16 in. How many tiles are used?

13. A dairyman delivered $4\frac{1}{2}$ gal. of cream at 15¢ a pint. What did it bring?

14. A market gardener packed $3\frac{1}{2}$ bu. of cherries in quart boxes. How many boxes did he use?

15. A teamster feeds to each horse 9 qt. of oats a day. How many bushels must he lay in to supply a four-horse team 3 days?

16. If you sleep 8 hours each night, how many days in duration do you sleep in the month of September?

17. There are 39 children and one teacher in a schoolroom whose dimensions are 30 ft. long, 20 ft. wide, and 12 ft. high. How much floor space does this allow for each person? How many cubic feet of space?

18. A family consumes 49 lb. of meat in 2 wk. Allowing $\frac{1}{2}$ lb. a day for each member, how many were there in the family?

19. A man bought a rectangular field 20 rd. by 80 rd. How many acres did it contain? What is the cost of inclosing it with a wire fence at 60¢ a rod?

20. If barbed wire runs a rod to a pound, and the fence around this field (Example 19) is 5 strands high, how many feet of wire are used in fencing it? How many pounds?

21. How many acres in a square field 40 rd. on a side? How many rods of woven wire fencing will be required to inclose it?

22. At the rate of 300 lb. an acre, how many tons of fertilizer will be required for a field 40 rd. by 60 rd.?

23. If a wheel is 7 ft. 4 in. in circumference, how many times will it turn in going $\frac{1}{2}$ of a mile?

24. A dealer has a coal bin 15 ft. long, 12 ft. wide, and 8 ft. high. How many tons of coal does it hold, allowing 35 cu. ft. to the ton?

25. An empty freight car weighs 15 tons. Loaded with wheat it weighs 33 tons. How many bushels of wheat does it contain?

26. A cedar block is 12 in. square at the end, and 10 ft. long. What is its weight, allowing 38 lb. to the cubic foot?

27. Florida oranges are usually shipped in boxes 12 in. by 12 in. by $26\frac{5}{8}$ in. How many cubic inches more than a bushel does such a box contain?

RELATED PROBLEMS

208. **1.** The picture represents a cotton field in picking time. If the field is 80 rd. by 100 rd., and averages 1200 lb. of seed cotton to the acre, what is the total yield in seed cotton? (The product is called seed cotton until it is ginned; then it is called lint.)

2. If each picker averages 150 lb. a day, how many laborers are needed to pick this cotton in 80 working days?

3. How many bales of 500 lb. each (lint) will this make? (Three pounds of seed cotton yield on an average 1 pound of lint and 2 pounds of seed.)

4. Find the cost of picking at 50¢ per 100 lb. (seed cotton).

5. At $10\frac{1}{2}$¢ a pound (lint), what will this crop bring?

6. If the cost of planting, cultivating, picking, ginning, and marketing the crop was $900, and storage and insurance 50¢ a bale, what are the net profits of the planter? How much per acre?

7. How many tons of seed come from this crop? What is the value of the seed at $12.50 a ton?

8. In a certain year, Texas produced 2,446,000 bales of cotton ; Georgia, 1,405,000 bales ; Mississippi, 1,385,000 bales ; Alabama, 1,040,000 bales ; Arkansas, 855,000 bales ; South Carolina, 845,000 bales ; and Louisiana, 832,000 bales. What was the total value of this cotton at 10 cents a pound, counting 500 lb. to the bale ?

9. In a certain year, 23,000,000 American spindles used 3,910,000 bales of cotton, averaging 500 lb. each. What was the average number of pounds to the spindle ?

10. This picture represents vessels at a wharf, ready to load with cotton. If one of the vessels takes on 3000 bales, averaging 492 lb. to the bale, how many tons will its cargo weigh ?

11. In fifty years the number of bales produced annually in this country increased from 2,930,000 bales to 10,124,000 bales. What was the average annual increase ?

12. In a certain year the total cotton crop of the United States was 10,002,000 bales, of which 6,032,000 bales were exported. How many bales were consumed in this country ?

13. What did it cost to compress this exported cotton at 40 cents a bale?

14. In a certain year southern mills had 8,210,000 spindles. If these mills took that year 2,140,000 bales, aggregating 1,067,300,000 lb., what was the average number of pounds to a spindle?

15. A Carolina mill consumes each working day 10 bales of 500 lb. each. Allowing 9 days for repairs and holidays, how many pounds does it consume in a common year? If each spindle consumes 125 lb. a year, what was the number of spindles in the mill?

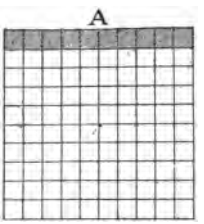
16. In a recent year southern mills consumed 2,140,000 bales, while northern mills consumed 2,029,000 bales. At $50 a bale, what was the difference in value between cotton taken by southern and northern mills?

17. What was the freight on the cotton taken by northern mills that year, if the bales averaged 489 lb. each, and the average freight rate was 55¢ per hundred pounds?

18. North Carolina manufactures a larger proportion of its cotton crop than any other cotton-growing state. In a certain year it consumed 602,400 bales, which was $\frac{6}{7}$ of the crop. How many bales were produced in North Carolina that year?

19. If the raw material costs a spinner 10¢ a pound net, and he sells his yarns at 16¢ a pound, how many pounds must he manufacture in a month in order to pay running expenses of $5220?

DECIMAL FRACTIONS



209. 1. What part of A is shaded? Into how many parts is each strip divided?

What part of each strip is one of the small squares? What part of the large square?

Then $\frac{1}{10}$ of one tenth of A is what part of A?

2. When anything is divided into ten equal parts, what is each part called?

When each of these ten parts is divided into ten equal parts, how many parts are there?

When each of these hundred parts is divided into ten equal parts, how many parts are there?

3. When anything is divided into tenths, hundredths, thousandths, etc., the parts are called *decimal parts*.

210. You have already learned that 10 hundredths make 1 tenth, and that 10 tenths make 1 one, just as 10 ones make 1 ten, 10 tens 1 hundred, and so on. You have also learned that tenths and hundredths may be written with the decimal point and without denominators. Such fractions are called **Decimals** or **Decimal Fractions.**

211. 1. In the number 12,345.6789, where are the tens and tenths written with reference to the ones? Hundreds and hundredths? In what place is the 8? What does it express?

2. What decimal place is occupied by tenths? By hundredths? By thousandths?

3. What occupies the second decimal place? The third? The fourth?

212. An integer and a decimal, written together as one number, is called a **Mixed Number.**

In writing mixed numbers the decimal point is placed between the integral part and the fractional part; thus, 2.05. What word should be read between these parts?

READING DECIMALS

213. In Articles 86 and 89 you have learned to read decimals to the hundredths' place. We shall now study further divisions of the whole into thousandths, ten-thousandths, etc.

.325 is read 325 thousandths.

716.0034 is read 716 and 34 ten-thousandths.

Read the decimal as a whole number, and give it the name of the right-hand order.

214. Read the following:

1. .8.	**4.** .07.	**7.** .610.	**10.** .1235.
2. .17.	**5.** .66.	**8.** .032.	**11.** 375.35.
3. .48.	**6.** .125.	**9.** .807.	**12.** 81.065.

215. Since *per cent* means *hundredths*, in reading decimals we may say per cent instead of hundredths. The symbol % means either per cent or hundredths. Thus,

.10 = 10 per cent = 10 %. .65 = 65 per cent = 65 %.

.25 = 25 per cent = 25 %. .12½ = 12½ per cent = 12½ %.

.50 = 50 per cent = 50 %. .05 = 5 per cent = 5 %.

Read the following:

1. 3.25.	6. 80.203.	11. 236.004.
2. 51.17.	7. 80.005.	12. 7000.005.
3. 13.03.	8. 35.075.	13. 3150.055.
4. 215.09.	9. 38.267.	14. 27,105.218.
5. 17.294.	10. 307.016.	15. 22,345.1234.

Read in two ways:

16. .35.	19. .01.	22. .37½.	25. 20%.
17. .17.	20. .05.	23. .66⅔.	26. 45%.
18. .40.	21. .75.	24. .62½.	27. 33⅓%.

WRITING DECIMALS

216. 1. Write 7 thousandths as a decimal.

Thousandths occupy the third place, hence we write the 7 and put ciphers in the first two decimal places; thus, .007.

2. Express decimally thirty-five ten-thousandths.

Ten-thousandths is the fourth decimal order, hence 4 places are needed to express the decimal. But 35 written as an integer occupies only 2 places, leaving 2 places on the left to be filled with ciphers. Hence the decimal is written .0035.

Pupils should become familiar with the names of the decimal orders to millionths.

Express decimally:

3. Five tenths. Six tenths. Seven tenths. Eight tenths.

4. Four hundredths. Fifty-six hundredths. Twenty-two hundredths. 6%. 8 per cent. 16⅔%.

5. Seven thousandths. Sixteen thousandths. Two hundred four thousandths. Twenty-five ten-thousandths. Five ten-thousandths. Fifty-five ten-thousandths.

6. Nine and nine hundredths. Two hundred-thousandths. One and six thousandths. Sixteen millionths. Four hundred-thousandths.

7. Write in words: .5; .05; .08; .015; .213; .25; .325; 8%; .506; .0025; .0215; .2415; .0052; .0012.

217. Annexing Ciphers to Decimals.

1. Express three tenths decimally. Express it as a common fraction. $.3 = \frac{3}{10}$.

2. Just as $\frac{3}{10} = \frac{30}{100} = \frac{300}{1000} = \frac{3000}{10000}$,

so $.3 = .30 = .300 = .3000$.

From this it is evident that —

Annexing ciphers to a decimal does not change its value.

218. Change the following numbers to millionths, and read the results; then begin at the right, cancel one cipher at a time, and read the successive results:

.8. .75. .023. .380. .1275. .17056.

219. To change Decimals to Common Fractions.

1. Change .75 to a common fraction.

$.75 = \frac{75}{100} = \frac{3}{4}$. .75 expressed as a common fraction is $\frac{75}{100}$, which reduced to its smallest denominator is $\frac{3}{4}$.

If the decimal is read aloud, the denominator is apparent. In every case, it is 1 followed by as many ciphers as there are decimal places.

Change to common fractions:

2. .35.	6. .37.	10. .375.	14. .0125.
3. .45.	7. .42.	11. .425.	15. .0375.
4. .85.	8. .38.	12. .325.	16. .8750.
5. .95.	9. .55.	13. .625.	17. .0575.

220. To change Common Fractions to Decimals.

1. How many tenths in 6 twentieths? In $\frac{8}{20}$? In $\frac{10}{20}$?
2. How many tenths in $\frac{1}{5}$? In $\frac{2}{5}$? In $\frac{4}{5}$?
3. How many hundredths in $\frac{3}{20}$? In $\frac{7}{20}$? In $\frac{4}{25}$?

221. To change a common fraction to a decimal is to change it to larger or smaller denominators, and to express it in the form peculiar to decimals.

Thus, $\frac{1}{2} = \frac{5}{10} = .5$; $\frac{3}{4} = \frac{75}{100} = .75$; $\frac{4}{200} = \frac{2}{100} = .02$.

1. Change $\frac{5}{8}$ to a decimal fraction.

Since the numerator may be considered as having a decimal point after it, we may annex ciphers, and write the fraction as $\frac{5.000}{8}$. Dividing the numerator by the denominator, as in changing fractional forms to whole or mixed numbers, and pointing off as in division of decimals, Art. 100, we get .625 as the required decimal.

$$\frac{5}{8} = \frac{5.000}{8} = .625.$$

2. Change $\frac{38}{200}$ to a decimal fraction.

$$(a) \quad \frac{38}{200} = \frac{19}{100} = .19$$

$$(b) \quad 200\overline{)38.00}\ .19$$

In many cases, changing a fraction to a smaller (or larger) denominator changes it to a decimal, as in (*a*). This example may also be solved as in (*b*).

Therefore, to change a common fraction to a decimal, *write a decimal point after the numerator, and divide the numerator by the denominator, annexing ciphers as necessary.*

When the division will not terminate, the remainder may be expressed as a common fraction, or the sign + may be placed after the decimal figures to show that the division is not complete. Thus, $\frac{2}{3} = .66\frac{2}{3}$, or .66+.

Change to decimals:

3. $\frac{1}{4}$.	7. $\frac{5}{6}$.	11. $\frac{22}{25}$.	15. $\frac{7}{8}$.	19. $\frac{22}{55}$.
4. $\frac{2}{5}$.	8. $\frac{3}{20}$.	12. $\frac{11}{20}$.	16. $\frac{11}{12}$.	20. $\frac{13}{20}$.
5. $\frac{3}{8}$.	9. $\frac{9}{25}$.	13. $\frac{5}{12}$.	17. $\frac{7}{12}$.	21. $\frac{3}{40}$.
6. $\frac{3}{4}$.	10. $\frac{9}{20}$.	14. $\frac{1}{3}$.	18. $\frac{18}{25}$.	22. $\frac{3}{50}$.

ADDITION AND SUBTRACTION

222. In addition and subtraction of decimals the operations are the same as the like operations in integral numbers.

1. What is the sum of .625, .238, .21, and .5555?

.625
.238
.21
.5555
1.6285

How do we make decimals of the same order stand in the same vertical column? Where do we place the decimal point in the sum?

2. From .35 subtract .1325.

(*a*)	(*b*)
.35	.3500
.1325	.1325
.2175	.2175

Why may .35 be written as in (*b*)? Is it necessary to annex ciphers to the minuend?

Add:

3.	4.	5.	6.
$142.35	33.25	$17.12	.047
25.07	16.095	15.74	.293
45.45	4.317	8.64	.07
11.08	18.94	23.08	.546
60.24	6.105	16.74	.008

Subtract:

7.	8.	9.	10.	11.
$27.21	52.65	$17.15	.888	.0127
13.62	38.33	9.47	.699	.0089

12.	13.	14.	15.	16.
$12.75	5.12	$16.3	.3071	.4823
10.83	3.85	12.14	.0545	.2348

Find the value of:

17. $472.5 + 3.24 + 5.17$.
18. $61.5 + 12.78 + .018$.
19. $56.9 + 141.3 + .842$.
20. $15.34 + .544 + 53.4$.
21. $9.016 + .169 + .028$.
22. $7.13 + .310 + 5.65$.
23. $.866 + .721 - .136$.
24. $9.78 + .812 - .753$.
25. $4.62 + 34.2 - .916$.
26. $.746 + 8.63 - 7.28$.
27. $10.27 - 3.48 + .784$.
28. $8.175 + 91.54 - 7.96$.

29. Jake paid, on the first day of school, \$1.25 for a geography, \$.54 for an arithmetic, \$.60 for a history, \$.10 for pencils, and \$.25 for paper. What was the total expenditure?

30. Four places are in a straight line. It is 15.4 mi. from A to B, 13.7 mi. from B to C, and 45.9 mi. from A to D. How far is it from C to D? (Draw a diagram.)

31. A man owes \$1250 to A, and \$1500.50 to B. He has in cash \$1875.75. If he pays A in full, how much will he lack of having enough to pay B?

32. In a bin that will hold 270.5 bu. of wheat there are 129.25 bushels. How many bushels are required to fill it?

33. A 750-gallon tank contains 250.5 gal. of oil. If 125.25 gal. are added, how much oil is still needed to fill the tank?

34. A man bought a horse for \$175 and a carriage for \$135.50. For what must he sell both to gain \$40?

35. A teacher is employed for \$675 for a term of 9 months. Her expenses for the entire 12 months are as follows: board, \$120; laundry, \$15; books, \$25; periodicals, \$8; postage and stationery, \$18; sundries, \$30; traveling expenses, \$11; and cost of attending summer normal, \$23. How much does she save in a year?

MULTIPLICATION AND DIVISION

223. We have already seen that multiplication and division of decimals are the same as the like operations in integers, the placing of the decimal point alone requiring special attention.

224. 1. How many tenths are 5×6 tenths? 30 tenths are equal to how many ones? 30 tenths $= 3$.

2. How many tenths are 5×9 tenths? In 45 tenths there are how many ones and tenths? 45 tenths $= 4.5$.

225. Multiply 4.2 by 6.

$$\begin{array}{r} 4.2 \\ 6 \\ \hline 25.2 \end{array}$$

6 × 2 tenths = 12 tenths, or 1.2. Where do we write the 2? What is done with the 1? How many decimal places are there in the factors? How many in the product?

226. 1. What is $\frac{1}{3}$ of 9 tenths? $\frac{1}{3}$ of $.9 =$

2. What is $\frac{1}{4}$ of 24 hundredths? $\frac{1}{4}$ of $.24 =$

3. What is $\frac{1}{2}$ of 18 tenths? $\frac{1}{2}$ of $1.8 =$

227. Divide 6.24 by 4.

$$4\overline{)6.24} \quad 1.56$$

$\frac{1}{4}$ of 6 ones = 1 one, and 2 ones remaining. 2 ones and 2 tenths = 22 tenths.

$\frac{1}{4}$ of 22 tenths = 5 tenths, and 2 tenths remaining. 2 tenths and 4 hundredths = 24 hundredths.

$\frac{1}{4}$ of 24 hundredths = 6 hundredths.

228. Find the products:

1. 3×5.7.	6. $2 \times .36$.	11. 3×1.24.
2. 5×2.6.	7. $6 \times .24$.	12. 5×4.35.
3. 6×6.6.	8. $9 \times .25$.	13. 7×6.73.
4. 7×1.9.	9. $7 \times .34$.	14. $12 \times .64$.
5. 8×4.5.	10. $8 \times .09$.	15. 11×2.3.

Find the quotients:

16. $24.3 \div 3$.	**20.** $4.72 \div 4$.	**24.** $64.8 \div 9$.
17. $33.6 \div 4$.	**21.** $13.5 \div 3$.	**25.** $53.6 \div 8$.
18. $65.1 \div 7$.	**22.** $51.6 \div 4$.	**26.** $17.64 \div 4$.
19. $64.5 \div 5$.	**23.** $44.8 \div 7$.	**27.** $14.52 \div 6$.

229. 1. What is the product of $\frac{1}{2}$ and $\frac{3}{5}$? Of $\frac{1}{10}$ and $\frac{1}{10}$? $\frac{1}{2} \times \frac{3}{5} = —$; $\frac{1}{10} \times \frac{1}{10} = —$.

2. What is the product of $\frac{3}{10}$ and $\frac{3}{10}$? Of .3 and .3? Then what is the product of tenths by tenths?

3. Compare:

$$\frac{3}{10} \times \frac{9}{10} = \frac{27}{100}, \text{ and } \frac{2}{10} \times \frac{7}{100} = \frac{14}{1000},$$

with $\quad .3 \times .9 = .27$, and $.2 \times .07 = .014$.

230. 1. Multiply .36 by .7.

(a)	(b)
36	.36
7	.7
252	.252

Multiplying without noticing the points, the result is 252, as shown in (a). And since hundredths multiplied by tenths equal thousandths, the required product is .252, as in (b). This is also seen from $\frac{36}{100} \times \frac{7}{10} = \frac{252}{1000} = .252$.

How many decimal places are there in the factors? How many have we in the product?

2. Multiply .036 by .27.

.036	$27 \times 36 = 972$.
.27	
.00972	$\frac{27}{100} \times \frac{36}{1000} = \frac{972}{100000}$.

What is the product of thousandths by hundredths? Then what is the product of $\frac{27}{100}$ and $\frac{36}{1000}$? How many decimal places have both factors? What must we do to make five places?

Restate in your own words the principle involved in multiplication of decimals, and compare with the statement given in Art. 97.

231. 1. Compare:

$10 \div 5 = 2$,	and	$100 \div 50 = 2$;
$\$3.50 \div 5 = \$.70$,	and	$\$350 \div 500 = \$.70$;
$240 \div 4 = 60$,	and	$24 \div .4 = 60$.

2. In $15 \div .3$, if we multiply both dividend and divisor by 10, we get $150 \div 3$. Does this change the value of the quotient? Compare the quotient of $150 \div 3$ with that of $15 \div .3$. State your conclusion.

Both dividend and divisor may be multiplied by 10, 100, 1000, etc., without changing the quotient.

232. Divide .00918 by .34.

$$\begin{array}{r} .027 \\ 34\overline{).918} \\ 68 \\ \hline 238 \\ 238 \\ \hline \end{array}$$

Since $.00918 \div .34 = .918 \div 34$, the work may be performed as indicated.

By what number were both dividend and divisor multiplied? How many places to the right did that move the decimal point in each?

Restate in your own words the principle involved in the division of decimals, and compare with its statement as found in Art. 100.

In the following pages, when the division is not exact, the work is carried to three decimal places.

233. Multiply:

1. 48.1×32.5.
2. $.462 \times .21$.
3. $.735 \times 500$.
4. $3.06 \times .065$.
5. 2.626×2.6.
6. $.873 \times 1.72$.
7. 8.42×2.6.
8. $\$32 \times 1.6$.
9. 3.46×2.008.
10. $.00202 \times .047$.

Divide, stating in each case by what number the dividend and divisor are multiplied to make the divisor an integer:

11. $4.62 \div 2.2$.
12. $625 \div .25$.
13. $27.63 \div .003$.
14. $69.76 \div .032$.
15. $865.8 \div 1.11$.
16. $.876 \div 1.46$.
17. $\$6.25 \div \$.125$.
18. $2458.624 \div 35.84$.
19. $9.5676 \div 3.57$.
20. $.001728 \div 1440$.

Multiply or divide as indicated:

21. $382.1 \div .125$.
22. $21.20 \div .625$.
23. $4.8 \times .375$.
24. $4.8 \times .125$.
25. $68.752 \times .75$.
26. $273.85 \div 47.825$.
27. $68.97 \div 3.25$.
28. 22.65×62.5.
29. 21.37×87.5.
30. $214.097 \div 82.345$.

31. A farmer picked 135.5 bu. of apples. He sold .25 of them to one man and .75 of them to another. How many did he sell to each?

32. Find the cost of 15.75 cords of wood at \$2.40 a cord.

33. A man sold .4 of his crop of 1250 bu. of wheat. At \$1.10 a bushel, what did the part sold bring?

34. I bought a phonograph for \$60. After using it 4 mo., I sold it for .75 of its cost. What did it bring?

35. Find the cost of 37.25 bu. of oats at .42 of a dollar a bushel.

36. At \$.34 a bushel, how many barrels of apples can be bought for \$27.20, allowing 2.5 bu. to the barrel?

37. How many pounds of butter can be made from 12 cows in 30 days, if each cow averages 3.5 gal. of milk a day, and each gallon makes .4 lb. of butter?

38. If 35.84 cubic feet of a liquid weigh a ton, how many tons will 537.6 cubic feet weigh?

39. If a bushel of oysters in the shell yields 1.25 gal., how many bushels of shell oysters must be opened to fill three 10-gallon cans?

40. A bushel of potatoes weighs 56 pounds. How much would the potatoes raised on a 5-acre lot weigh, the average yield being 185.5 bu. per acre?

41. One sixteenth of an acre in strawberries yielded 300 qt. of berries, which brought 10¢ a quart. If fertilizer cost \$2.50, labor \$5, and boxes \$1, what was the profit?

42. The cost of raising a wheat crop was as follows: 12 days' plowing at \$2.50 per day, 4 days' harrowing at \$2.50 a day, 5 days' seeding at \$3.50 a day, 20 bu. of seed wheat at \$1 a bushel, 2 tons of fertilizer at \$18 a ton, harvesting the crop \$12, and threshing \$15. The crop yielded 460 bushels. If wheat was worth \$.95 a bushel, how much more was the wheat worth than it cost the farmer to raise it?

43. What is the total cost of 61,900 lb. of steam coal at \$1.05 a ton at the mines, if freight costs \$1.60 a ton additional?

44. A sack of coffee weighing 125 lb. is invoiced at $16\frac{2}{3}$¢ a pound. The freight is 35¢ a hundred pounds. What is the total cost, including 25¢ drayage?

45. Find the cost of 1500 feet of flooring at \$32 a thousand feet.

46. The expenses of a student at college were as follows:

Board, nine months, at	\$14.50 a month
Books and stationery,	18.75
Tuition,	75.00
Traveling expenses,	12.45
Incidentals,	27.00

How much did he have left from an allowance of \$350 for a 9 months' session? What were his average monthly expenses?

BILLS AND RECEIPTS

234. Mr. Richard Roe bought of John Doe & Co. the following articles: 2 dozen tablets at $.30 a dozen, 5 gross of pens at $.65 a gross, 25 geographies at $.80 each, and 30 arithmetics at $.54 each. The bill read:

JACKSON, MISS., May 1, 1906.

MR. RICHARD ROE,

Bought of JOHN DOE & CO.

2 doz. tablets	@ $.30		60
5 gross pens	@ .65	3	25
25 geographies	@ .80	20	
30 arithmetics	@ .54	16	20
		$40	05

When this bill was paid, it was receipted thus:

"*Received payment,*

JOHN DOE & CO."

235. A **Debt** is the amount which one person owes another. A **Debtor** (Dr.) is a person that owes a debt.

236. A **Credit** is the amount paid on a debt. A **Creditor** (Cr.) is a person to whom a debt is due.

In the transaction mentioned above, after the goods are delivered and before the bill is paid, who is the debtor? Who is the creditor?

237. A **Bill** is a creditor's written statement of the quantity and price of each item in his account with a debtor, together with the credits and the net amount due. Bills are sometimes called *invoices*.

An item like "5 books @ $.60" means "$.60 *each*"; an item like "3 doz. eggs @ $.20" means "$.20 a *dozen*."

WRITTEN EXERCISES

238. **1.** Mrs. R. D. White bought the following from Russell & Russell, Raleigh, N.C.:

18 yd. Scotch Gingham	@ 21¢,
10 yd. Persian Lawn	@ 12½¢,
12¼ yd. India Silk	@ 48¢,
25 yd. Cashmere	@ $1.25.

Make out her bill and receipt it.

Make out and receipt the following bills, supplying dates and names as needed:

2. Mr. W. F. Kyle bought of Anderson & Co. 25 lb. of coffee at 18¢ a pound, 76 lb. of sugar at 5½¢ a pound, and 20 lb. of prunes at 8¢ a pound.

3. Mrs. R. B. Jones bought of Scott & Co. 12 yards of taffeta silk at $1.25 a yard, 16 yards of lawn at 8 cents a yard, 15 yards of flannel at 45 cents a yard, and 3 pairs of shoes at $2.75 a pair.

4. Bill of groceries: 10 lb. granulated sugar @ 5½¢, 10 lb. butter salt @ 1¢, 6 1-pound cartons seeded raisins @ 7½¢, 2 4-ounce bottles vanilla extract @ 25¢, 2 lb. chocolate @ 25¢, 3 lb. baking powder @ 20¢, 5 lb. Java coffee @ 28¢.

5. Bill of dry goods: 16 yd. velvet @ $2.25, 12½ yd. suiting @ 48¢, 14 yd. sheeting @ 15¢, 6½ yd. dimity @ 24¢, 7⅛ yd. ribbon @ 28¢.

6. Bill of groceries: 4 lb. prunes @ 12½¢, 6 lb. apricots @ 12½¢, 3½ lb. raisins @ 16¢, 2¼ lb. figs @ 20¢, 3 qt. currants @ 8¢, 1¾ lb. cheese @ 16¢, 1½ lb. English walnuts @ 18¢.

7. Bill for:

2 boxes sapolio @ $2.25,
1 box soap @ $3.10,
100 lb. copperas @ 2¢,
3 cases washing powder @ $3,
64 packages laundry starch @ 5½¢.

8. J. R. Rucker bought of Payne & Co:

900 fire bricks @ $30 a thousand,
4 barrels fire clay @ $1.90,
6 barrels cement @ $1.95.

RELATED PROBLEMS

239. 1. If it costs $1 to transport 1000 lb. of wheat from St. Louis to New Orleans by water, how much does it cost at the same rate to transport 124.5 tons?

2. In a certain year it was estimated that there were 17,572,000 milch cows in this country. If the average amount of butter produced annually by each cow was 110 pounds, what was the value of the butter at 20¢ a pound?

3. The wheat crop of the United States in a certain year was 638,000,000 bu. If this country consumed $\frac{4}{5}$ of this product, and the remainder was exported, how much was exported?

4. If 516,600,000 bu. of wheat were consumed by the people of this country in one year, how many bushels were consumed *per capita*, taking the population at 82,000,000?

5. The production of rice in the rice-producing states of this country in a certain year was 3.2 lb. to each inhabitant of the whole country. If the population at that time was 76,303,000, what was the total number of pounds produced?

6. Of the $1,435,179,000 worth of exports from the United States in a certain year, .6 of the total were farm products. What was the value of the farm products exported that year?

7. If we sold in one year to Great Britain $37,800,000 worth of breadstuffs, $\frac{1}{4}$ as much to Holland and Belgium, $\frac{1}{5}$ as much to Germany, and $\frac{1}{10}$ as much to France as to Germany, what was the total value of the breadstuffs sold to these countries?

8. If New York City uses 1000 tons of hay a day, how many tons will it use in 1910 at that rate? How many car loads of 20 tons each will be used?

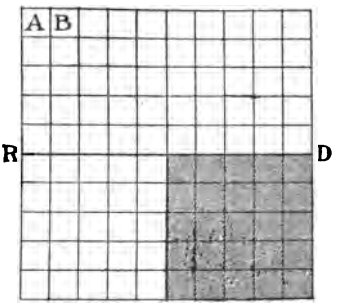
9. In a recent year 26,882,400 tons of bituminous coal were mined in West Virginia. If the value of this coal at the mines was $1.05 a ton, what was the total value? How many 40-ton car loads would the total product make?

10. A dealer buys 540 barrels of apples, paying $2.40 per barrel, and 30 cents per hundredweight freight. Allowing 135 lb. to the barrel, what did the shipment cost him delivered at his depot?

11. If the average annual consumption of sugar is 75.3 lb. *per capita*, how many pounds would be required to supply a population of 150,000 for a year?

12. A certain city handled in a season 150 car loads of oranges, averaging 300 boxes to the car. If each box contained 200 oranges, how many oranges were handled? What were they worth at $2.25 a box?

PERCENTAGE



240. 1. This square is divided into 100 equal parts. Each small square is what part of the large square?

2. What part of the square is $A + B$? How many hundredths?

3. Each strip is what part of the square? How many hundredths of the square are there in each strip?

241. Instead of hundredths we may use the term *per cent*, because per cent means *hundredths*.

What per cent of the square is A? $A + B$? How many per cent are there in the square? How many hundredths make a whole?

242. One hundred per cent of a number is the number itself. Thus, 100 per cent of \$50 is \$50.

243. 1. What per cent of the square is above the line RD? What part of the square is 50 per cent of it?

2. What part of the square is shaded? What per cent of it is shaded?

244. The symbol % stands for the term *per cent*, and means either *per cent* or *hundredths*. Thus, 4% of the square means 4 per cent, or 4 hundredths of it.

1. Since per cent is so many hundredths, how may we express *per cent?*

2. Compare the values of 20%, .20, and $\frac{1}{5}$ of anything.

3. Express 10 per cent in three ways. 5 per cent in three ways.

245. 1. Draw a rectangle and divide it so as to show 50% of it. 25% of it. 75% of it.

2. Draw a line of any convenient length, and another line 50% as long. Draw a line 100% as long.

3. What part of a number is 10 hundredths of it? 10% of it?

4. What per cent of a number is $\frac{1}{10}$ of it? $\frac{1}{2}$ of it? All of it?

5. How do you find 1 hundredth, or 1%, of a number? What is the effect of changing the decimal point two places to the left?

6. When you have found 1% of a number, how do you find 5% of it? 8% of it?

7. Find 1% of \$300. Of \$600. Then find 5% of \$300. Of \$600.

246. Note carefully the following:

100% of a number = all of it.
50% of a number = $\frac{1}{2}$ of it.
$33\frac{1}{3}$% of a number = $\frac{1}{3}$ of it.
25% of a number = $\frac{1}{4}$ of it.
20% of a number = $\frac{1}{5}$ of it.
$16\frac{2}{3}$% of a number = $\frac{1}{6}$ of it.
$12\frac{1}{2}$% of a number = $\frac{1}{8}$ of it.
10% of a number = $\frac{1}{10}$ of it.
$8\frac{1}{3}$% of a number = $\frac{1}{12}$ of it.

247. The number of which some per cent is taken is called the **Base.**

248. The per cent taken is called the **Rate.** Thus, in 6% of 50 = 3, the 6% is the rate. The 6 alone is sometimes called the *rate per cent.*

249. The result obtained by taking a certain per cent of a number is called the **Percentage.** Observe that —

The percentage is the product of the base and the rate.

ORAL EXERCISES

250. 1. How much is 50% of \$5? Of 4 ft.? Of 2 gal.?

2. What is 25% of 36 in.? Of 18 ft.? Of 12 ct.?

3. How much is $33\frac{1}{3}\%$ of 60 ct.? Of 9 yd.? Of 6 bu.?

What is —

4. 1% of 30?
5. 10% of \$30?
6. $12\frac{1}{2}\%$ of 64?
7. $16\frac{2}{3}\%$ of 72?
8. $8\frac{1}{3}\%$ of 84?
9. 30% of \$120?
10. 5% of \$90?
11. 20% of \$50?
12. 50% of 100 quarts?
13. 75% of 120 sheep?
14. $33\frac{1}{3}\%$ of 24 yards?
15. 100% of \$100?

WRITTEN EXERCISES

251. 1. Find $3\frac{1}{2}\%$ of \$500.

(*a*)

1% of \$500 = \$5.00.

$3\frac{1}{2}\%$ of \$500 = $3\frac{1}{2}$ × \$5, or \$17.50.

(*b*)

\$500
$.03\frac{1}{2}$
1500
250
\$17.50

The pupil should apply the method best suited to each example.

Find:

2. $2\frac{1}{4}\%$ of \$1600.
3. $12\frac{1}{2}\%$ of \$720.
4. $16\frac{2}{3}\%$ of 1200 bu.
5. 15% of \$580.
6. $33\frac{1}{3}\%$ of \$2700.
7. $3\frac{1}{5}\%$ of \$2500.
8. $\frac{1}{2}\%$ of \$2000.
9. $\frac{3}{4}\%$ of \$3000.

10. A man who had \$120 spent 40% of it. How much had he left?

11. A farmer has 400 acres of land. If he plants 15% of it in corn, how many acres does he plant?

12. What is 20% of \$124.50?

13. A farmer had 80 sheep and sold 45% of them. How many had he left?

14. Ann's age is 75% of the age of Mary, who is 16 years old. How old is Ann?

15. A jeweler bought watches at \$60 a dozen, and sold them at a gain of 75%. Find the selling price of each watch.

16. The distance between A and B is 75 miles. If the distance from C to D is 20% of this distance, how far is it from C to D?

17. I had \$20, but spent \$10 of it for a suit. What part of my money did I spend? What per cent of it?

To find what part the second of two numbers is of the first, divide the second by the first. For example, 10 is $\frac{10}{20}$, or $\frac{1}{2}$ of 20.

18. A farmer who had 16 cows sold 12 of them. What part of them did he sell? What per cent of them?

19. A man bought 40 sheep at \$4 a head, and sold them at a gain of 25%. How much did he get for the flock?

20. A man paid \$50 for a typewriter, and sold it at a loss of 20%. What did he get for it?

21. At what price must coffee which cost 15¢ a pound be sold to make a profit of 20%?

22. How much must be paid for a book marked $2.50, if 20% is deducted from the marked price?

23. A piano which was marked $175 was sold for 10% less. What did it bring?

24. A man who earned $75 a month had his salary increased $33\frac{1}{3}$%. How much did he then earn a month?

25. If my income is $3000, and I spend 35% of it, how much do I save?

26. A farmer bought 30 head of cattle at $40 a head, and sold them at a profit of $4\frac{1}{2}$%. What did he make?

27. A farmer raised 1600 bu. of corn, and sold $6\frac{1}{4}$% of it. How many bushels did he sell?

28. A dealer bought 25 lambs at $4.50 each, and sold them at a profit of 12%. How much did he gain?

29. A village school has an enrollment of 160 pupils. If 25% of them are in the second grade, how many are in that grade?

30. An agent sells 10 automobiles at $1250 each, and is allowed 15% of the receipts. How much does he receive?

31. A farmer raised 136 bu. of beans. He sold 25% of them at $1.50 a bushel, and the remainder at $1.60 a bushel. What did he get for the beans?

32. A dealer buys a 30-lb. bucket of candy for $3, and sells it at a profit of 50%. At what price per pound does he sell it?

33. A dealer bought 100 tons of coal at $4.25 a ton, expecting to sell it at 10% profit. Owing to a decline in the market price, he had to sell it for $17.50 less than he expected. What did he get for the coal?

34. A dealer bought 100 barrels of salt at \$1.15 a barrel. Freight and drayage amounted to \$65. At what price per barrel did he sell it, if he gained 10% on cost and carriage?

35. A stenographer's salary is \$16 a week. If she spends 25% of it for board, 10% for clothes, and 15% for other expenses, how much does she save each week?

TRADE DISCOUNT

252. 1. The list price of a book is \$1, but copies are sold to dealers at 25% reduction. What is the price to dealers?

2. A steel range sells for \$50 on 3 months' credit, or 10% less for cash. What is the cash price?

3. A bill of goods was sold for 10% less than list prices, which amounted to \$80. What was the discount and the selling price?

253. An allowance or deduction from a list price, or from the amount of a bill, is called a **Discount.**

The per cent deducted is called the *rate of discount.*

WRITTEN EXERCISES

254. 1. A merchant buys goods listed at \$1500, and is allowed a discount of 25%. Find the discount and the cost.

$.25 \times \$1500 = \$375$, the discount.

$\$1500 - \$375 = \$1125$, the cost.

2. A bill of \$750 was discounted at 8%. What were the discount and the net price?

3. What is the cash price of a carriage listed at \$130, if 5% is deducted for cash?

4. A school district bought 500 grammars listed at \$.60, at a discount of 20%. What sum must be remitted?

5. At a reduction sale a \$15 sofa was sold at 20% discount. For how much did it sell?

6. A \$27.50 suit of clothes is marked down 8%. What is the selling price?

7. A \$10 reed rocker is marked "25% off" at a closing-out sale. Find the selling price.

8. I bought a \$100 typewriter at 10% discount, and then sold it at 10% less than it cost me. What did I receive for it?

9. I buy a piano listed at \$450, but get a discount of 5% for cash. What does the piano cost me?

10. A shipment of bananas valued at \$180 was received in a damaged condition. On this account the bill was discounted 30%. What did the buyer pay for them?

11. A dinner set listed at \$24 is offered at $12\frac{1}{2}$% off. What is the selling price?

12. A teacher bought a \$60 encyclopedia on the instalment plan, at $16\frac{2}{3}$% less than list price. Later, by paying cash, he was allowed a further discount of 10%. What was the net cost of the set?

13. A dressmaker bought for a customer a 14-yd. dress pattern at \$2 a yard. From this price she was allowed a discount of 10% for cash. What was the amount of the bill?

14. By paying cash this bill was further discounted 5%. What did she pay for the pattern?

15. A dealer buys 12 doz. composition books for \$9, and is allowed 5% off for cash. He retails the books at 10 cents each. What does he gain?

INTEREST

255. 1. If I borrow $100 and agree to pay back the money at the end of a year, and 6% additional for its use, how much do I pay for the use of the money, or how much *interest* do I pay? How much do I pay in all at the end of the year?

2. How much should I receive for the use of $300 for 1 year at 6%? For 2 years?

256. Money paid for the use of money is called **Interest.**

Thus, if I borrow $100 for a year, and pay $6 for its use, the $6 is the interest.

257. The sum on which interest is paid is called the **Principal.**

258. The **rate of interest** is expressed as a certain per cent.

Thus, interest at 6% means that the charge for a year is 6% of the principal.

259. The sum of the principal and interest is called the **Amount.**

WRITTEN EXERCISES

260. 1. If you borrow $225 at 4%, how much interest will you owe at the end of the year?

2. What is the interest on $275 for 1 year at 5%?

3. At 6%, what is the interest on $250 for 1 year? On $600?

4. Find the interest on \$375 for 3 years at 5%.

5. At 6%, what is the interest on \$450 for 6 months?

6. At 5%, what is the interest on \$600 for 3 months ($\frac{1}{4}$ yr.)?

7. At 6%, what is the interest on \$650 for 1 year and 6 months ($1\frac{1}{2}$ yr.)?

8. If you have \$81 in a bank that pays 3% on deposits, how much interest will you receive for a year?

9. A man has \$275 in a savings bank that pays 4%. How much interest will he receive in 2 years?

261. Find the interest on \$350 for:

1. 2 yr. at 6%.
2. 3 yr. at 6%.
3. 4 yr. at 4%.
4. 5 yr. at 5%.
5. 6 mo. at 6%.
6. 3 mo. at 6%.
7. 2 yr. at 8%.
8. 4 yr. at 5%.
9. 3 yr. at 3%.
10. 2 mo. at 6%.
11. 4 mo. at 3%.
12. 9 mo. at 4%.

13. Find the interest on \$250 for 2 yr. 3 mo. at 6%.

\$2.50 = interest for 1 yr. at 1%.
6
\$15.00 = interest for 1 yr. at 6%.
$2\frac{1}{4}$
\$30.00 = interest for 2 yr.
3.75 = interest for 3 mo. ($\frac{1}{4}$ yr.).
\$33.75 = interest for 2 yr. 3 mo.

Find the interest on \$240 for:

14. 2 yr. at 5%.
15. 3 yr. at 6%.
16. 2 mo. at 4%.
17. 9 mo. at 4%.

Find the interest on:

18. $360 for 2 yr. at 6%.
19. $325 for 3 yr. at 4%.
20. $475 for 4 yr. at 5%.
21. $860 for 3 yr. at 6%.
22. $600 for 1 yr. 3 mo. at 6%.
23. $420 for 2 yr. 6 mo. at 5%.
24. $500 for 2 yr. 4 mo. at 6%.
25. $375 for 3 yr. 8 mo. at 3%.
26. $400 for 1 yr. 3 mo. at $3\frac{1}{2}$%.
27. $800 for 2 yr. 1 mo. at $4\frac{1}{2}$%.
28. $840 for 4 yr. 9 mo. at 6%.
29. $376 for 2 yr. 3 mo. at 5%.
30. $725.50 for 3 yr. 4 mo. at 6%.

Find the amount of:

31. $200 for 2 yr. at 4%.
32. $160 for 3 yr. at $4\frac{1}{2}$%.
33. $500 for 2 yr. at $3\frac{1}{2}$%.
34. $300 for 1 yr. 6 mo. at 5%.
35. $225 for 2 yr. 3 mo. at 6%.
36. $325 for 3 yr. 4 mo. at 6%.

37. A man borrowed $350 for 4 mo. at 6%. What amount, principal and interest, did he owe when the debt fell due?

38. On the first day of June, 1905, a man borrowed $250 at 6%. What was the amount due on the first day of September following?

39. A state owes a debt of $22,500,000 bearing 5% interest. By reducing the rate from 5% to $3\frac{1}{2}$%, how much will the state save annually?

RELATED PROBLEMS

262. **1.** The picture represents a field of peanuts in the shock. If the field is 80 rd. long and 40 rd. wide, and yields 45 bu. to the acre, what is the total yield? At $3\frac{1}{2}¢$ a pound, what is the crop worth? (1 bu. = 22 lb.)

2. A neighbor has 30% more acres in peanuts, but his yield per acre is 20% less. What is his crop?

3. A grocer buys 10 bu. of peanuts at 7¢ a pound. If roasting costs 60¢ for the lot, at what price per pint must he sell them in order to gain 100% on the cost?

4. If North Carolina had 95,856 acres in peanuts in one year, and the average yield was 35 bu. per acre, what was the value of the crop at 60¢ a bushel?

5. When Virginia had 116,042 A. in peanuts and produced 3,713,344 bu., what was the average yield per A.?

6. In a recent year the peanut crop of five leading states was: Virginia, 3,713,344 bu.; North Carolina, 3,460,439 bu.; Alabama, 1,021,708 bu.; Georgia, 1,967,927 bu.; and Tennessee, 747,668 bu. What was the total value of the crop at 3¢ a pound?

FUNDAMENTAL OPERATIONS REVIEWED

263. 1. Write numbers from one to fifty in Roman and Arabic characters in parallel columns. Count the characters used in each column, and compare results.

2. Try to add the column of Roman characters. Can you do it?

3. Can you best compute in Roman or Arabic characters?

4. Write the year of Independence in both systems. Count and compare the number of characters used.

5. Which system is best suited to writing large numbers?

6. For what purposes have you seen the Roman System used?

264. 1. Write in our common (Arabic) system the number one hundred twenty-three million, four hundred fifty-six thousand, seven hundred eighty-nine. Give the place value of each figure, and the name of each period.

2. What is the use of 0 in our common system?

3. How many different figures are used in the Arabic system? How many fingers have you, thumbs included? Why is a figure sometimes called a *digit?*

265. The system of counting by *tens* is called a *decimal system*, from the Latin *decem* meaning "ten." In our system *ten* of any place or order make *one* of the next higher order; hence it is a decimal system.

266. Since 10 units of any order equal 1 of the next higher order, the scale in *simple numbers* is *uniform*.

In United States money the scale is also uniform; in compound numbers it is varying. United States money is the connecting link between simple and compound numbers, as it is between integers and decimals.

Compare the following:

Uniform Scale	Uniform Scale	Varying Scale
10 ones =1 ten.	10 mills =1 cent.	2 pints =1 qt.
10 tens =1 hundred.	10 cents =1 dime.	8 quarts=1 pk.
10 hundreds=1 thousand.	10 dimes=1 dollar.	4 pecks =1 bu.

267. 1. Tell why a uniform scale is better, in general, than a varying scale.

2. What is meant by the word *integer?* By the word *fraction?*

3. What is meant by the expression *simple number?* By the expression *compound number?* Give an example of each.

268. 1. In counting dollars, 1 dollar is the unit; in counting thirds, $\frac{1}{3}$ is the unit; and in counting tenths, .1 is the unit. In the first example, the unit is integral; in the others, fractional.

2. What is the principal unit of length? State the other units of the table.

3. Express 15 yd. in the next smaller unit. Express 24 in. in the next larger unit. Express 5 yd. 2 ft. as yards and a fraction of a yard.

4. In the fraction $\frac{7}{24}$, what is the fractional unit? What does the denominator express about the fraction? What does the numerator tell?

ADDITION AND SUBTRACTION

269. In addition and subtraction, the general principles are the same, whether we are dealing with integers, decimals, common fractions, or compound numbers. The numbers are not written in exactly the same form, but the processes are virtually the same.

270. 1. Compare the following additions:

(*a*)	(*b*)	(*c*)	(*d*)
78	7.8	$7\frac{8}{10}$	7 bu. 2 pk.
23	2.3	$2\frac{3}{10}$	2 bu. 3 pk.
101	10.1	$10\frac{1}{10}$	10 bu. 1 pk.

Is the number of units in the sum the same in each case? In these additions why do we add 1 to the next higher order? In which column is the lowest order written? With which column do we begin to add? Why?

2. Also compare these subtractions:

(*e*)	(*f*)	(*g*)	(*h*)
81	8.1	$8\frac{1}{10}$	8 bu. 1 pk.
43	4.3	$4\frac{3}{10}$	4 bu. 3 pk.
38	3.8	$3\frac{8}{10}$	3 bu. 2 pk.

Before subtracting, why do we increase the 1 in each minuend by a unit of the next higher order?

ORAL EXERCISES

271. State only the sums:

1.	2.	3.	4.	5.
63	42	83	70	86
24	28	26	32	48

6.	7.	8.	9.	10.
67	75	58	87	49
31	62	45	60	53

11.	12.	13.	14.	15.
90	89	110	130	144
45	36	25	75	36

Repeat and time yourself on Examples 1–15.

State only the differences, timing yourself:

16.	17.	18.	19.	20.
53	78	59	84	96
21	29	47	26	34

21.	22.	23.	24.	25.
85	94	57	89	91
58	71	29	37	24

26.	27.	28.	29.	30.
105	134	203	380	165
37	80	95	270	89

31. What is necessary before one fraction can be added to another? Before one fraction can be subtracted from another? Why is this first step necessary in each case?

32. Add 2 pk., 3 pk., 5 pk., and 9 pk. Express the result as bushels and pecks. As bushels and a fraction of a bushel.

33. Can you add 3 ft. and 24 in.? What change is first necessary? Express the sum as inches, and then as feet. Show that the results are equal.

WRITTEN EXERCISES

272. Add, checking each result by adding from the top downward:

1.	2.	3.	4.
$56.50	$12.30	98.75	$50.00
73.28	56.75	28.47	85.70
35.75	91.23	32.10	87.76
87.97	45.65	55.00	49.38
48.85	89.24	90.99	60.48

5.	6.	7.	8.
$73.25	218.85	963.70	$491.00
29.34	641.55	349.10	632.80
62.50	827.30	192.39	493.75
37.80	468.32	269.45	860.40
29.36	827.36	282.35	495.75
48.75	316.30	467.65	328.81

9.	10.	11.	12.
94.80	$147.96	$339.30	$638.72
67.65	237.94	460.50	368.45
39.84	182.50	680.95	683.70
98.75	375.80	840.60	273.25
84.60	250.75	395.40	337.65
75.80	160.84	453.70	954.38

Is the above method of checking results an absolute test of accuracy?

Subtract, testing each result by adding subtrahend and remainder:

13.	14.	15.	16.
$732.50	321.75	$640.37	305.25
232.80	278.90	450.65	187.65

17.	18.	19.	20.
543.20	$756.25	231.56	$936.06
123.45	467.45	179.65	470.76

21.	22.	23.	24.
$1962.50	2123.45	$7300.00	5437.05
873.69	1745.75	200.95	2465.50

Is the above method an absolute test of accuracy in subtraction?

Add or subtract as indicated:

25. $5\frac{1}{3} + 3\frac{1}{4}$.

26. $\frac{11}{16} + \frac{7}{8}$.

27. $21\frac{1}{3} + 16\frac{3}{5}$.

28. $\frac{7}{8} + \frac{5}{12} + \frac{5}{6}$.

29. $\frac{3}{8} + \frac{5}{12} + \frac{3}{4}$.

30. $75\frac{1}{8} - 57\frac{1}{24}$.

31. $36\frac{3}{8} - 14\frac{2}{3}$.

32. $67\frac{5}{6} - 16\frac{8}{9}$.

33. $94\frac{1}{6} - 36\frac{5}{16}$.

34. $289\frac{3}{8} - 161\frac{5}{12}$.

Add:

35.			36.	37.
$	d.	ct.		
7	2	0	7.20	$7\frac{1}{5}$
3	5	0	3.50	$3\frac{1}{2}$
2	7	5	2.75	$2\frac{3}{4}$
				$13\frac{9}{20} = 13\frac{45}{100}$.

Compare the processes in Examples 35, 36, and 37; note that they involve the same principles and that reductions are automatic in United States money and decimals, though they demand attention in common fractions. Why are decimals more easily added than common fractions?

Add:

38.	39.		
	yd.	ft.	in.
7225	5	2	8
3870	7	1	6
2765	8	1	9

Why are simple numbers more easily added than compound numbers?

Add:

40.

gal.	qt.	pt.
3	2	1
7	3	
8	1	1
5	2	1

41.

bu.	pk.
3	1
9	3
12	2
10	4

42.

ft.	in.
6	5
4	3
7	9
8	4

43.

yr.	mo.
2	3
4	5
5	6
1	2

44.

hr.	min.	sec.
12	40	34
7	8	50
18	25	26

45.

rd.	yd.	ft.
40	5	2
85	2	1
216	4	

46.

bu.	pk.	qt.
17	3	1
85	1	7
104	2	3

Subtract:

47.

bu.	pk.
5	1
2	3

48.

gal.	qt.
13	2
8	3

49.

lb.	oz.
9	4
6	9

50.

ft.	in.
10	3
7	10

51.

yd.	ft.	in.
5	1	9
3	2	11

52.

T.	lb.
3	2
1	4

53.

hr.	min.
24	30
16	45

54.

yr.	mo.	da.
1905	10	7
1900	8	15

55.

yr.	mo.	da.
1910	3	13
1905	6	18

56. Write the dates in Examples 54 and 55 as dates are usually written.

Subtract:	57.	58.
	8.2	$8\frac{1}{5}$
	2.5	$2\frac{1}{2}$

Note the "borrowing" and subtraction in the decimal and compare with the same in the fractional form. Which is the easier, subtraction of decimals or subtraction of common fractions? Why?

59. On a certain railway the divisions are as follows: 129.4 mi., 949.9 mi., 403.1 mi., 323 mi., and 790.2 mi. What is the total length of the road?

60. A farmer sold corn as follows: to A, 75 bu. 1 pk.; to B, 37 bu. 3 pk.; to C, 110 bu. 2 pk.; to D, 18 bu. 3 pk. How much did he sell to all?

61. How long was it from March 12, 1890, to July 4, 1905?

62. From a cheese which weighed 45 lb., a grocer cut 12 lb. 8 oz. How much remained?

63. From a barrel containing 41 gal. 2 qt. of vinegar, 10 gal. 3 qt. were sold at one time and 23 gal. 1 pt. at another. How much remained in the barrel?

64. From a piece of cloth $37\frac{5}{8}$ yd. long a merchant sold to one customer $15\frac{1}{4}$ yd., to another $3\frac{1}{8}$ yd., and to a third $12\frac{3}{4}$ yd. How many yards remained unsold?

65. The value of the wool clip in the five states named below in a certain year was as follows:

Montana	$7,479,000
Wyoming	4,859,000
Ohio	3,396,000
Idaho	2,878,000
Oregon	2,472,000

What was the total value of the clip in the five states?

MULTIPLICATION

273. In multiplication we multiply the denominations or orders separately, and then reduce and add the partial products.

Compare:

(*a*)	(*b*)	(*c*)
		gal. qt.
\$4.30	$18\frac{3}{5}$	4 3
9	$\frac{2}{3}$	9
$2.70 = 9 \times \$0.30$	$\frac{6}{15} = \frac{2}{3} \times \frac{3}{5}$	$27 = 9 \times 3$ qt.
$36 = 9 \times \$4.$	$12 = \frac{2}{3} \times 18$	$36 = 9 \times 4$ gal.
$\$38.70 = 9 \times \$4.30$	$12\frac{2}{5} = \frac{2}{3} \times 18\frac{3}{5}$	$42 \quad 3 = 9 \times 4$ gal. 3 qt.

Which is easiest, the multiplication of decimals, common fractions, or compound numbers? Why?

ORAL EXERCISES

274. State only the products:

1.	**2.**	**3.**	**4.**	**5.**
16	28	35	47	54
3	2	4	5	6
6.	**7.**	**8.**	**9.**	**10.**
15	23	44	55	60
7	8	8	11	12
11.	**12.**	**13.**	**14.**	**15.**
\$2.50	\$3.50	\$5.40	\$7.50	\$12.50
6	7	5	9	8

16. $\frac{3}{5} \times \frac{5}{6}$.
17. $\frac{3}{4} \times \frac{4}{5}$.
18. $\frac{4}{9} \times \frac{3}{5}$.
19. $\frac{3}{4} \times \frac{7}{8}$.
20. $\frac{3}{4} \times \frac{8}{9}$.
21. $\frac{2}{3} \times \frac{5}{8}$.
22. $\frac{3}{4} \times \frac{2}{3}$.
23. $\frac{2}{3} \times \frac{3}{8}$.
24. $\frac{7}{12} \times \frac{5}{14}$.

25. State a rule for multiplying one fraction by another.

26. What will $\frac{3}{4}$ of a yard of gingham cost at 16¢ a yard?

27. What will $\frac{5}{8}$ of a yard of damask linen cost at 96¢ a yard?

WRITTEN EXERCISES

275. Multiply:

1.	2.	3.	4.
578	327	8076	4110
30	245	670	385
5.	**6.**	**7.**	**8.**
7340	5896	8645	5798
465	309	975	606
9.	**10.**	**11.**	**12.**
$150.40	$296.50	$270.64	$890.25
225	728	325	576
13.	**14.**	**15.**	**16.**
$375.50	$628.70	$296.95	$825.45
240	606	888	905

Repeat and time yourself on Examples 1–16. See how many of these you can solve in 5 minutes.

Find the products:

17.		18.		19.	
yd.	in.	hr.	min.	bu.	pk.
6	9	8	20	3	2
	5		12		8

20.		21.		22.	
ft.	in.	lb.	oz.	gal.	qt.
15	8	16	9	8	2
	24		22		9

23. 34.6×6.5.

24. $3.75 \times .35$.

25. 2.36×1.25.

26. $.236 \times .25$.

27. $6.42 \times .245$.

28. 6.07×6.07.

29. $8.25 \times .625$.

30. $1.325 \times .44$.

31. 2.703×1.6.

32. $\frac{9}{16} \times \frac{8}{9}$.

33. $2\frac{1}{2} \times \frac{5}{8}$.

34. $\frac{7}{9} \times \frac{5}{16}$.

35. $2\frac{5}{24} \times \frac{11}{12}$.

36. $\frac{13}{16} \times \frac{11}{12}$.

37. $2\frac{9}{20} \times \frac{5}{16}$.

38.	39.	40.
156	128	$112\frac{1}{2}$
$14\frac{3}{4}$	$37\frac{3}{8}$	$37\frac{1}{2}$

41. What will 56 lb. of sugar cost at $6\frac{1}{2}¢$ a pound?

42. At $1.20 a yard, what will $\frac{5}{8}$ yd. of silk cost?

43. At 30¢ a dozen, what will 4 doz. bottles of mucilage cost?

44. What will 16 T. of hay cost at $9.75 a ton?

45. A grocer bought 8 bags of chestnuts, each containing 3 bu. 2 pk. 5 qt. How many did he buy? How much did he pay for them, if they cost him $2.25 a bushel?

46. A dealer paid $1.25 a bushel for onions, and sold them at $.50 a peck. How much did he gain on 18 bushels?

47. There are 16.5 ft. in a rod. How many feet are there in 320 rd., or 1 mile?

48. A bushel of apples will make 2 gal. of cider. At that rate, what will a load of 45 bu. of apples bring if made into cider that sells at 3¢ a pint?

DIVISION

276. 1. As we have seen, division is the inverse of multiplication. That is, it is the operation of finding one factor when the product and the other factor are given. For example,

$25 \times \$50 = \$1250$. $\therefore \$1250 \div 25 = \$50$, and $\$1250 \div \$50 = 25$.

2. We have also seen that division includes two cases, viz.: finding one of the equal parts of a number, involving the idea of *partition;* and finding how many times one number contains another of the same kind, involving the idea of *measuring.* Point out the two cases in the foregoing example.

3. In division examples, two of the three numbers involved (dividend, divisor, and quotient) are always alike, or of the same denomination. Which two are alike in problems involving partition? In problems involving measurement?

277. Compare the following:

(*a*)	(*b*)
$\$312 \div \$12 = 26$.	$78 \text{ bu.} \div 12 \text{ pk.} = 312 \text{ pk.} \div 12 \text{ pk.} = 26$.

Which is easier? In what other way can you solve (*b*)?

Also compare these:

(*c*)	(*d*)
$.75 \div .625 = 1.2$.	$625\overline{)750.0}$

Tell why $.75 \div .625 = 750 \div 625$.
State the principle for division of decimals.

$$\begin{array}{r} 1.2 \\ 625\overline{)750.0} \\ \underline{625} \\ 1250 \\ \underline{1250} \end{array}$$

And compare these:

(*e*)

$$\frac{3}{4} \div \frac{5}{8} = \frac{18}{24} \div \frac{15}{24} = \frac{18}{15}, \text{ or } 1\tfrac{1}{5}.$$

(*f*)

$$\frac{3}{4} \div \frac{5}{8} = \frac{3}{\cancel{4}_{1}} \times \frac{\cancel{8}^{2}}{5} = 1\tfrac{1}{5}.$$

Explain why (*f*) is easier than (*e*). Which of these methods do we usually follow in division of fractions?

Note carefully the following:

(*g*)

$$50 \div 2.5 = 500 \div 25 = 20.$$

(*h*)

$$50 \div 2\tfrac{1}{2} = 50 \div \tfrac{5}{2} = 50 \times \tfrac{2}{5} = 20.$$

(*i*)

	gal.	qt.	pt.
9	43	3	1
		31	9
	4	3	1

(*j*)

	gal.	qt.	pt.
9)	43	3	1
	4	3	1

Which involves more thought and work, (*g*) or (*h*)? Explain the processes in (*i*) and (*j*).

ORAL EXERCISES

278. State only the quotients:

1. $125 \div 5$.	**6.** $672 \div 6$.	**11.** $264 \div 12$.
2. $164 \div 4$.	**7.** $656 \div 8$.	**12.** $242 \div 11$.
3. $156 \div 3$.	**8.** $287 \div 7$.	**13.** $384 \div 12$.
4. $324 \div 4$.	**9.** $434 \div 7$.	**14.** $363 \div 11$.
5. $225 \div 9$.	**10.** $729 \div 9$.	**15.** $552 \div 12$.

Repeat and time yourself on Examples 1–15.

Name quotients and remainders:

16. $5\overline{)38}$	**19.** $4\overline{)47}$	**22.** $6\overline{)77}$	**25.** $7\overline{)83}$
17. $8\overline{)49}$	**20.** $9\overline{)78}$	**23.** $8\overline{)101}$	**26.** $9\overline{)103}$
18. $9\overline{)86}$	**21.** $7\overline{)107}$	**24.** $12\overline{)117}$	**27.** $11\overline{)138}$

State quotients as rapidly as possible:

28.	29.	30.	31.
$6)\$15$	$7)\$24.50$	$9)67.50$	$8)\$100$

32. $\frac{5}{8} \div \frac{3}{4}$.

33. $\frac{7}{9} \div \frac{2}{3}$.

34. $\frac{2}{3} \div \frac{8}{9}$.

35. $\frac{2}{3} \div \frac{5}{6}$.

36. $\frac{5}{12} \div \frac{2}{3}$.

37. $\frac{5}{24} \div \frac{7}{8}$.

38. How do you divide one fraction by another?

39. At $12\frac{1}{2}¢$ a yard, how many yards of cloth can be bought for \$1?

40. If Harry can walk 3 miles an hour, in what time can he walk 10 miles?

WRITTEN EXERCISES

279. Divide as indicated:

1. $4515 \div 21$.
2. $8268 \div 52$.
3. $64{,}554 \div 203$.
4. $10{,}725 \div 75$.
5. $35{,}344 \div 47$.
6. $1089 \div 33$.
7. $64{,}108 \div 94$.
8. $10{,}388 \div 28$.
9. $162{,}944 \div 76$.
10. $178{,}416 \div 144$.
11. $\$17{,}616 \div \$48$.
12. $\$84{,}231 \div 63$.
13. $\$25{,}568 \div 68$.
14. $\$8008 \div \$308$.
15. $\$1140.48 \div \$35.64$.

16.			17.		18.	
T.	cwt.	lb.	ft.	in.	da.	hr.
5)11	16	80	7)19	10	12)51	12

19.		20.		21.	
bu.	pk.	gal.	qt.	lb.	oz.
15)52	2	25)312	2	21)102	6

22. 13 ft. 4 in. $\div$ 1 ft. 8 in.
23. 40 bu. 2 pk. $\div$ 6 bu. 3 pk.
24. 372 gal. 3 qt. $\div$ 15 qt.
25. 46 lb. 14 oz. $\div$ 25 oz.

26. $78.13 \div 5$.

27. $79.38 \div 3$.

28. $\$2 \div \$.05$.

29. $27.45 \div 1.5$.

30. $27.63 \div .06$.

31. $.052 \div .0013$.

32. $\$75.11 \div \$14.50$.

33. $\$27.50 \div \$2.50$.

34. $\$285.12 \div \$8.91$.

35. $\frac{4}{5} \div \frac{3}{8}$.

36. $\frac{7}{15} \div \frac{3}{5}$.

37. $\frac{9}{12} \div \frac{5}{8}$.

38. $\frac{3}{8} \div \frac{5}{12}$.

39. $\frac{27}{32} \div \frac{5}{16}$.

40. $\frac{2}{15} \div \frac{7}{8}$.

41. $\frac{7}{48} \div \frac{3}{16}$.

42. $\frac{3}{8} \div \frac{9}{10}$.

43. $\frac{17}{32} \div \frac{11}{12}$.

44. $42 \div 3\frac{1}{4}$.

45. $66 \div 7\frac{2}{5}$.

46. $51\frac{1}{14} \div 7\frac{3}{7}$.

47. $37\frac{1}{4} \div 9\frac{5}{8}$.

48. $11\frac{3}{7} \div \frac{9}{14}$.

49. $4\frac{1}{4} \div 102\frac{1}{8}$.

50. At $12\frac{1}{2}¢$ a quart, how many quarts of molasses will $4 buy?

51. At $33\frac{1}{3}¢$ a yard, how many yards of cloth can be bought for $198?

52. How many dress patterns of $12\frac{1}{2}$ yd. each can be cut from a piece of cloth containing $62\frac{1}{2}$ yd.?

53. If 25 bu. 3 pk. of chestnuts fill 8 bags of the same capacity, how much does each bag contain?

54. How many pieces 1 ft. 6 in. long can be sawed from a board 19 ft. 6 in. long?

55. Four men and 2 boys weigh 791 lb. 8 oz. If the two boys together weigh 141 lb. 4 oz., what is the average weight of the men?

56. How many quart bottles will it take to hold $9\frac{1}{2}$ gal. of ink?

57. A man feeds his horse 6 qt. of oats 3 times a day. How long will 9 bu. of oats last?

58. If 3 ducks, each weighing 5 lb., are worth as much as a 9-pound turkey which sells for $1.65, what is the price of the ducks per pound?

PRACTICAL MEASUREMENTS

BUILDING MATERIALS

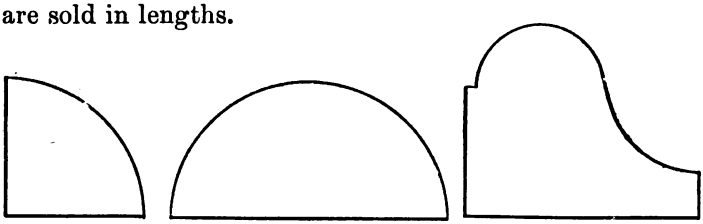
280. Lumber is bought and sold by the thousand board feet; in small quantities, by the hundred.

A **board foot** (bd. ft.) is a rectangular solid 1 ft. × 1 ft. × 1 in. (read, 1 foot by 1 foot by 1 inch). It contains 144 cubic inches.

In lumber transactions, board feet are usually referred to as feet.

281. Lumber less than an inch thick is counted as if it were an inch thick. Pieces less than 2 inches thick are called boards, or planks.

282. Certain building materials such as quarter-round, half-round, and moldings of different shapes for finishing are sold in lengths.



The cuts show three different shapes. Can you find any of these used in the school building? Which cut represents the quarter-round? Which represents the half-round?

ORAL EXERCISES

283. 1. How many board feet in a plank 16 ft. long, 12 in. wide, and 1 in. thick?

2. How many board feet in a board 10 ft. long, 1 ft. wide, and $\frac{3}{8}$ in. thick?

3. How many board feet in 10 such boards?

WRITTEN EXERCISES

284. 1. How many board feet in a joist 10 ft. long, 1 ft. wide, and 2 in. thick? (Such a joist is described as "2 in. × 12 in., 10 ft. long"; it is sometimes expressed in symbols, as $2'' \times 12'' \times 10'$.)

To find the number of board feet, we divide the number of cubic inches by 144; thus, $\frac{2 \times \cancel{12} \times \overset{10}{\cancel{120}}}{\underset{12}{\cancel{144}}} = 20$, number of board feet.

This process is conveniently shortened by expressing two of the dimensions in feet, thereby dividing by 144 in advance. Thus, $2 \times 1 \times 10 = 20$, number of board feet.

2. How many feet in a board 1 in. × 16 in., 15 ft. long?

$1 \times 1\frac{1}{3} \times 15 = 20$, number of board feet.

3. What will 7500 ft. of lumber cost at \$1.50 per M? (Per M = by the thousand.)

7500 = 7.5 M.
∵ 1 M costs \$1.50,
7.5 M costs 7.5 × \$1.50, or \$11.25.

4. How many feet in 12 oak beams 12 in. × 12 in., 20 ft. long?

5. What is the cost of 27 joists 2 in. × 10 in., 12 ft. long, at \$20 per M?

6. Find the cost of 15,600 feet of flooring at $27.50 a thousand. (Unless otherwise stated, flooring is 1 in. thick.)

7. A square lot, each side of which is 180 feet long, is to be inclosed by a 4-board fence. What will be the cost of the lumber at $15 a thousand, each board being 6 in. wide and 1 in. thick?

8. James Watt bought of Franklin Mfg. Co. the following bill of lumber:

Rough lumber,

40 pieces 2 in. × 8 in., 16 ft. long, @ $15;
80 pieces 2 in. × 4 in., 12 ft. long, @ $20.

Dressed lumber,

250 boards 1 in. × 10 in., 14 ft. long, @ $30.

Make a bill showing extensions and calculations.

9. Find the cost of quarter-round for 8 rooms (base only), each 16 ft. × 18 ft., at $1\frac{1}{2}$¢ per lineal foot.

Twice 16 ft. + twice 18 ft. = 68 ft., perimeter.
8 × 68 = 544, number of feet required.
544 × $1\frac{1}{2}$¢ = $8.16, cost.

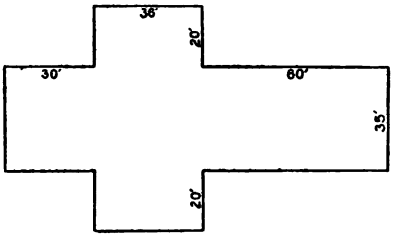
10. Find the cost of 170 ft. of half-round at $1\frac{3}{4}$¢ a running foot.

11. Find the cost of 2300 ft. of molding at $2.25 per 100 ft.

12. Find the cost of the flooring required to cover a library floor 36 ft. by 70 ft., at $44 per M. (In estimating flooring, the surface to be covered is increased by one third of itself to include the tongue.)

36 × 70 = 2520, number of square feet.
$2520 + \frac{2520}{3} = 3360$, number of sq. ft., or 3.36 M.
3.36 × $44 = $147.84, cost.

13. Allowing $\frac{1}{3}$ of the cost of material for labor and nails, what will it cost at $27 per M to renew the floor of the church whose ground plan is given in the cut?



14. If 1000 laths will cover 70 square yards of surface, how many will be needed to cover the side walls of this church, if the walls are 21 feet high?

15. If the roof of this church has an area of 7000 sq. ft., what will it cost to cover it with shingles at $4 per M, allowing 800 shingles to a square (100 sq. ft. of surface)?

16. How many feet (board measure) in a bridge timber 8 in. × 8 in., 30 ft. long?

BRICK, STONE, AND CEMENT

285. Bricks are bought by the thousand. In estimating the number required for a wall, a convenient method is as follows:

For a wall 1 brick (8 in.) thick, allow 14 bricks for each square foot of surface; for a wall $1\frac{1}{2}$ bricks (12 in.) thick, allow 21 bricks; for a wall 2 bricks (16 in.) thick, allow 28 bricks. Take outside measurements.

This method is sufficiently accurate for ordinary contracts; but when the exact amount of material is desired, as in large contracts, allowance must be made for corners and openings.

In the problems that follow, the size of a brick is considered to be 8 in. × 4 in. × $2\frac{1}{2}$ in. Other sizes are made.

286. Stone work is estimated most conveniently by the cubic yard or cubic foot. Other units may be used; for example, the perch, which contains $24\frac{3}{4}$ cu. ft.

287. Cement is used in many kinds of construction; for example, pavements, foundations, piers, abutments, retaining walls, etc. Cement pavements are sometimes called granolithic. Cement blocks are extensively used for buildings of all kinds in place of brick and stone.

WRITTEN EXERCISES

288. 1. Find the number of bricks needed for the walls of a building 60 ft. long, 24 ft. wide, and 15 ft. high, if the walls are $1\frac{1}{2}$ bricks (12 in.) thick.

168 ft. = perimeter, outside measurement.
15 × 168 = 2520, number of square feet in surface of walls.
2520 × 21 bricks = 52,920 bricks.

Find the number of bricks needed for:

2. A building 40 ft. long, 20 ft. wide, and 12 ft. high, if the walls are two bricks thick.

3. A retaining wall a brick and a half thick, 10 yd. long, and 10 ft. high.

4. A rectangular schoolhouse 36 ft. × 72 ft. and 30 ft. high, if the walls are 12 in. thick. Allow for two doors each 5 ft. × $7\frac{1}{2}$ ft. and 24 windows each 3 ft. × 6 ft.

5. A brick partition 36 ft. × 30 ft., and 8 in. thick.

6. Find the cost of brick in this partition at $7.50 per M.

7. Two masons with 3 helpers can lay 4000 bricks a day. Bricks cost $7.50 per M; lime and sand, $1.50 for each thousand bricks used. A mason is paid $4.05 a day; a helper $1 a day. Estimate the cost of labor and material required to build the walls of a building which requires 120,000 bricks.

8. How many cubic yards of masonry in the walls of a foundation 40 ft. × 23 ft., 8 ft. high, and 18 in. thick?

126 ft. = perimeter.

$$\frac{126 \times 8 \times 1\frac{1}{2}}{27} = 56, \text{ number of cubic yards.}$$

9. Find the number of cubic yards of masonry in a dam 60 ft. long and 12 ft. high, 8 ft. thick at the bottom, and battered to a thickness of 3 feet at the top.

$$\frac{8+3}{2} = \frac{11}{2}, \text{ number of feet of average thickness.}$$

$$\frac{\frac{11}{2} \times 60 \times 12}{27} = \frac{11 \times \overset{10}{\cancel{60}} \times \overset{4}{\cancel{12}}}{\cancel{2} \times \underset{9}{\cancel{27}}} = 146\tfrac{2}{3}, \text{ number of cubic yards.}$$

$$3$$

10. Find the number of cubic yards in the wall of a dam 120 ft. long, 20 ft. high, 12 ft. thick at the bottom, and 3 ft. thick at the top.

11. A mixture for making a granolithic pavement was composed of $\frac{1}{12}$ best cement, $\frac{1}{3}$ hard sand, and $\frac{7}{12}$ broken stone. How many barrels of cement were needed if the pavement was 302.4 ft. long, 10 ft. wide, and 3 in. thick? (3 cu. ft. of cement = 1 barrel.)

12. Three abutments of the same size are made to support an iron bridge. The average length of each abutment is 22.6 feet, average width 6 feet, and height 15 feet. In the mixture used the cement is $\frac{1}{9}$ of the volume. Find the number of barrels used in the construction.

13. What is the cost of erecting the 3 abutments in the preceding problem at $5.40 a cubic yard?

14. How many cubic yards of masonry in a foundation whose outside measurements are 40 ft. long and 36 ft. wide, if it is 3 ft. high and 18 in. thick? What will it cost to build the wall at $5.10 a cubic yard?

15. What will it cost to lay a cement floor 2 in. thick in a basement 24 ft. × 18 ft., at $4.20 a cubic yard?

16. A wing to a house is to be built out from a wall so that three walls are needed, sides 16 ft. each and end 24 ft. Taking these as outside measurements, estimate the number of brick for walls 35 ft. high, 12 in. thick for 25 ft., and 8 in. thick for the upper 10 ft.

17. How many bricks will lay a city pavement 9 feet wide and 100 yards long, if 9 bricks will cover 2 square feet? (Use 8 × 4 faces.)

PAINTING AND PLASTERING

289. A gallon of liquid paint covers from 200 sq. ft. to 300 sq. ft. of surface with two coats.

WRITTEN EXERCISES

290. **1.** A town hall is 60 ft. long by 40 ft. wide and 27 ft. high. How much paint must be bought to give the outer walls two coats, allowing 1 gal. to 300 sq. ft.?

2. Find the cost of painting the outside of this hall at $12\frac{1}{2}¢$ a square yard, making no allowance for openings.

3. Again making no allowance, what will it cost to plaster the inner walls of this hall at 30¢ a square yard?

Find the cost of plastering the sides and ceiling of the following rooms at 27¢ per square yard:

4. 16 ft. × 14 ft., 11 ft. high, allowing 13 sq. yd. for openings.

5. 18 ft. × 16 ft., 12 ft. high, allowing $16\frac{2}{3}$ sq. yd. for openings.

6. 17 ft. × 13 ft., 10 ft. high, allowing 12 sq. yd. for openings.

PAPERING

291. Wall paper is put up in double rolls 16 yd. long and (unless otherwise stated) 18 in. wide.

A double roll contains 72 sq. ft., and when trimmed will cover about 70 sq. ft. of surface.

292. In estimating the number of double rolls for side walls, divide the number of square feet in the surface, after deducting for the openings, by 70. Estimate the number of double rolls required for the ceiling in the same way.

Paper is sometimes sold by the single roll, but nothing less can be bought.

293. Borders are also put up in double rolls of 16 yd. each. The number of yards required for a room is the same as its perimeter.

WRITTEN EXERCISES

294. **1.** How many double rolls of paper are required for the walls and ceiling of a room 15 ft. long, 12 ft. wide, and 10 ft. high, deducting 60 sq. ft. for openings?

54 ft. = perimeter of room.

$10 \times 54 = 540$, number of square feet in walls.

540 sq. ft. $-$ 60 sq. ft. $=$ 480 sq. ft.

$480 \div 70 = 6\frac{8}{7}$. Therefore 7 double rolls of side-wall paper are required.

Area of ceiling = 180 sq. ft.

$180 \div 70 = 2\frac{4}{7}$. Therefore 3 double rolls of ceiling paper are required.

The border required is readily found to be 18 yd.

2. What is the cost of the paper for the room mentioned in the preceding example, at 16¢ per double roll for side wall, 25¢ per double roll for ceiling, and 5¢ per yard for border?

3. Making no allowance for openings, what will be the cost of paper and border for 3 rooms, each 16 ft. square and 12 ft. high, at 20¢ per double roll for side wall, ceiling, and border?

4. At 20¢ per double roll, what will the hanging of this paper cost?

5. A library is 24 ft. long, 18 ft. wide, and 14 ft. high. Make an estimate of the cost of paper for this room at 24¢ per double roll for side wall, and 25¢ per double roll for ceiling and border, deducting 2 double rolls for windows and doors.

6. Estimate the cost of papering your schoolroom at 18¢ per double roll for side wall, ceiling, and border. Deduct for the area of doors and windows, and allow 25¢ per double roll for hanging.

CARPETING

295. In carpeting a room regard must be had for the dimensions of the room, the width of the carpet, and waste in matching patterns. In the problems that follow, waste in matching is neglected. A yard of carpet means a yard in length, and not necessarily a square yard.

WRITTEN EXERCISES

296. 1. A dining room is 27 feet long and 20 feet wide. How would you run the strips for the most economical laying of a carpet a yard wide? How many yards would be required to cover the floor? (Make a diagram.)

2. A hall is 12 ft. wide and 100 ft. long. How much will it cost to cover $\frac{3}{4}$ of the floor with carpeting $\frac{3}{4}$ yd. wide, at 90¢ a yard?

3. A room is 16 ft. × 18 ft. How much will it cost to carpet the room with carpeting 27 in. wide, at 75¢ a yard?

4. What length of carpeting will be needed to run one width down the center of a stairway, if there are 20 steps each 10 in. wide, and the rise is 8 in. to the step?

5. What will it cost to carpet two stairways like the one in Example 4, with carpeting at $1.25 a yard, including a charge of 5¢ a yard for the laying?

6. If a rug 6 ft. × 9 ft. costs $30, what will a similar rug 3 ft. × 6 ft. cost, at the same rate per square yard?

7. A school dormitory has 50 rooms, each 14 ft. × 15 ft. The dealer will sell druggets 9 ft. × 12 ft. at $12 each, or yard-wide carpeting at 75¢ a yard. Which will cost the more — druggets or carpeting — and how much? Which way should the strips be laid, if carpeting is used?

TANKS, BINS, RESERVOIRS, ETC.

297. You have already learned how to compute the volumes of rectangular solids. The tanks, bins, and reservoirs considered in the following problems are rectangular in shape, and their volumes are found in the same manner.

298. A liquid gallon contains 231 cu. in. A standard bushel contains 2150.42 cu. in.; in most calculations we may discard the decimal and use 2150.

WRITTEN EXERCISES

299. 1. Show that a cubic foot contains 7.48 + gallons.

2. Find the number of gallons in a rectangular tank 8 ft. × 8 ft. × 16 ft., allowing $7\frac{1}{2}$ gallons to a cubic foot.

$8 \times 8 \times 16 = 1024$, number of cubic feet.

$1024 \times 7\frac{1}{2} = 7680$, number of gallons.

Allowing $7\frac{1}{2}$ gallons to the cubic foot, find the capacity in gallons of :

3. A rectangular tank 12 ft. long, 10 ft. wide, and 4 ft. deep.

4. A rectangular vat 15 ft. long, 4 ft wide, and 4 ft. deep.

5. A mill race 100 ft. long, 4 ft. wide, and 4 ft. deep.

6. A rectangular cistern 15 ft. × 18 ft., 12 ft. deep.

7. A rectangular reservoir 120 ft. long, 80 ft. wide, and 15 ft. deep.

8. A reservoir whose length is 60 ft., width 32 ft., and depth 12 ft.

9. A bin is 16 ft. long, 8 ft. wide, and 8 ft. deep. How many bushels of wheat will it hold?

$$\frac{8 \times 8 \times 16 \times 1728}{2150} = 823+, \text{ number of bushels.}$$

10. An elevator is to contain 100,000 bushels of wheat. Its ground flour is 25 ft. × 36 ft. Find its height.

100,000 × 2150 cu. in. = 215,000,000 cu. in. required.
36 × 25 = 900, number of square feet in ground floor.
900 sq. ft. = 129,600 sq. in.
215,000,000 ÷ 129,600 = 1659 (nearly), number of inches in height.
1659 in. = $138\frac{1}{4}$ ft.

Find how many bushels each of the following will contain:

11. A bin 18 ft. × 15 ft. × 12 ft.

12. A bin 12 ft. × 10 ft. × 4 ft.

13. A crib 20 ft. × 15 ft. × 6 ft.

14. A bin 21 ft. × 10 ft. × 6 ft.

15. What is the depth of a reservoir 20 ft. square that holds 120,000 gal., counting 7.5 gal. to a cubic foot?

16. A cord of wood is 8 ft. long, 4 ft. wide, and 4 ft. high. How many cubic feet does it contain?

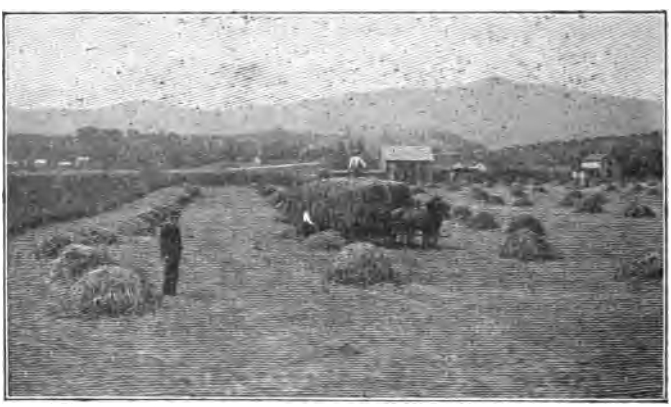
17. How many cords in a pile of wood 16 ft. long, 8 ft. wide, and $3\frac{1}{2}$ ft. high?

18. A wagon body is 12 ft. long and $3\frac{1}{3}$ ft. wide. How high must it be to hold a cord?

19. What part of a cord will a wagon body contain, if it is 3 ft. $\times$ $3\frac{1}{3}$ ft. $\times$ 8 ft.?

20. At \$3 a cord, a man paid \$18 for a pile of wood 3 ft. high and $21\frac{1}{3}$ ft. long. How wide was it?

RELATED PROBLEMS



300. 1. The picture represents a harvest scene on a farm. The crop in the foreground was a mixture of cow peas and millet. If the field was a rectangle 40 rd. by 48 rd. and the yield $3\frac{1}{2}$ T. per acre, how many tons were harvested?

2. What was the value of this hay at \$12.50 a ton?

3. The farmer sowed 1 bu. of each to the acre. Peas and millet seed cost respectively $2 and $1.50 a bushel. If the cost of preparing the land, seeding, and harvesting was $6.50 an acre, what were his net profits?

4. A 10-acre field on the same farm yielded 24½ bu. of wheat per acre. At 95¢ a bushel, how much did the crop bring?

5. If the yield of wheat was 20% greater the preceding year, what was the yield per acre that year?

6. Another field 20 rd. by 64 rd. yielded 42 bu. of oats per acre. If the oats brought $.37½ a bushel, what was the value of the oat crop?

7. If the number of acres planted in potatoes was 62½% less than the number sown in oats, how many acres were planted in potatoes?

8. If the yield of potatoes was 120 bu. per acre, what was the value of the potato crop at 65¢ a bushel?

9. Find the cost of the potatoes required to plant this field, allowing 10 bu. to the acre, the seed potatoes being worth $1.25 a bushel.

10. If in addition to the foregoing there were on this farm a vegetable garden 16 rd. by 20 rd., and a corn field 40 rd. square, how many acres were in cultivation on the entire farm?

11. If 300 lb. of fertilizer were used on each acre in cultivation, how many tons were required? What did it cost at $18 a ton?

12. The owner used as a pasture a rectangular field containing 15 acres. If this field was 30 rd. wide, how long was it?

SOLUTIONS AND PRACTICE PROBLEMS

301. You are now ready to make some further applications of what you have learned about numbers to the solution of practical problems. (See Art. 144.)

In making solutions you should endeavor to secure facility and accuracy. You also need to understand clearly each step of the operation.

302. In written solutions the work should be arranged in a neat and orderly manner, so as to suggest the reasons for the various steps. Diagrams are often helpful and suggestive. When practicable, shorten the work by cancellation. To avoid mistakes, always test your results.

303. 1. The area of a rectangle is 120 sq. in., and the width is 10 in. What is the length?

GIVEN: the width and area. We also know that the area is the product of the numbers measuring the length and width.

TO FIND: the length. If the product and one factor are given, how is the other factor obtained?

ANALYSIS

10×1 sq. in. $= 10$ sq. in., area of a strip 10 in. long and 1 in. wide.

120 sq. in. $\div$ 10 sq. in. $= 12$, no. of strips, and hence the no. of inches in the length.

Test by ascertaining if the area of a rectangle 10 in. by 12 in. is 120 sq. in.

CALCULATION

$$10\overline{)120}\quad 12$$

2. A 5-acre lot is 20 rd. wide. How long is it?

3. A field containing 20 acres is 32 rd. wide. What is its length?

4. A farmer covered his barn with galvanized iron. What did it cost him at \$7 per square (100 sq. ft.), if each side of the roof is 75 ft. by 25 ft.?

ANALYSIS

$25 \times 75 = 1875$, no. of sq. ft. in one side.

2×1875 sq. ft. $= 3750$ sq. ft., area of roof.

3750 sq. ft. $\div$ 100 sq. ft. $= 37.5$, number of squares.

$37.5 \times \$7 = \$262.50$, the cost.

WORK INDICATED

$$\frac{2 \times 25 \times 75 \times \$7}{100} = \$262.50$$

$$4$$
$$2$$

What is given in this problem? What is to be found? Show how this is obtained from what is given. How is the work shortened?

5. Each side of a roof is 50 ft. long and 20 ft. wide. What is the cost of roofing it at \$8.75 a square?

6. If 1 gal. of paint covers 300 sq. ft. of surface with 2 coats, how many gallons are required to paint (with 2 coats) the outer walls of a building 35 ft. long, 28 ft. wide, and 20 ft. high?

7. If a man earns \$1600 a year, and spends \$790, in how many years can he save enough to buy a farm of 135 acres, at \$24 an acre?

$135 \times \$24 = \$3240$, value of farm.

$\$1600 - \$790 = \$810$, saving in 1 yr.

$\$3240 \div \$810 = 4$, no. of years.

135	1600
24	790
540	810
270	

$$810)3240(4$$
$$3240$$

8. A man earns \$75 a month. He spends \$10 a month for rent, \$20 a month for board, and \$140 a year for other expenses. In how many years will he save \$2000 at this rate?

9. A dealer bought 200 barrels of apples at \$3.50 a barrel. He sold 120 barrels at \$4 a barrel, but owing to a decline in the market price he was compelled to sell the rest at \$3.40 a barrel. How much did he gain on the transaction?

$200 \times \$3.50 = \$700$, the cost.
$120 \times \$4 = \$480$, received for 120 bu.
$80 \times \$3.40 = \$272$, received for 80 bu.
$\$480 + \$272 = \$752$, total receipts.
$\$752 - \$700 = \$52$, the gain.

What is given in the problem? What is to be found? Explain the different steps in the solution.

10. A dealer bought 120 arithmetics at 42¢ each, and sold them at 54¢ each. How much did he gain on the whole number?

11. A grocer bought 300 lb. of prunes for \$27, and sold 160 lb. at 11¢ a pound, and the rest at 7¢ a pound. Did he gain or lose, and how much?

12. A grocer bought a barrel of sugar (320 lb.) at $4\frac{1}{2}$¢ a pound. It lost 5% of its weight by drying, and was then sold at $6\frac{1}{4}$¢ a pound. What did he gain?

$320 \times \$.04\frac{1}{2} = \$14.40$, the cost.
$.05 \times 320$ lb. $= 16$ lb., loss in weight.
320 lb. $- 16$ lb. $= 304$ lb., left.
$304 \times \$.06\frac{1}{4} = \$19$, selling price.
$\$19 - \$14.40 = \$4.60$, the gain.

13. A farmer bought 30 head of cattle at \$50 a head, and sold them at a profit of $2\frac{1}{2}$%. What did he gain?

14. If it costs a farmer \$4.25 per 100 bu. to have his wheat threshed, how many bushels will he have to give as toll for threshing a crop of 280 bushels, when the market price is 85¢ a bushel?

$\$4.25 \div \$.85 = 5$, number of bushels for threshing 100 bu.
5 bu. out of 100 bu. = 5%.
$.05 \times 280$ bu. $= 14$ bu., amount required.

15. A farmer who owed \$250 gave in part payment 150 bu. of wheat at 90¢ a bushel, and paid the remainder with flour at \$5 a barrel. How many barrels did he give?

16. The depth of a certain rainfall was 1.25 inches. What weight of water fell on a lot 40 feet by 120 feet, if a cubic foot of water weighs 1000 ounces?

(*a*)

1.25 × 480 × 1440 = 864,000, number of cubic inches of water.
864,000 cu. in. ÷ 1728 cu. in. = 500, number of cubic feet.
500 × 1000 oz. = 500,000 oz., the weight.

(*b*)

$$\frac{1.25 \times 40 \times 120 \times 144 \times 1000}{1728} = \text{——}.$$

Explain the advantage of first indicating the work, as in (*b*). Cancel and find the result.

17. A 2-acre pond is covered with ice 6 in. thick. If a cubic foot of ice weighs 920 oz., how many tons of ice are on the pond?

18. A and B rent a pasture for \$24. A puts in 4 cows for 2 weeks, and B puts in 5 cows for 8 weeks. How much should each pay?

4 cows for 2 wk. = 2 × 4 cows, or 8 cows, for 1 wk.
8 cows for 5 wk. = 5 × 8 cows, or 40 cows, for 1 wk.
8 cows for 1 wk. + 40 cows for 1 wk. = 48 cows for 1 wk.
48 cows for 1 wk. cost \$24.
1 cow for 1 wk. costs $\frac{1}{48}$ of \$24, or \$.50.
8 cows for 1 wk. cost 8 × \$.50, or \$4, A's part.
40 cows for 1 wk. cost 40 × \$.50, or \$20, B's part.

19. Three men pasture horses in the same field, the first pasturing 6 horses for 9 weeks, the second 9 horses for 8 weeks, and the third 12 horses for 7 weeks. They pay \$105. How much of the rent should each pay?

20. A can build a fence in 4 days, and B can build it in 5 days. How long will it take both together?

A can build $\frac{1}{4}$ of the fence in 1 da.
B can build $\frac{1}{5}$ of it in 1 da.
Both can build $\frac{1}{4} + \frac{1}{5}$, or $\frac{9}{20}$, of it in 1 da.
$\because$ $\frac{9}{20}$ of the work require 1 da.,
$\frac{1}{20}$ of it requires $\frac{1}{9}$ da.,
and $\frac{20}{20}$ of it require $\frac{20}{9}$ da., or $2\frac{2}{9}$ da.

21. A can do a piece of work in 2 days, B in 4 days, and C in 6 days. How long would it take them to do it together?

22. If 12 lb. of tea cost \$6, how much must be paid for 18 lb.?

23. If an automobile goes 54 miles in 3 hours, how far will it go in 8 hours at the same rate?

---

24. If 1 reaper can cut 15 acres a day, how long will it take 3 reapers to cut a field of 900 acres at the same rate?

25. If $\frac{3}{4}$ of a cord of wood costs \$3, how much will 5 cords cost at the same rate?

26. If 8 barrels of flour cost \$52.80, what will 3 barrels cost at the same rate?

27. A 3-acre field of potatoes yields 165 bu. At the same rate, how many bushels will a 20-acre field produce?

28. If 5 people consume on an average 375 lb. of sugar in a year, how much does a family of 7 persons consume in the same time?

29. If 12 bales of cotton weigh 5760 lb., what do 8 bales of the same average weigh?

30. A lady bought a dress pattern for \$11.25. If she paid 90¢ a yard for the material, how many yards were in the pattern?

31. How many 3-inch bandages can be torn crosswise from $5\frac{1}{2}$ yd. of medicated cloth?

32. A bushel of wheat weighs 60 lb. How many bushels of wheat are there in a shipment of 5 car loads, each containing 22,200 lb.? What is the value of this wheat at 95¢ a bushel?

33. If a cubic foot of granite weighs 165 lb., what is the weight in tons of a block of granite 6 ft. long, 4 ft. wide, and 3 ft. thick?

34. A dealer buys 700 lb. of meat at 14¢ a pound, and sells it at 18¢ a pound. How much profit does he make, allowing 5% for loss in weight?

35. A merchant pays $144 for 20 dozen pairs of gloves, and sells them at 75¢ a pair. How much profit does he make?

36. I bought a watch for $60, and paid $\frac{1}{3}$ of the price in cash. How much must I pay per week to pay the remainder in 8 weeks?

37. A dealer bought 304 lb. of rice for $19. If $\frac{1}{4}$ of it was destroyed, and he sold the remainder at $8\frac{1}{2}$¢ a pound, how much was his gain?

38. A grocer bought 30 bu. of sweet potatoes at 90¢ a bushel, and sold them at 35¢ a peck. How much did he gain?

39. A man had 12 hives of bees. Four hives yielded in a year 35 lb. of honey each, which he sold at 10¢ a pound. The rest yielded 40 pounds each, which he sold at 12¢ a pound. What were the bees worth to him that year?

40. What is the value of the fruit from an orchard of 130 pear trees, if the trees average 12 bu. each, worth $1.25 a bushel?

41. A field of 14 acres produces 26 bu. of wheat per acre, and each bushel of wheat makes 42 lb. of flour. How many barrels of flour does the crop make? (1 barrel = 196 lb.)

42. Find the cost of the ties for a mile section of single-track railway, the ties being placed 2 ft. from center to center, if the ties cost 40¢ each.

43. A man bought an equal number of cows and horses. For the cows he paid $35 a head and for the horses $125 a head. The cows and horses together cost him $1920. How many of each did he buy?

44. A and B hired a carriage for which they paid $12. A used it 1 day, and B used it 2 days. How much should each pay?

45. Harry worked 40 days, and after spending $\frac{2}{5}$ of the money earned he had $48 left. How much did he receive per day for his labor?

46. A farmer has 5 horses to each of which he feeds $\frac{1}{2}$ pk. of oats three times a day. If oats are worth 40¢ a bushel, what is the cost of the oats required to feed them 30 days?

47. How many fire bricks, each 8 in. by 4 in. by 2 in., can be packed in a box 32 in. by 16 in. by 20 in.?

48. A man wishes to make a bin to contain 144 bu. of grain. If he makes it 8 ft. long and 4 ft. wide, how deep must it be, allowing 2150 cu. in. to a bushel?

49. What will it cost to floor a room 30 ft. by 16 ft., with flooring 1 in. thick, at $30 per thousand, if $\frac{1}{3}$ is added for tongue?

50. A mechanic received $3 a day for his labor and paid $5 a week for expenses. He saved $100 in 10 weeks. How many days was he idle?

51. A physician made 40 calls and saw 23 office patients in one day. If his fees were \$3 each for calls and \$1 each for office patients, what was the total amount of his fees that day?

52. In a farmer's hennery there are 40 chickens; $\frac{1}{5}$ as many geese; $\frac{1}{4}$ as many ducks as chickens and geese; and twice as many turkeys as chickens, ducks, and geese together. How many turkeys are there?

53. A cuts corn in shocks 12 hills by 18 hills at 8¢ a shock, B cuts shocks 6 hills by 20 hills at 6¢ a shock, and C cuts shocks 16 hills square at 10¢ a shock. What would each receive for cutting a field of corn containing 103,680 hills?

54. A boy can read 2400 words in 1 hour. How many hours will it take him to read a book of 450 pages, 30 lines on a page, and 10 words in a line?

55. A farmer has an orchard of 120 apple trees; $\frac{2}{5}$ of them bear 8 bu. each, $\frac{1}{2}$ of them bear 15 bu. each, and the rest bear 20 bu. each. What is his crop worth at \$1.50 per barrel for half of them and \$1.80 per barrel for the other half? (3 bu. = 1 bbl.)

56. A farmer wishes to exchange 100 bu. of wheat with his neighbor for an equal weight of corn. How many bushels of corn should he receive for his 100 bu. of wheat? (A bushel of wheat weighs 60 lb., and a bushel of corn weighs 56 lb.)

57. A business man bought postage stamps as follows: 7 fractional sheets of 2-cent stamps, containing 4 rows of 7 stamps each; 5 sheets of 1-cent stamps, containing 10 rows of 10 stamps each; and 3 fractional sheets of 5-cent stamps containing 8 stamps square. What did they all cost him?

58. A grocer bought 200 barrels of flour at \$4.90 a barrel. He sold 95 barrels at \$5 a barrel, 75 barrels at \$5.25 a barrel, and the rest at \$4.25 a barrel. Did he gain or lose, and how much?

59. A merchant bought goods amounting to \$3000. He sold half of them at a profit of 25%, and the other half at a profit of 15%. Find his total gain.

60. What is the value of a rectangular field 120 rd. long and 40 rd. wide, at \$60 an acre?

61. A and B can build a house in 3 months. B alone can build it in 8 months. In what time can A build the house?

62. A dealer bought 60 young turkeys at 2 for a dollar from one party, and 60 at 3 for a dollar from another. He sold all at 5 for \$2. Did he gain or lose, and how much?

63. A stationer bought 3 gross of pens at 75¢ a gross, and retailed them at 10¢ a dozen. How much did he gain? (12 dozen = 1 gross.)

64. A teacher bought a ream of examination paper for \$1.50, and sold it to pupils at the rate of 3 sheets for a penny. Did he gain or lose, and how much? (24 sheets = 1 quire; 20 quires = 1 ream.)

65. A dealer bought 5 boxes of letter paper at 90¢ a box, and sold it at 30¢ a hundred sheets. How much did he gain? (Letter paper is usually put up in boxes containing 500 single or half sheets. These boxes are sometimes called reams.)

66. A rectangular rice field is 40 rd. by 50 rd. If the yield is 16 sacks of 162 lb. each, in the rough, to the acre, what is the total yield? What is the value of the crop at 2¢ a pound? If it cost \$10 an acre to raise the crop, what is the net profit per acre?

67. The cost of building a greenhouse was as follows: excavation \$20, stone and brick \$119, glass \$60, lumber and labor \$240, hardware \$35, and paint \$50. If the builder's profit was 10% on the above cost, and heating and plumbing cost \$165, what was the total cost?

68. Allowing 8 bu. to the acre, what is the cost of the seed potatoes required to plant a rectangular lot 20 rd. by 24 rd., if the potatoes are worth \$1.75 a bushel?

69. A ton of a certain fertilizer contains 750 lb. of acid phosphate, worth \$12 a ton; 900 lb. of cottonseed meal, worth \$25 a ton; 100 lb. of nitrate of soda, worth \$48 a ton; and 250 lb. of sulphate of potash, worth \$40 a ton. What does a ton of the fertilizer cost?

70. A hotel required 40 lb. of steak for a meal. If 2 lb. was required for 9 guests, how many guests were served?

71. A college which had 300 students graduated 20% of them. How many undergraduates were left on its rolls?

72. At a telephone office the number of calls received on Friday was 1500. On Saturday the number of calls showed an increase of 20% over the previous day. How many calls were received on Saturday?

73. At \$12 per M, find the cost of the following framing materials:

4 pieces, 6 in. × 8 in., 16 ft. long, for sills;
35 pieces, 2 in. × 8 in., 16 ft. long, for joists;
4 pieces, 4 in. × 4 in., 16 ft. long, for studding;
8 pieces, 2 in. × 4 in., 16 ft. long, for plates;
42 pieces, 2 in. × 4 in., 14 ft. long, for rafters;
150 pieces, 1 in. × 3 in., 16 ft. long, for sheathing.

RELATED PROBLEMS

304. 1. This picture represents the entrance to a coal mine. The picture on the next page shows a train carrying the coal to market. The average output of this mine was 5600 tons a day. How many trains a day would be required to haul this product, if each train consisted of 35 cars and each car carried 40 tons of coal?

2. If each miner with his helper got out 10 tons a day, how many miners and helpers were employed in the mine?

3. The coal traffic on a certain railroad was 766,250 tons in one month. How many car loads of 50 tons each did it take to haul this coal? If this traffic was handled in 25 working days, what was the average number of cars handled per day?

4. At 50¢ a ton, what did the mining of this coal cost? What did each miner clear a day, if he paid his helper $1 a day and their average daily output was 10 tons?

5. If the output of bituminous coal in the United States in a recent year was 277,077,000 tons, and the output of anthracite the same year was 27% of the output of bituminous coal, how much anthracite was mined that year?

6. If 30,000,000 tons of coal yield 19,000,000 tons of coke, how many tons of coal were coked in a year, when the coke supply of the United States was 25,262,400 tons?

7. If the value of the coke that year was $66,440,112, what was it worth a ton?

8. If this country contains 41% of the coal fields of the world, and the latter aggregate 472,000 sq. mi., what is the area of our coal fields?

9. If a vein of coal yields 1 ton to the square yard of surface, what is the value of the coal that underlies 1 sq. mi., at $1.25 a ton?

10. The number of tons of bituminous coal mined in the five leading coal-producing states in a recent year, and the value of the same at the mines, was as follows:

Pennsylvania,	103,000,000 T.,	worth $1.12 per ton;
Illinois,	34,955,073 T.,	worth $1.04 per ton;
West Virginia,	26,882,526 T.,	worth $1.05 per ton;
Ohio,	24,573,266 T.,	worth $1.20 per ton;
Alabama,	11,700,753 T.,	worth $1.25 per ton.

What was the total value of the product in the states named?

PERCENTAGE REVIEWED

305. As we have seen, the *percentage* is found by taking a certain number of hundredths of a number.

306. That part of arithmetic which deals with the use of hundredths or per cents is also called **Percentage.** Hence, the finding of so many hundredths of a number is the elementary problem of percentage.

The *rate per cent* is the number of hundredths to be found.

307. As we have seen, there are three ways of writing a given per cent. Thus, $\frac{2}{100} = .02 = 2\%$.

308. Write each of the following in two other ways:

.25.	.075.	16%.	$\frac{1}{8}$.
.14.	22%.	$6\frac{1}{2}\%$.	$\frac{1}{2}$.
.15.	31%.	$12\frac{1}{2}\%$.	$\frac{1}{5}$.
.40.	66%.	$\frac{1}{10}$.	$\frac{1}{20}$.

309. Familiarity with the per cents and their equivalents given in Art. 246, is essential to facility in percentage calculations. Hence, they should be reviewed at this point.

310. To find the percentage.

You have learned in Art. 251 how to find any required per cent of a number. We now wish to pursue the subject farther.

ORAL EXERCISES

311. 1. What is $12\frac{1}{2}\%$ of 800?

$12\frac{1}{2}\%$ of 800 = $\frac{1}{8}$ of 800, or 100.

2. What is $87\frac{1}{2}\%$ of \$1600?

$87\frac{1}{2}\% = \frac{7}{8}$.

What is —

3. 75% of 80?
4. 80% of \$25?
5. 10% of \$47.80?
6. $33\frac{1}{3}\%$ of \$30?
7. 10% of 2540?
8. $16\frac{2}{3}\%$ of 6666?
9. $33\frac{1}{3}\%$ of 300?
10. $62\frac{1}{2}\%$ of \$800?
11. 30% of \$200?
12. 50% of \$250?

13. If you lose 5% of your property, what per cent will you have left?

14. How much is 100% of the value of a 10-dollar gold piece?

15. What is 100% of 29 horses?

1% of a number is easily found by moving the decimal point 2 places to the left. (See Art. 245.)

Give at sight 1% of each of the following:

16. \$5786.
17. 28,500 bu.
18. 200 head of cattle.
19. 675 quarts.
20. \$960.
21. \$10.
22. 1000 soldiers.
23. .001.
24. 6500 miles.
25. 10,000 lb.

26. If a man's income averages \$1000 a year, and he saves 10% of it, how much will he save in 10 years?

27. A drover bought 150 head of cattle. If $33\frac{1}{3}\%$ of them were steers and the rest cows, how many cows did he buy?

WRITTEN EXERCISES

312. 1. Find 21% of \$8965.

From Art. 251 we have:

(*a*)

1% of 8965 = \$89.65.

21% of \$8965 = 21 × \$89.65, or \$1882.65.

(*b*)

\$8965
.21
8965
17930
\$1882.65

2. Find $66\frac{2}{3}$% of 61,584 bricks.

$66\frac{2}{3}\% = \frac{2}{3}$.

$\frac{2}{3}$ × 61,584 bricks = 41,056 bricks.

3. Find $\frac{2}{3}$% of \$600.

Calculate, using the method best suited to each example:

4. 75% of 1200 horses.
5. 220% of 525 men.
6. 50% of $\frac{1}{10}$.
7. $33\frac{1}{3}$% of 458,796 bushels.
8. 12% of \$251.
9. 150% of 356 yards.
10. 90% of 1820 men.
11. 25% of \$2500.

12. What is the difference between 50% of \$175 and 175% of \$50?

13. If a rope when wet shrinks 4% of its length, to what length would 525 feet of rope shrink?

14. A bankrupt settled his indebtedness of \$12,500 by paying his creditors 65%. How much money did his creditors lose by the settlement?

15. A farmer raised 1600 bu. of grain. If 30% of it was wheat, 50% corn, and 20% oats, how many bushels of each did he raise?

16. A man bought 42 head of young cattle at \$30 a head. He sold the herd the next year at an advance of 15%. What was the selling price?

17. On account of damage to my new surrey in a wreck, the railroad company paid me $33\frac{1}{3}\%$ of its value. How much did I recover, if it was invoiced at $312?

18. A hunter shooting partridges used 36 cartridges. He then found that the number of birds bagged was 75% of the number of cartridges used. How many birds did he bag?

19. A gray horse sells for 85% as much as a bay of the same grade, which costs $150. Find the cost of the gray.

20. A reservoir whose capacity is 1,200,000 gallons averages 56% of its capacity during a drought. Find the average number of gallons it contains.

21. A haberdasher whose capital is $8500 determines to spend 22% of it in hats, 53% in suitings, 15% in underwear, and 10% in other furnishings. What amount is expended in each class of goods?

22. A man has $1500 with which to furnish a house. He decides to expend it as follows: for bedrooms, 25%; for parlor and library, 45%; for halls, $5\frac{1}{2}\%$; for kitchen and pantry, $4\frac{1}{2}\%$; for dining room, 20%. How much money does he expend in the different kinds of furnishings?

23. In a certain month a publisher sold 100,000 textbooks. The sales were distributed as follows: readers, 40%; arithmetics, 25%; spellers, 15%; histories, 12%; grammars, 8%. How many of each kind were sold?

313. To find the rate per cent.

In finding the percentage in the previous article, the base and the rate per cent were two factors, which multiplied together gave the percentage as a product.

By what operation can you find one of the factors if the product and the other factor are given?

ORAL EXERCISES

314. Supply the missing factors:

1. $3 \times (\quad) = 12$.
2. $.03 \times (\quad) = 12$.
3. $(\quad) \times \$600 = \$18$.
4. ( ) per cent of $\$600 = \$18$.

5. What fractional part of 24 is 12? How many hundredths of 24 is 12? Then what per cent of 24 is 12?

The percentage divided by the base gives the rate.

What per cent of —

6. \$25 is \$5?
7. 100¢ is 60¢?
8. 10¢ is 5¢?
9. 1 is $\frac{1}{5}$?
10. 2 is $\frac{1}{5}$?
11. 4 oz. is 1 oz.?

WRITTEN EXERCISES

315. 1. 6 is what per cent of 30?

(*a*)

$\because$ $1 = \frac{1}{30}$ of 30,

$6 = \frac{6}{30}$ of 30, or $\frac{1}{5}$ of 30.

$\frac{1}{5}$ of $30 = \frac{20}{100}$ of $30 = 20\%$ of 30.

TEST: 20% of $30 = 6$.

(*b*)

$$30)6.00 \quad .20$$
$$60$$
$$0$$

In practice, the rate is readily found as in (*b*).

Find what per cent —

2. \$8 is of \$40.
3. 15 is of 60.
4. 5 is of 40.
5. \$30 is of \$75.

6. A bankrupt has assets of \$25,000 and liabilities of \$100,000. What per cent of his liabilities can he pay?

7. A school has 65 pupils. If 52 are present, what is the per cent of attendance?

8. The income of a business in 1905 was \$75,000; in 1906, \$82,500. Find the per cent of increase of income.

9. A drover starts to market with a herd of 1500 cattle. If 10 stray and 5 die, what per cent of them will he be able to deliver?

10. A man bought a farm for $6500, and paid $2990 cash, $1660 in 1 year, and $1850 in 2 years. What per cent of the purchase price was deferred?

11. The annual expenses of a central heating plant were $4850. If $970 was paid for labor, what per cent of the annual expense was for labor?

12. From a barrel of molasses containing $51\frac{1}{2}$ gallons, 25 gallons 3 quarts are drawn off. What per cent of the contents remain?

13. What per cent of the day does a man work who works 9 hours each day?

14. The area of North America is 9,350,000 square miles, and the Mississippi watershed has an approximate area of 1,215,500 square miles. What per cent of the area of this continent is drained by the "Father of Waters"?

15. If a steer weighing 1235 lb. furnishes 741 lb. of salable beef (net weight), what per cent of the gross weight is the net weight?

16. In a certain state a candidate for governor won over his competitor by a majority of 30,000 out of 150,000 votes polled. What per cent of the total vote did the loser receive?

316. To find the base.

Again we have the finding of one factor when the other factor and the product are given. In the preceding article, the rate was the missing factor; in this, the base.

ORAL EXERCISES

317. State the missing factor:

1. $\frac{5}{100} \times (\ ) = 3.$

$3 \div \frac{1}{100} = 60.$

2. $.05 \times (\ ) = 55.$
3. $.03 \times (\ ) = 78.$
4. $.03 \times (\ ) = 96.$
5. $.25 \times (\ ) = 48.$
6. $.02 \times (\ ) = 50.$
7. $.04 \times (\ ) = 52.$

The percentage divided by the rate gives the base.

State the number of which —

8. 75 is 5%.
9. \$75 is 20%.
10. 40 is 25%.
11. 25 is 50%.
12. 10 is $12\frac{1}{2}\%$.
13. 60 is $66\frac{2}{3}\%$.
14. 35 is $33\frac{1}{3}\%$.
15. \$20 is $16\frac{2}{3}\%$.

WRITTEN EXERCISES

318. 1. \$155 is 5% of what number?

(*a*)

∵ 5% of the number = \$155,

1% of the number = $\frac{1}{5}$ of \$155, or \$31.

100% of the number = 100 × \$31, or \$3100.

(*b*)

$5\overline{)15500}$

3100

TEST: \$155 = 5% of \$3100.

In practice, the work is usually performed as shown in (*b*).

2. 60 is 3% of what number?
3. \$152.64 is 4% of what number?
4. 224 is 8% of what number?

Find the number of which —

5. 1194 is 6%.
6. 1075 is 25%.
7. 3624 is $16\frac{2}{3}\%$.
8. 2849 is $33\frac{1}{3}\%$.
9. 4786 is 25%.
10. 296 is $8\frac{1}{3}\%$.

11. 273 is 20%.
12. 9264 is $16\frac{2}{3}\%$.
13. 2800 is 75%.
14. 1800 is $66\frac{2}{3}\%$.
15. 4722 is $37\frac{1}{2}\%$.
16. 8265 is $12\frac{1}{2}\%$.

17. The owner of a threshing machine charges $7\frac{1}{2}\%$ toll. What was A's crop if he paid $130\frac{1}{2}$ bushels for threshing?

18. A charity concert realized $490.75 as the net proceeds. If the expenses were 35% of the whole, how much was received?

19. Of the passengers on an ocean liner 231 pay cabin fare. The rest of the passengers are in the steerage, and constitute 58% of the whole number. How many passengers are there all together?

20. A man died leaving an estate to be divided among three children. The first was to get 42% of it, the second 32%, and the third the balance, or $6661.20. What was the value of the estate?

319. To find a number when the number increased (or diminished) by a certain per cent is given.

ORAL EXERCISES

1. What number increased by $\frac{1}{5}$ of itself equals 24? By 20% of itself equals 24?

2. What number decreased by $\frac{1}{4}$ of itself equals 12? By 25% of itself equals 12?

3. After increasing my money by 25% of itself I have $50. How much did I have at first?

4. If I spend $33\frac{1}{3}\%$ of my money and then have $20, how much had I at first?

WRITTEN EXERCISES

320. **1.** What number increased by 10% of itself equals 1210?

100% of the number + 10% of it = 110% of the number.

110% of the number = 1210,

1% of the number = $\frac{1}{110}$ of 1210, or 11,

and 100% of the number = 100 × 11, or 1100.

2. What number diminished by 60% of itself equals 480?

100% of the number − 60% of it = 40% of the number.

∵ 40% of the number = 480,

1% of the number = $\frac{1}{40}$ of 480, or 12,

and 100% of the number = 100 × 12, or 1200.

3. What sum increased by 5% equals $210?

4. What number increased by 32% equals 1980?

5. What number diminished by 50% equals 1800?

6. What sum diminished by 12% equals $660?

7. A man sold a horse for $275, thereby gaining 10%. Find the cost of the horse.

8. After a 10% advance in wages, the pay roll of a factory was $16,500. What was it before the advance?

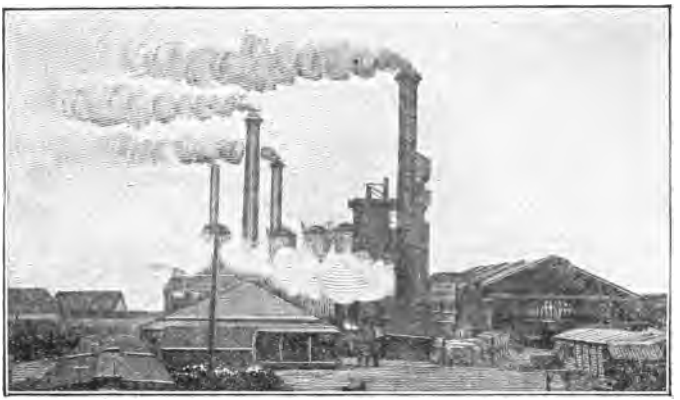
9. A farmer sold $33\frac{1}{3}$% of his farm, and had 240 acres left. How many acres had he before the sale?

10. A high school has 368 pupils, an increase of 15% over last year. How many pupils had it last year?

11. By the use of fertilizers a farmer increased the yield of a 10-acre corn field by 40%. After the increase it was 420 bu. What was the yield per acre without fertilizing?

12. By exercise an athlete reduced his weight 15%, thereby bringing it down to 178.5 lb. What was it before he went into training?

RELATED PROBLEMS



321. 1. In blast furnaces like that shown in the picture, iron ore is freed from some of its impurities and converted into pig iron. In the foreground the pieces of "pig" ready for the cars resemble sticks of wood. If the capacity of the furnace is 250 tons a day, how many tons does it turn out in a common year, allowing 43 days for repairs?

2. What is the value of this yearly product on the yard, at \$16.50 a ton? What is the freight on this product to market at \$2.12$\frac{1}{2}$ a ton?

3. If this furnace requires 3 tons of ore, 1 ton of coke, and 1 ton of limestone for each ton of pig iron, how many tons of each are required in a day? (The stack is charged at short intervals with ore, coke, and limestone. The coke is ignited by hot blasts from below; the limestone makes the melted iron run or flow into molds when the furnace is tapped or opened.)

4. On the basis of the time given in Example 1, how many car loads of 10 tons each are required to haul in the raw material (ore, coke, and limestone) used by this furnace in a year? How many of 50 tons each are required to haul out the pig iron?

5. If the cars average 40 ft. in length, including couplings, engines, and tenders, how many miles would all these cars reach if made up in one train?

6. If the 250 tons are made in 4 "runs" a day, and the pigs average 100 lb. each, how many pigs are turned out at each run? (2240 lb. of pig iron = 1 ton, called a long ton.)

7. In a recent year the United States produced 17,816,000 long tons of pig iron. If this pig iron was worth \$16.05 a ton, what was the value of the product?

8. In the same year the steel output was $80\frac{3}{5}\%$ of that of iron. How many tons of steel were produced in the United States that year?

9. In 1850 this country exported \$1,953,700 worth of iron and steel, and in 1904, \$111,949,540 worth. What was the average yearly increase?

10. The total value of the manufactured products of the United States in a recent year was \$13,000,000,000. The value of the iron and steel products was $6\frac{2}{5}\%$ of the whole. What was the value of the iron and steel products that year?

11. In a certain year this country produced 3,301,200 tons of steel rails. If these rails averaged 90 lb. to the yard, how many miles would they fall short of making a single-track railroad around the earth? (Use 25,000 miles for the circumference of the earth.)

COMMISSION

322. When a man transacts business for another, he expects to be paid for the service rendered. When a laborer is hired to plow the field, cut the corn, carry the hod, etc., he is paid a daily or weekly wage. When a clerk, bookkeeper, or stenographer is employed, he is paid a salary. When a man handles money for you, or buys and sells property, the amount which he receives as a remuneration is called **Commission**, and is often calculated by percentage methods.

For example, a farmer sends his products to a merchant who sells for him on 5% commission. If they bring $1250, the commission merchant will get 5% of $1250, or $62.50. The farmer will receive the balance.

323. When an agent collects money, he usually deducts his commission, and then remits the balance, called *net proceeds*.

ORAL EXERCISES

324. A real estate agent collects rents and charges 10% commission. State what he receives for collecting each of the following amounts:

1. $150.	5. $2150.	9. $829.
2. $2755.	6. $1875.	10. $5450.
3. $1158.	7. $285.	11. $5432.
4. $2925.	8. $6286.	12. $682.

13. A merchant sells $100 worth of produce. At the rate of $3\frac{1}{2}\%$, what was his commission?

14. A lawyer charged 10% for collecting a claim of $500. How much did he remit to his client?

WRITTEN EXERCISES

325. **1.** A broker sold a car load of wheat for \$724, charging $2\frac{1}{2}\%$ commission. What was his commission, and how much did he send the owner of the wheat?

2. A commission merchant sold 140 barrels of flour at \$5.20 a barrel. Find the commission and net proceeds, if $3\frac{1}{2}\%$ was allowed for selling.

3. Find the commission, at $2\frac{1}{2}\%$, on 500 bu. of wheat sold at 93¢ a bushel.

4. Find the net proceeds of a bill of 150 boxes of oranges sold at \$2.50 a box and 200 boxes of grape fruit sold at \$4 a box, if the rate of commission was 5%.

5. A collector's commission for collecting taxes is $1\frac{1}{2}\%$. How much must he collect in a month to make \$75?

6. A real estate agent sold 10 lots at an average price of \$755. He paid \$4.75 each for examination of title, and charged a commission of 2% on the price of the lots. What were the net proceeds?

7. Anderson, Sloane & Co. are agents, and remit monthly their collections after subtracting their commission of 10%. Find how much they remit each month, if collections are as follows:

(1) January, \$150.
(2) February, \$2755.
(3) March, \$1158.
(4) April, \$2925.
(5) May, \$829.
(6) June, \$5450.
(7) July, \$2150.
(8) August, \$1875.
(9) September, \$285.
(10) October, \$6286.
(11) November, \$5432.
(12) December, \$682.

8. Make a statement showing total amount of collections and commissions of this firm (Example 7) for the year.

RELATED PROBLEMS

326. 1. A dealer bought 150 barrels of apples at \$1.60 a barrel, and sold them at \$1.90. How much did he gain by the transaction, if he paid 10¢ a barrel for buying and selling?

2. A grocer bought 500 crates of Elberta peaches at 90¢ a crate. Picking and packing cost 15¢ a crate; freight and drayage charges amounted to \$47.50. How much did he gain by selling them for \$650, if he paid a commission merchant 2% for selling them?

3. A woolen mill buys 10,000 lb. of wool at 28¢ a pound in one lot, and 15,000 lb. at 32¢ in another. If drayage, storage, and other expenses amount to \$380, what is the total cost, including a commission of $1\frac{1}{2}\%$?

4. A produce dealer bought 250 barrels of pork at \$12.35 a barrel and sold it at \$12.50 a barrel. If he paid the commission merchant through whom he bought and sold 5¢ a barrel for his services, how much did he gain?

5. What is the cost of 1600 bu. of corn, quoted at $62\frac{5}{8}$¢ a bushel, if bought through a broker whose charge for buying is $\frac{1}{8}$¢ a bushel?

6. A man buys 3000 bu. of wheat at $83\frac{1}{4}$¢ a bushel and sells it at $87\frac{1}{2}$¢. If he pays $\frac{1}{8}$¢ a bushel to a broker for buying and the same for selling, how much does he gain?

7. A jobber buys 250 sacks of coffee (130 lb. to the sack) at $8\frac{1}{2}$¢ a pound, and sells it at $10\frac{1}{4}$¢ a pound. If he pays a commission of \$10 to an agent through whom he buys, what is his gain?

8. A merchant buys 5 car loads of flour, 150 barrels to the car, at \$4.60 a barrel. If freight is \$82 a car, for how much must he sell it in order to gain 25¢ a barrel?

PROFIT AND LOSS

327. In transactions involving a purchase and a sale, there may be a profit or a loss. When the selling price exceeds the cost, there is a *profit;* when the selling price is less than the cost, there is a *loss.*

To understand the subject clearly, we must remember that (1) cost price = the base; (2) profit or loss = the percentage; (3) selling price = the base + the percentage, or the base − the percentage.

ORAL EXERCISES

328. 1. Marvin paid $100 for a pony, and sold it for $120. How much did he gain? What per cent?

2. A man bought an organ for $100, and sold it at a profit of 25%. How much did he gain?

3. A merchant paid $20 for an overcoat, and sold it at a loss of 10%. How much did he lose?

4. Owen bought a bobsled for $15, and sold it for $24. What per cent on the cost did he gain?

5. If a hat cost $2, what per cent is gained by selling it for $3?

329. The per cent of profit or loss is always estimated *on the cost* as the *base.*

WRITTEN EXERCISES

330. 1. A man bought a town lot for $250, and sold it for $300. What per cent did he gain?

$300 − $250 = $50, the gain.

$50 ÷ $250 = .20, or 20%. (See Art. 314.)

Observe that we find the per cent of gain by dividing the gain by the cost. How would you find the per cent of loss?

2. A dealer bought 10 cows at $30 each, and sold them at $35 each. How much did he gain? What per cent did he gain?

3. A man bought an automobile for $650, and after using it 30 days sold it at a loss of 15%. How much did he get for it?

4. A druggist buys toilet soap at $1.80 a dozen, and sells it at 25¢ a cake. What per cent does he gain?

5. A dealer bought 12 lb. of tea for $7.20, and sold $\frac{1}{3}$ of it at 65¢ a pound and the remainder at 55¢ a pound. Did he gain or lose, and what per cent?

6. At the close of a season a milliner sold 5 hats at $1.50 each. If they cost $30 a dozen, what per cent did she lose?

7. What per cent does a merchant gain by selling cloth at $1.20, if it cost 90¢ a yard?

8. If a merchant buys 25 lb. of evaporated apricots at 10¢ a pound, at what price per pound must he sell them in order to gain 25%?

9. A stationer bought 1 ream of paper for $1.20, and sold it at 10¢ a quire. What per cent did he gain?

10. If a gross of pens cost 75¢, at what price per dozen must the dealer sell them to gain 60%?

11. A drover bought a car load of horses (20 head) for $1500. If the freight, feed, and other expenses amounted to $250, what per cent did he gain on the total cost by selling them at $100 a head?

12. A merchant bought 40 yd. of cashmere at 60¢ a yard. He sold 24 yd. at a profit of $16\frac{2}{3}$%, and the remainder at a profit of 25%. What per cent did he gain on the whole?

13. A merchant estimates the expense of doing business at 10% of the invoice price of goods. What per cent does he net by selling goods invoiced at $100 for $132?

14. A butcher buys 400 lb. of beef at 7½¢ a pound, and retails it at an average of 10¼¢ a pound. How much does he gain? What per cent?

15. A furniture dealer sold a suit of oak furniture for $12 less than it was marked, a reduction of 10%. What was it marked?

16. A grocer paid $2.40 for a box of laundry soap containing 100 bars. At what price must he sell each bar in order to gain 25%?

17. A merchant bought 500 bu. of apples at 70¢ a bushel. He stored them 3 months at a cost of 5¢ a bushel per month. If 10% of them rotted, did he gain or lose, and how much, by selling them at $1 a bushel?

18. If a dealer sells a plow for $4.80, thereby making 20%, what did he pay for it?

19. If the cost of manufacturing this plow was $3.20, what per cent did the factory gain by selling it to the dealer?

20. A dealer bought druggets at $90 a dozen, and sold them at $10 each. What per cent did he gain?

Find the marking prices of single pieces of the following goods:

21. Handkerchiefs costing $3 a dozen, to gain 40%.

22. Cravats costing $1.80 a dozen, to gain 60%.

23. Shirts costing $12.96 a dozen, to gain 16⅔%.

24. Waists costing $36 a dozen, to gain 20%.

25. Skirts costing $51 a dozen, to gain 20%.

REVIEW WORK

WRITTEN EXERCISES

331. 1. A dealer bought 100 grammars listed at 60¢ each. If allowed a discount of 20%, what was the amount of the bill?

2. What per cent did this dealer gain by selling these books at list price?

3. A grocer bought 3 cases of peaches (2 dozen cans to the case) at $4.50 a case, and sold them at 25¢ a can. What per cent did he make?

4. A dealer buys 4-oz. bottles of vanilla extract at $3 a dozen. At what price per bottle must he sell it in order to gain 40%?

5. A dealer bought 6 dozen pairs of shoes for $180, and sold them at $3 a pair. What per cent did he gain?

6. A grocer bought 40 bu. of sweet potatoes at 60¢ a bushel, and sold them at 25¢ a peck. If 10% of them rotted, what per cent did he gain?

7. A man bought a farm and sold it the same day for $7540, thereby gaining 16%. Find the cost.

8. An architect drew plans and superintended the building of a residence that cost $3750, for which he charged a commission of $3\frac{1}{2}\%$. What was his commission?

9. A merchant discounted a bill of $48.54 at the rate of $16\frac{2}{3}\%$. Find the discount.

10. The Daisy Grocery Co. advertises a discount of 2% for cash payments on all sales. Find the discount on Mr. Coop's bill of $165.

11. After marking a set of furniture, a dealer was compelled to deduct 10% from his marked price to meet competition. If he sold the set for $36, what was the marked price?

12. An agent sells a buggy for $62.50, thereby gaining 25%. What was the cost?

13. A newsboy subscribes for 100 copies of a paper published every day in the year. The subscription price is $5 a year. If he sells each copy at 2¢, how much does he gain in a year? What per cent?

14. A fertilizer for potatoes contains 8% phosphoric acid, 3½% ammonia, and 6% potash. If it takes 1500 lb. per acre, how many pounds of each of these ingredients are used to the acre?

15. A ton of cotton fertilizer consists of 1200 lb. of acid phosphate, 700 lb. of fish scrap, and 100 lb. of muriate of potash. What per cent of each does it contain?

16. A man spends $525 a year for living expenses. If this is 75% of his annual income, what is his monthly income?

17. During the last season John Smith played 160 games of baseball. His record was 550 times at bat, and 154 hits. What was his batting average, or per cent of hits to times at bat?

18. A country merchant bought 5 cases of eggs (30 dozen to the case) at 16¢ a dozen, and sold them to a city dealer at 25% advance. If the latter retailed these eggs at an advance of 20%, what did they cost the consumer a dozen?

19. A retail dealer advertises a cut of 10% on all clothing. What would be the selling price of (*a*) a suit marked \$22; (*b*) an overcoat marked \$28; (*c*) a pair of trousers marked \$7.50?

20. A furniture house makes a standing offer of 10% off for payment within 10 days, or 5% for payment within 30 days. Find the amounts of the following bills:

(*a*) 1 dining table at \$25, paid in 5 days;
(*b*) 12 dining chairs at \$2.65 each, paid in 25 days;
(*c*) 1 sofa at \$12.50, cash on delivery;
(*d*) 1 sideboard at \$20, paid in 8 days.

21. If the capital invested in fisheries in the Gulf States is \$4,710,000, and the value of the annual catch is \$3,485,400, the value of the catch is what per cent of the capital?

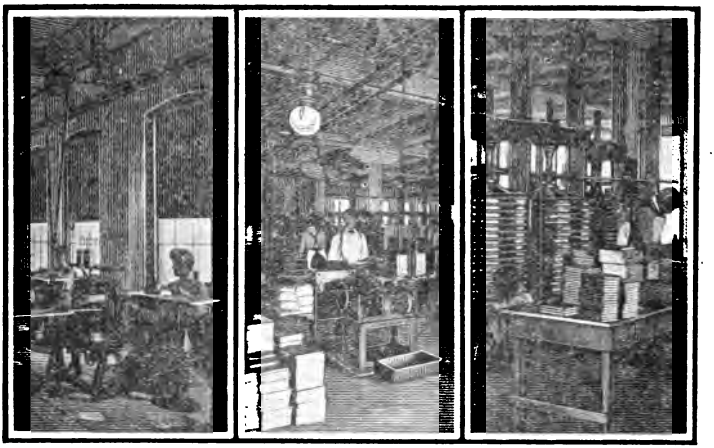
22. In 1890 there were 4,560,000 farms in the United States; in 1900 there were 5,700,000. What per cent did the number of farms increase during the decade?

23. A man engaged to buy produce for a city dealer on a 5% commission. In one week he bought 250 dozen eggs at 21¢ a dozen, 125 lb. of butter at 20¢ a pound, 100 chickens at 30¢ each, and 200 12-pound turkeys at 15¢ a pound. What commission did he earn that week?

24. A mechanic's regular charge for his services is \$4.50 a day. In order to secure a 6 months' job, he agreed to discount his daily wage 20%, and to work 24 full days each month. How much did he earn?

25. The census of 1900 showed for the United States (including Alaska and Hawaii) a population of 76,303,000. If 13.7% of these were of foreign birth, what was our native population?

RELATED PROBLEMS



332. **1.** This picture shows three scenes in a book bindery — a sewing machine, a case-making machine, and a book press. This bindery employs 100 operatives, and turns out an average of 10,000 books for each work day. What is the value of the weekly output if the books are sold at 45¢ each? What is the weekly pay roll if the average daily wage to each operative is $2.10?

2. The sewing machine used in this factory sews 16 pages every 2 seconds. How many hours will be required to sew 9000 copies of a 320-page book, if no allowance is made for waste?

3. The paper on which the book mentioned in Example 2 is printed weighs 80 lb. to the ream of 500 sheets, 64 pages to the sheet. What is the cost of the paper, at 6¢ a pound, for an edition of 10,000, if the amount of paper required is increased 5% for waste?

4. The cost of manufacturing a book that retailed at 25¢ was as follows: paper, 3¢, printing, 3¢, and binding, 9¢. If the publisher sold the books to dealers at 16⅔% off the retail price, and paid to the author a royalty of 6% on the net price, how much did the publisher clear on an edition of 40,000?

5. The number of different books produced in the United States in a certain year was 7850. Of these, 5652 were written by American authors. What per cent of the total product did American authors write?

6. In a certain year 744,000 sewing machines were made in this country for family use, and 56,000 for use in factories. What per cent of the entire number was made for family use? What per cent for use in factories?

7. In a recent year 2,124,000 high-grade watch movements were made in this country, and 1,210,680 cheap movements. The number of cheap movements was what per cent of the number of high-grade movements?

8. In a recent year there were made in this country 145,000 typewriters worth $5,655,000 at the factories. If 50% of these machines were sold through agents at an average of $75 each, what did they bring? What was the average margin on each machine between the factory price and the selling price?

9. The number of bicycles manufactured in the United States in one year was 1,113,000. What population would this supply at the rate of 15 wheels to the thousand persons?

10. In a year when there were made in this country 1,764,000 gross of steel pens, the number of lead pencils made was 6% less. How many pencils were made?

11. If this country produced in a certain year 4,509,750,000 sq. yd. of cotton goods, how many square yards were produced *per capita*, if the population was 75,162,500?

12. If we made in another year 399,000,000 sq. yd. of woolen and worsted goods, how many square yards were produced *per capita* for a population of 76,000,000?

13. In the census year this country produced 75,532,000 running yards of carpet of all grades. Estimating the circumference of the earth at 25,000 miles, how many yards would this lack of girdling the earth twice?

14. The same year there were made 219,000,000 pairs of boots, shoes, and slippers. There were also made 49,275,000 pairs of rubber boots and shoes. What per cent was the latter of the former?

15. The total floor space occupied by the shoe factories of the United States is estimated at 23,958,000 sq. ft. How many acres are so occupied?

16. If a dealer or middleman buys pianos at $212.50 each, and sells them at an advance of 40%, what does he get apiece for them? Allowing $15 per instrument for expenses, what would be the middlemen's profit on 171,000 pianos, the output of this country for one year?

17. The total value of raw materials consumed in manufactories in one year was $2,389,140,000. Of this sum agriculture supplied 81.2%; the mines, 13.4%; the forests, 5%; and the fisheries, .4%. Find the value supplied by each of these great industries.

18. In a recent year the census showed the net value of manufactured products to be $8,370,000,000. This represented the cost of raw material, together with the cost

of manufacturing. If the value of the raw material was 28½% of the net value, what was the cost of manufacture?

19. In 1900 the production of cheese in this country was 300,000,000 lb. If the farmers made 5½% of this, and the factories the remainder, how many pounds were made by each?

20. In the same year the factory product of butter was 421,026,000 lb. out of a total product of 1,493,000,000 lb. What per cent of the whole product did the farmers produce?

21. In 1890 the total horse power employed in manufacturing industries was 5,950,000; in 1900 it was 11,305,000. What per cent was the increase in 10 years?

22. If 1 horse power is equivalent to the labor of 10 men, the horse power employed in manufactures in the year last named was equal in producing capacity to the labor of how many persons?

23. The gross value of manufactured products in the United States in 1900 was $13,014,570,000, an increase of 39% in ten years. What was the value of these products 10 years before?

24. The total number of wage earners employed in manufacturing industries in our last census year was 5,316,750; 10 years earlier, 4,250,000 were so employed. What per cent was the increase during the decade?

25. The value of domestic manufactures exported from the United States in a recent year was $452,600,000, which was 31% of the total value of exports that year. Find the total value of the exports.

SHORT METHODS

333. Rapid Addition.

The best way to learn to add rapidly is to master thoroughly the 45 combinations used in addition, which are given in Art. 16, 4. To secure accuracy as well as rapidity, accountants use a method, which is explained below.

	387
	452
	688
	981
30	573
41	829
39	3910

In adding the right-hand column upward, say, "Nine twelve, thirteen, twenty-one, twenty-three, **thirty.**" Write 30 at the side of the column, or on a separate sheet of paper, for checking your work. Similarly, say in adding the second column upward, "Carry three, five, twelve, twenty, twenty-eight, thirty-three, forty-one." Write 41 below 30. In adding the third column, say, "Carry four, twelve, seventeen, twenty-six, thirty-two, thirty-six, **thirty-nine.**" Write 39 below 41.

CHECK. Add the first column downward saying, "Seven, nine, seventeen, eighteen, twenty-one, **thirty.**" What would you say in adding the second column downward? The third column?

Time yourself in adding by the foregoing method:

1.	2.	3.	4.
179	5968	99242	851464
465	7842	31868	395466
539	3273	18783	153245
884	5834	38797	951023
263	7917	22315	618192
892	6531	44733	395468
675	8653	96847	134565
726	2946	65645	739876
358	1722	46738	996327

334. Short Method in Subtraction.

The only practical method of shortening subtraction is the method of making change, referred to in Art. 30. By this method, the remainder is found by adding. It may be applied to all subtractions.

1. From 2793 subtract 1671.

Minuend	2793
Subtrahend	1671
Remainder	**1122**

Recalling that 1 and **2** make 3, we write **2** ones in the remainder. Again, recalling that 7 tens and **2** tens make 9 tens, we write **2** tens. In like manner we may obtain and write **1** hundred and **1** thousand, thereby completing the remainder.

2. From 18,782 subtract 17,858.

Minuend	18782
Subtrahend	17858
Remainder	**924**

Recalling that 8 and **4** make 12, we write **4** ones in the remainder; that 1 ten carried and 5 tens plus **2** tens make 8 tens, we write **2** tens; that 8 hundreds and **9** hundreds make 17 hundreds, we write **9** hundreds.

Subtract by the above method, and prove:

3.	4.	5.	6.	7.
718	871	781	1875	38756
643	463	364	783	29878

335. By extending the foregoing method the sum of several numbers may be readily subtracted from a single number.

1. From 87,643 subtract 43,256 + 17,432 + 586 + 9374.

Min.	87643	The mental steps to be taken are indicated in the following outline:
Sub.	43256	4, 10, 12, 18 plus **5** make 23, with 2 tens to carry.
	17432	2, 9, 17, 20, 25 plus **9** make 34, with 3 hundreds to carry.
	586	
	9374	3, 6, 11, 15, 17 plus **9** make 26, with 2 thousands to carry.
Rem.	**16995**	2, 11, 18, 21 plus **6** make 27, with 2 ten-thousands to carry.
		2, 3, 7 plus **1** make 8.

2. From 26,785 subtract 5643 + 7328 + 3576 + 8796.

3. From 18,750 subtract 7777 + 5432 + 236 + 1856 + 95.

4. The balance in bank to the credit of a customer at the beginning of June was $286.50. During the month he gave checks for the following amounts: $12.25, $3.50, $6.75, $25.75, $85.60. Find his balance on July 1.

336. In articles 143 and 246 you learned something about shortening computations by the use of the *equal parts of a hundred.* It will now be shown that they may be used to great advantage in solving certain problems that involve multiplication or division.

The equal parts of a hundred are sometimes called *aliquot parts.*

The equal parts of a hundred are as follows:

$\frac{1}{2}$ of 100 = 50.	$\frac{1}{10}$ of 100 = 10.
$\frac{1}{3}$ of 100 = $33\frac{1}{3}$.	$\frac{1}{12}$ of 100 = $8\frac{1}{3}$.
$\frac{1}{4}$ of 100 = 25.	$\frac{1}{16}$ of 100 = $6\frac{1}{4}$.
$\frac{1}{5}$ of 100 = 20.	$\frac{1}{20}$ of 100 = 5.
$\frac{1}{6}$ of 100 = $16\frac{2}{3}$.	$\frac{1}{25}$ of 100 = 4.
$\frac{1}{8}$ of 100 = $12\frac{1}{2}$.	$\frac{1}{50}$ of 100 = 2.

337. To multiply by the equal parts of a hundred.

1. Multiply 5786 by 50.

$2)\overline{578600}$
289300

Since 50 = $\frac{1}{2}$ of 100, 5786 × 50 = 5786 × $\frac{1}{2}$ of 100.

Hence, to multiply by 50, we may annex two 0's and divide the result by 2. The arrangement of the work is shown at the left.

2. Multiply 6270 by $33\frac{1}{3}$.

$3)\overline{627000}$
209000

Since $33\frac{1}{3}$ = $\frac{1}{3}$ of 100, 6270 × $33\frac{1}{3}$ = 6270 × $\frac{1}{3}$ of 100.

Hence, to multiply by $33\frac{1}{3}$, we may annex two 0's, and divide the result by 3.

3. Multiply 5257 by 25.

$4)\overline{525700}$
131425

In this case why may we annex two 0's, and divide the result by 4?

4. Multiply 72.996 by $16\frac{2}{3}$.

$6)\overline{7299.6}$
1216.6

Note that in this case instead of annexing two 0's, we move the decimal point two places to the right. Why?

Multiply:

5. 3879 by $33\frac{1}{3}$.
6. 4220 by 20.
7. 5148 by 25.
8. 2644 by 25.
9. 5624 by $12\frac{1}{2}$.
10. 2886 by $16\frac{2}{3}$.
11. 3888 by $8\frac{1}{3}$.
12. 2848 by $12\frac{1}{2}$.
13. 3246 by $16\frac{2}{3}$.
14. 21.495 by 20.
15. 89.64 by 25.
16. 864.6 by $33\frac{1}{3}$.

Now solve examples 5–16 by inspection. Having in mind the divisor, begin at the left and name or write the digits of the result in order, as if the 0's were written or the decimal point moved.

338. To divide by the equal parts of a hundred.

1. Divide 2147 by 50.

21.47
$\underline{\quad 2}$
42.94

Since $50 = \frac{1}{2}$ of 100, $2174 \div 50 = 2147 \div \frac{100}{2} = 2147 \times \frac{2}{100} = 42.94$.

Hence, to divide by 50, we may move the decimal point two places to the left, and multiply by 2. The work is shown at the left.

2. Divide 2271 by $33\frac{1}{3}$.

22.71
$\underline{\quad 3}$
68.13

In this case why may we move the decimal point two places to the left and multiply by 3?

3. Divide 42.7 by $16\frac{2}{3}$.

.427
6 Explain the steps.
2.562

Make the following indicated divisions:

4. $2982 \div 50$.
5. $3771 \div 33\frac{1}{3}$.
6. $482 \div 25$.
7. $2145 \div 16\frac{2}{3}$.
8. $2236 \div 12\frac{1}{2}$.
9. $214 \div 8\frac{1}{3}$.
10. $6217 \div 20$.
11. $214.8 \div 16\frac{2}{3}$.
12. $12.34 \div 33\frac{1}{3}$.
13. $478 \div 25$.
14. $561 \div 50$.
15. $249 \div 16\frac{2}{3}$.
16. $729 \div 33\frac{1}{3}$.
17. $194 \div 16\frac{2}{3}$.
18. $205 \div 50$.
19. $6.28 \div 25$.
20. $72.3 \div 20$.
21. $.486 \div 8\frac{1}{3}$.
22. $87.95 \div 12\frac{1}{2}$.
23. $58.75 \div 6\frac{1}{4}$.

339. Other Fractional Parts of a Hundred.

$\frac{2}{3}$ of $100 = 66\frac{2}{3}$. $\frac{3}{8}$ of $100 = 37\frac{1}{2}$.
$\frac{3}{4}$ of $100 = 75$. $\frac{5}{8}$ of $100 = 62\frac{1}{2}$.
$\frac{5}{6}$ of $100 = 83\frac{1}{3}$. $\frac{7}{8}$ of $100 = 87\frac{1}{2}$.

340. 1. Multiply 2787 by $66\frac{2}{3}$.

$2787 \times 66\frac{2}{3} = 2787 \times \frac{2}{3}$ of $100 = 2787 \times \frac{2}{3} \times 100 = 185{,}800$.

In multiplying by $66\frac{2}{3}$, explain why we may find $\frac{2}{3}$ of the multiplicand, and annex two 0's to the result.

2. Divide 8250 by $37\frac{1}{2}$.

$8250 \div 37\frac{1}{2} = 8250 \div \frac{3}{8}$ of $100 = 8250 \times \frac{8}{300} = 220$.

Multiply:

3. 278 by 75.
4. 2925 by $66\frac{2}{3}$.
5. 3972 by $83\frac{1}{3}$.
6. 1514 by $87\frac{1}{2}$.
7. 5962 by $37\frac{1}{2}$.
8. 7814 by $62\frac{1}{2}$.
9. 8942 by $37\frac{1}{2}$.
10. 7802 by 75.
11. 27,825 by $83\frac{1}{3}$.
12. 8248 by $87\frac{1}{2}$.
13. 2896 by $37\frac{1}{2}$.
14. 4782 by $66\frac{2}{3}$.
15. 8233 by 75.
16. 7848 by $66\frac{2}{3}$.

Divide:

17. 3783 by $37\frac{1}{2}$.
18. 4827 by 75.
19. 5895 by $62\frac{1}{2}$.
20. 7822 by $66\frac{2}{3}$.
21. 8421 by $37\frac{1}{2}$.
22. 8375 by $83\frac{1}{3}$.
23. 4263 by 75.
24. 2786 by $66\frac{2}{3}$.
25. 4940 by $62\frac{1}{2}$.
26. 8750 by $83\frac{1}{3}$.

Fractional parts of a hundred involving fifths and tenths are not included in these lists for the reason that multiplying or dividing by 20, 30, etc., cannot be made simpler than by the direct method.

Solve the following, using short methods:

27. At 75¢ a bushel, what will 320 bushels of rye cost?

28. At $83\frac{1}{3}$¢ a yard, what will 156 yards of cloth cost?

29. At $87\frac{1}{2}$¢ a pair, what will 48 pairs of rubbers cost?

30. If cloth is worth 75¢ a yard, how many yards can be bought for $18?

31. At the rate of $33\frac{1}{3}$ miles a day, how many days will it take a man to walk 200 miles?

32. James missed $12\frac{1}{2}$% of his 16 examination questions. How many did he miss? If 100 represented the highest grade possible, what was his mark?

33. A man who owned 25 % of an estate sold 20 % of his share to his brother. What per cent of the estate did he sell?

34. Mr. Perry owned a fifth interest in a storehouse, and sold 25 % of his share to each of his four associates. What part of it did each buy?

35. In an orchard $\frac{1}{2}$ of the trees bear apples, 40 % peaches, and the remaining 50 cherries. How many trees are in the orchard?

36. At $12\frac{1}{2}$ ¢ a pound, what will a 500-pound bale of cotton bring?

37. A man paid \$5075 for $87\frac{1}{2}$ acres of land. How much was that an acre?

38. If $6\frac{1}{4}$ pounds of tea cost \$5, how many pounds of tea can be bought for \$ 9.60?

39. If a dealer pays $37\frac{1}{2}$ ¢ for 16 boxes of toothpicks, how many boxes can he buy for \$ 2.25?

40. How much would be gained by buying 320 lb. of sugar at 5 ¢ a pound and selling it at $6\frac{1}{4}$ ¢ a pound?

41. A man bought 16 bu. of corn at $62\frac{1}{2}$ ¢ a bushel and paid for it with lard at $12\frac{1}{2}$ ¢ a pound. How many pounds of lard were required?

42. At $62\frac{1}{2}$ ¢ a rod, what is the cost of fencing a rectangular field that is 195 rd. long and 65 rd. wide?

43. A man receives a salary of \$1860 a year, and $8\frac{1}{3}$ % of his salary equals $16\frac{2}{3}$ % of his savings. What sum does he save per annum?

44. James's school report showed that he had 5 studies, and that his marks in them were 75, 85, 90, 95, 93. Find his average grade, using a short method based on $5 = 10 \div 2$.

THE EQUATION

341. Hitherto in all computations you have used figures to represent numbers. It is sometimes more convenient to represent numbers by letters. When we speak of 3 dollars or of 5 gallons, we mean a definite number of dollars or gallons; but when we speak of n dollars or of x gallons, the number of dollars or gallons is indefinite. The reasoning, however, is the same whether numbers are represented by letters or by figures. Thus, if x stands for the number of dollars in your purse, $2x$ stands for twice that number, $3x$ for three times that number, etc. (Note that x is used for $1x$.)

1. If n represents the number of marbles you have, what does $5n$ represent? $7n$? $10n$?

2. If one bucket holds a quarts and another $3a$ quarts, how many quarts do both hold? $a + 3a =$

3. If Elizabeth has $5n$ yards of ribbon and gives $2n$ yards to Ann, how many yards has she left? $5n - 2n =$

4. At x dollars per head, what will 10 sheep cost? $10 \times x =$ What will they cost at $2x$ dollars per head? $10 \times 2x =$

5. What is $\frac{1}{3}$ of 6 dollars? Of $6x$ dollars? Of $12x$?

6. If 5 sheep cost $10x$ dollars, what will 1 sheep cost? $\frac{1}{5}$ of $10x =$

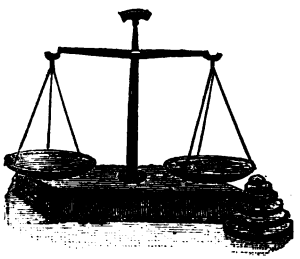
7. The expression $2(4+7)$ means that the sum of 4 and 7 is to be multiplied by 2. What does $6(5-3)$ mean? Indicate that the sum of $2x$ and $3x$ is to be multiplied by 5.

342. The **symbol of equality**, $=$, is read *equals* and indicates that the numbers or expressions before and after it are equal in value. Thus, $4 + 5 = 9$ means that the sum of 4 and 5 equals 9; and $9 - 4 = 5$ means that the difference between 9 and 4 equals 5.

Such statements of equality are called **Equations.**

For example, $3 + 5 = 8$, and $4 + 2 = 6$ are equations. Is $5 + 7 = 7 + 9$ an equation? Why not?

343. 1. When does a scale beam balance?



2. When balanced, if a weight is added to one scale pan, what must be added to the other to preserve the balance?

3. When balanced, if a weight is taken from one pan, what must be done to the other to preserve the balance?

4. How can the balance be maintained, if the weights in one pan are doubled? If halved?

344. You may consider the equal numbers in an equation as measuring the equal or balancing weights in the two scale pans.

1. If we add 5 to the first side of the equation, $7 + 3 = 15 - 5$, how much must be added to the second side to make the sides equal again? $7 + 3 + 5 = 15 - 5 + 5$; that is, $15 = 15$.

2. If we take 5 from the first side, what must be done to the second to preserve the equality? $7 + 3 - 5 = 15 - 5 - 5$; that is, $5 = 5$.

3. How much is 5 times $(7 + 3)$? How much is 5 times $(15 - 5)$? $5\,(7 + 3) = 5\,(15 - 5)$; that is, $50 = 50$.

4. What is the quotient of $(7 + 3)$ divided by 5? Of $(15-5)$ divided by 5? Are the quotients equal? $(7+3) \div 5 = (15 - 5) \div 5$; that is, $2 = 2$.

345. From the foregoing it will be seen that the following principles apply to every equation:

I. The same number may be added to both sides.

II. The same number may be subtracted from both sides.

III. Both sides may be multiplied by the same number.

IV. Both sides may be divided by the same number.

346. The problems in this book can be solved without the use of letters to represent numbers, but the use of letters in equations is of great advantage in solving many problems. Consider the following problem and solutions:

Harry has 3 times as many apples as John, and both have 24 apples. How many has John?

(*a*)

A certain number = the number John has.
3 times that number = the number Harry has.
Hence, 4 times that number = the number both have.
$\therefore$ 4 times that number = 24,
and the number = 6, the number John has.

(*b*)

Let x = the number John has.
Then $3x$ = the number Harry has,
and $4x$ = the number both have.
$\therefore 4x = 24$,
and $x = 6$, John's number.

Note that when we let x = the number John has, it follows from the problem that $3x$ = the number Harry has, and that $4x$ = the number both have. But both have 24. Hence we write $4x = 24$, thus forming an equation.

The solution marked (*b*) follows what is called the *equation method* of solving problems.

347. The expression of the facts or conditions of a problem in the form of an equation, as $4x = 24$, is called *stating the equation.* The letter x usually represents a number to be found.

348. In an equation like $4x = 24$, the equality exists only for a particular value of x, which is 6 in this case. Finding this value of x is called *solving the equation.*

In solving equations we apply one or more of the principles stated in Art. 345.

349. Find by inspection the missing (unknown) number for which x stands in each of the following:

1. $9 + x = 17$.
2. $x + 35 = 50$.
3. $40 - x = 15$.
4. $24 \times x = 72$.
5. $96 \div x = 8$.
6. $x \div 4 = 5$.

350. Solve the following problems by the equation method:

1. A and B together have 120 sheep. B has twice as many as A. How many has A?

Let x = A's number.
Then $2x$ = B's number.
$\therefore x + 2x = 120$, (Why?)
or $3x = 120$.
Dividing both sides by 3 (Art. 345, IV),
$x = 40$, A's number.

2. There are 30 pupils in a class. There are 5 times as many girls as boys. How many boys are in the class?

(1) What is to be found? (Number of boys.)
(2) What shall we let x equal? Then what will $5x$ equal? What will $x + 5x$ or $6x$ equal?
(3) State the equation and solve it.

3. What number increased by 6 times itself equals 21?

4. A coat and a hat cost $15. If the coat cost 4 times as much as the hat, what did the hat cost?

5. Five times a certain number is equal to twice the number increased by 36. Find the number.

Let x = the number.
Then $5x$ = 5 times the number,
and $2x$ = 2 times the number.
$\therefore 5x = 2x + 36.$

Subtracting $2x$ from each side (Art. 345, II),

$$5x - 2x = 36,$$
or $$3x = 36.$$
$$\therefore x = 12, \text{ the number.}$$

Solve this problem without the use of x, and compare the two solutions. Which method is more convenient?

6. If a certain number is doubled and the result diminished by 3, the sum is 10 more than the original number. Find the number.

Let x = the number.

Doubling x, diminishing the result by 3, and placing the remainder equal to 10 more than the number, we have

$$2x - 3 = 10 + x.$$

Subtracting x from both sides,

$$2x - x - 3 = 10 + x - x,$$
or $$x - 3 = 10.$$

Adding 3 to both sides (Art. 345, I),

$$x = 13, \text{ the number.}$$

TEST. Double 13, subtract 3, and see if the result is 10 more than 13.

7. After spending $\frac{2}{5}$ of his money, John had $48 left. How much had he at first?

Let x = number of dollars at first.
Then $x - \frac{2}{5}x = 48,$
or $\frac{3}{5}x = 48.$
$\frac{1}{5}x = 16.$ (Why?)

Multiplying both sides by 5 (Art. 345, III),

$x = 80$, number of dollars at first.

8. A, B, and C have together \$270. If B has twice as much as A, and C has 3 times as much as B, how much has each?

Let x = number of dollars A has.
Then $2x$ = number of dollars B has,
and $6x$ = number of dollars C has.
$\therefore x + 2x + 6x = 270$,
or $9x = 270$.
$\therefore x = 30$, number of dollars A has,
$2x = 60$, number of dollars B has,
and $6x = 180$, number of dollars C has.

To avoid fractions let x represent the smallest number to be found, and express the others in terms of x.

9. What principal at 6% will amount to \$159 in one year?

Let x = number of dollars in principal.
Then $.06x$ = number of dollars in interest.
$\therefore x + .06x = 159$,
or $1.06x = 159$.
Dividing both sides by 1.06,
$x = 150$, number of dollars in principal.

TEST. \$150 + 6% of \$150 = \$159.

10. If A and B together have \$500, and A has four times as much as B, how much money has each?

11. A man is three times as old as his son, and the sum of their ages is 60 years. How old is each?

12. The greater of two numbers is eight times the less, and their sum is 72. Find the numbers.

13. A horse and carriage cost \$450. If the carriage cost twice as much as the horse, what was the cost of each?

14. If twice a number is added to four times the same number, the sum will be 54. What is the number?

15. Divide 64 into two parts such that one shall be three times the other.

16. What number increased by $\frac{1}{2}$ of itself equals 36?

17. What number must be added to 24 to make 120?

18. The sum of two numbers is 50. One is 10 more than the other. What are they?

19. One boy has \$1 more than twice as much as another. Together they have \$7. How much has each?

20. At a picnic there were 270 persons. There were twice as many women as men, and 3 times as many children as women. Find the number of each.

21. A has 3 times as many horses as B has. If he has 8 more than B, how many has each?

22. Ralph and Keller together have \$80, and Keller has \$20 more than Ralph. How much has each?

23. In a certain menagerie there were 7 times as many monkeys as elephants, and 3 times as many horses as monkeys. If there were 87 of these animals, how many were there of each kind?

24. A trader having increased his capital by 10% found that it amounted to \$25,245. How much was it at first?

25. A city gained 15% in population in five years, and then had 92,000. What was the population before the gain?

26. Tom, Will, and Frank together caught 48 trout. Tom caught twice as many as Will, and Frank caught as many as both Tom and Will. How many trout were caught by each?

27. A school gives \$84 for three prizes. The first is double the second, and the second is double the third. What is the value of each prize?

28. I think of a certain number. I double it, add 5 to it, and find that the result is 8 more than the original number. What is the number?

29. I think of a number, multiply it by 4, and add 6. The result is 27 more than the original number. What is the original number?

30. A rectangular field is twice as long as it is wide. The distance around it is 120 rods. What is its width?

31. Two trains run from St. Louis in opposite directions, one going twice as fast as the other. After 1 hour they are 72 miles apart. How far is each train from St. Louis?

32. A passenger train and a freight train leave Washington at the same time, and go in the same direction. At the end of an hour they are 30 miles apart. If the passenger train runs three times as fast as the freight, how far has the latter run?

33. A newsboy sold four times as many papers on Wednesday as he sold on Tuesday. On Tuesday he sold 10 more than he sold on Monday. If he sold 80 papers in all, how many did he sell each day?

34. A second reader costs 10¢ more than a first reader, and a third reader 10¢ more than a second. If the three cost $1.05, what is the cost of each?

35. A man lost 22% of his capital, and had $19,500 left. How much had he at first?

351. Percentage Formulas.

If we use the initials of the words "base," "rate," and "percentage," letting b = base, r = rate, and p = percentage, we may write (Art. 249),

$$p = b \times r.$$

An equation like this which embodies a principle is called a **formula**, and may be used to advantage in solving problems.

1. A man bought a farm for $4000, and made a cash payment of 25% of the purchase price. What was the cash payment?

We have the formula,

$$p = b \times r.$$

Using 4000 for b, and .25 for r, we have $p = 4000 \times .25 = 1000$.
Therefore the cash payment was $1000.

2. A man bought a farm for $4000, and made a cash payment of $1000. What per cent of the purchase price was the cash payment?

Again we have,

$$p = b \times r.$$

Dividing both sides by b (Art. 345, iv),

$$\frac{p}{b} = r, \text{ or } r = \frac{p}{b} = \frac{1000}{4000} = .25.$$

Therefore the cash payment was 25% of the purchase price.

State the formula $r = \frac{p}{b}$ as a principle. Compare with the statement under Art. 314.

3. A man bought a farm, making a cash payment of $1000, which was 25% of the purchase price. What was the purchase price?

Dividing both sides of $p = b \times r$ by r,

$$\frac{p}{r} = b, \text{ or } b = \frac{p}{r}.$$

Use 1000 for p and .25 for r, and find b, the base, or purchase price.

State the principle embodied in the formula $b = \frac{p}{r}$, and compare with that found in Art. 317.

4. If a certain ore yields 55% of iron, how many pounds will a long ton (2240 lb.) yield?

5. A pine sill lost 30 lb. in seasoning, which was 15% of the original weight. What was the original weight?

6. Pearl answered 49 out of 50 questions asked in an examination. What per cent of them did she answer?

RELATED PROBLEMS

352. 1. In a certain city \$3,240,000 is raised by taxation, of which the public schools get $16\frac{2}{3}\%$. How much goes to the schools?

2. In another city \$3,800,000 is raised by taxation, of which \$1,140,000 goes to the schools. What per cent of the amount raised in taxes do the schools get?

3. At the close of the January term 17,340 pupils were promoted in the various grades of the elementary schools of a certain city. If 85% of the pupils were promoted, how many were not promoted?

4. A rural school with a session of 5 months of 20 school days each had an enrollment of 40 pupils. The number of absences for the months in order were 125, 150, 160, 180, 185. What was the per cent of attendance for the session?

5. In a year when the population of the United States was 75,690,000, $81\frac{2}{3}\%$ of the population was over 10 years of age, and 10% of those over 10 years old were illiterate (unable to write). How many illiterates were there in this country that year?

6. In a recent year the number of pupils enrolled in the common schools of this country was 16,256,000. If the enrollment was 20% of the entire population, what was the population that year?

7. The average daily attendance in the schools for the same year was 69.6% of the total number enrolled. What was the average daily attendance?

8. The amount expended for public schools that year was \$273,216,227. Find to the nearest cent the amount expended *per capita* of total population. Of school enrollment.

INTEREST REVIEWED

353. In Articles 255–261 you learned something about computing interest, *i.e.*, money paid for the use of money. You are now ready for a more extended treatment of the subject.

1. What is the principal? What is the amount?

2. What do we mean by the rate of interest? What does interest at 6% mean? At 4%?

ORAL EXERCISES

354. 1. What is the interest on \$500 for one year at 6%? \$500 × .06 =

2. If the interest for one year is \$30, how much is it for 2 years? For 3 years? For 5 years?

3. If the interest for one year is \$36, how much is it for one month? For two months? For 6 months?

4. If the interest for 1 month of 30 days is \$3, how much is it for 15 days? For 10 days? For 1 day?

5. What is the interest on \$500 for 3 yr. at 6%? \$500 × .06 × 3 =

6. What is the interest on \$400 for 2 yr. 6 mo. at 5%? \$400 × .05 × $2\frac{1}{2}$ =

State the interest on the following:

7. \$500 for 3 yr. at 5%.
8. \$100 for 1 mo. at 6%.
9. \$200 for 6 mo. at 8%.
10. \$400 for 4 mo. at 6%.
11. \$250 for 8 mo. at 6%.
12. \$500 for 10 da. at 6%.
13. \$600 for 15 da. at 5%.
14. \$100 for 1 yr. 6 mo. at 6%.

15. What is the interest on \$600 for 6 years and 6 months, at 5%?

16. A man paid interest on \$1000 for 2 years and 3 months, at 6%. How much interest did he pay?

WRITTEN EXERCISES

355. 1. Find the interest on \$1200 for 3 yr. 6 mo. at 4%.

$\$1200 \times .04 \times 3\frac{1}{2} = \$168.$

Find the interest on the following:

2. \$350 for $2\frac{1}{3}$ yr. at 6%.

3. \$750 for 8 mo. at 6%.

4. \$250 for 3 mo. at 8%.

5. \$900 for 2 yr. 6 mo. at 5%.

6. \$420 for 2 yr. 3 mo. at 5%.

7. \$700 for 1 yr. 4 mo. at 6%.

8. Find the interest on \$450 for 1 yr. 3 mo. 18 da. at 6%.

6% of \$450 = \$27, int. for 1 yr.
$\frac{1}{12}$ of \$27 = \$2.25, int. for 1 mo.
3 × \$2.25 = \$6.75, int. for 3 mo.
$\frac{18}{30}$ of \$2.25 = \$1.35, int. for 18 da.
\$27 + \$6.75 + \$1.35 = \$35.10, total interest.
Since 1 year 3 mo. 18 da. = 15.6 mo., the calculation is conveniently made as at the right.

\$450
.06
12)27.00
2.25
15.6
1350
1125
225
\$35.100

Find the interest at 6% on the following:

9. \$300 for 3 mo. 15 da.

10. \$480 for 5 mo. 10 da.

11. \$850 for 3 mo. 24 da.

12. \$640 for 7 mo. 27 da.

13. \$960 for 9 mo. 13 da.

14. \$420 for 1 yr. 6 mo. 6 da.

15. \$656 for 2 yr. 3 mo. 15 da.

16. \$200 for 1 yr. 3 mo. 21 da.

17. \$300 for 2 yr. 2 mo. 20 da.

18. \$120 for 1 yr. 4 mo. 5 da.

19. A man borrowed $500 on November 25, 1905, at 6%. How much interest was due April 5 of the next year?

yr.	mo.	da.
1906	4	5
1905	11	25
	4	10

The time is found as indicated. In computing interest, if the third decimal figure is 5 or over, it is counted as a cent; otherwise it is neglected.

Find the interest and amount of —

20. $1500 for 2 yr. 3 mo. 6 da. at 5%.

21. $1225 for 3 yr. 5 mo. 7 da. at $4\frac{1}{2}$%.

22. $3250 for 1 yr. 4 mo. 11 da. at 6%.

23. $1875 for 1 yr. 2 mo. 22 da. at $5\frac{1}{2}$%.

24. $1440 for 2 yr. 8 mo. 9 da. at $4\frac{1}{2}$%.

25. $720 for 1 yr. 1 mo. 1 da. at 6%.

26. $350.50 for 1 yr. 4 mo. 20 da. at 8%.

27. $185.60 for 3 yr. 10 mo. 24 da. at 7%.

28. $1800 for 2 yr. 7 mo. 16 da. at 5%.

29. $2500 for 1 yr. 5 mo. 5 da. at 6%.

30. $2856 for 1 yr. 3 mo. 15 da. at 6%.

31. $1130 for 1 yr. 5 mo. 25 da. at 7%.

32. $2750.50 for 2 yr. 4 mo. 6 da. at $3\frac{1}{2}$%.

33. $10,050.50 for 9 mo. 18 da. at $2\frac{1}{2}$%.

34. A man borrowed $1750 on January 1, 1906, at 5%. What amount was due him on the 6th of the following June?

35. Joseph Dupuy borrowed $1480, at 6%, from Henry Payne, on October 10, 1904. What amount was due on February 24, 1905?

36. If I borrowed $750, at 6%, on September 15, 1906, what amount was due on July 10, 1907?

356. The Six Per Cent Method.

The method of computing interest already given may be applied in all cases; the same results, however, can be obtained by using a shorter method, called the *six per cent method.* It is based on the fact that at the rate of 6 % a year the interest for 2 months ($\frac{1}{6}$ yr.) is $\frac{1}{6}$ of 6 %, or 1 %.

This method is especially convenient for finding the interest for 30 da. (1 mo.), 60 da. (2 mo.), or 90 da. (3 mo.), periods for which many loans are made. Moreover, 6 % is the legal rate in about two thirds of the states.

From the interest at 6 % the interest at any other rate is readily found.

1. Find the interest on $210 for 7 mo. 18 da. at 6 %. At 5 %.

Interest for 2 mo. = 1 % of $210, or $2.10.
Therefore,
Interest for 6 mo. = 3 × $2.10 = $6.30.
Interest for 1 mo. = $\frac{1}{2}$ of $2.10 = 1.05.
Interest for 18 da. = $\frac{6}{10}$ of $1.05 = .63.
Interest for 7 mo. 18 da. = $7.98.

The interest at 5 % is readily found as follows:

Interest at 6 % = $7.98.
Interest at 1 % = $1.33.
Interest at 5 % = $6.65.

Find the interest at 6 % on $1256 for —

2. 2 mo. 6 da.
3. 3 mo. 27 da.
4. 5 mo. 24 da.
5. 7 mo. 18 da.
6. 9 mo. 18 da.
7. 10 mo. 15 da.
8. 1 yr. 1 mo. 9 da.
9. 2 yr. 9 mo. 5 da.
10. 3 yr. 5 mo. 10 da.
11. 2 yr. 1 mo. 18 da.

Find the interest on \$665 at 8% for —

12. 6 mo. 21 da.
13. 1 yr. 18 da.
14. 1 yr. 3 mo. 6 da.
15. 2 yr. 5 mo. 9 da.

Apply the method best suited to each case, and find the interest on \$3875 for —

16. 3 mo. 3 da. at 9%.
17. 1 yr. 8 mo. at 4%.
18. 1 yr. 7 mo. 24 da. at $4\frac{1}{2}$%.
19. 2 yr. 3 mo. at 3%.
20. What is the interest on \$865 for 100 da. at 6%?

Interest for 60 da. = 1% of \$865 = \$8.65.
Interest for 30 da. = $\frac{1}{2}$ of \$8.65 = \$4.33.
Interest for 10 da. = $\frac{1}{3}$ of \$4.33 = \$1.44.
Interest for 100 da. = \$14.42.

Find the interest and amount at 6% on —

21. \$565 for 75 days.
22. \$780 for 80 days.
23. \$827 for 65 days.
24. \$925 for 50 days.
25. \$1025 for 40 days.
26. \$1675 for 110 days.
27. \$1730 for 100 days.
28. \$820 for 95 days.
29. \$750 for 96 days.
30. \$620 for 84 days.

357. Exact Interest.

In an exact calculation of interest it should be borne in mind that a month is not an equal part of a year and that any strictly theoretical system must be founded upon days and years.

Even the relation between a day and a year is different in leap years and common years. The laws of the United States provide that all contracts of the national government bear interest by the year which has 365 or 366 days. This method is known as *exact interest*, but it is little used by individuals and banks.

358. The following table gives the exact number of days from any day of any month to the corresponding day of any other month within 365 days:

	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
January . . .	365	31	59	90	120	151	181	212	243	273	304	334
February . .	334	365	28	59	89	120	150	181	212	242	273	303
March	306	337	365	31	61	92	122	153	184	214	245	275
April	275	306	334	365	30	61	91	122	153	183	214	244
May	245	276	304	335	365	31	61	92	123	153	184	214
June	214	245	273	304	334	365	30	61	92	122	153	183
July	184	215	243	274	304	335	365	31	62	92	123	153
August . . .	153	184	212	242	273	304	334	365	31	61	92	122
September .	122	153	181	212	242	273	303	334	365	30	61	91
October . . .	92	123	151	182	212	243	273	304	335	365	31	61
November .	61	92	120	151	181	212	242	273	304	334	365	30
December . .	31	62	90	121	151	182	212	243	274	304	335	365

To find the number of days from June 3 to February 16 following, we look opposite June and under February and find 245, the number of days from June 3 to February 3. Then $245 + 13 = 258$, the number of days to February 16. To find the number of days from June 3 to February 1, subtract 2 from 245.

359. 1. Find the exact interest on \$1277.50 for 68 days at 4%.

Interest for 1 year $= 4\%$ of \$1277.50 $=$ \$51.10.
Interest for 1 day $= \frac{1}{365}$ of \$51.10 $=$ \$.14.
Interest for 68 days $= 68 \times$ \$.14 $=$ \$9.52, exact interest.

Find the exact interest on —

2. \$1130 for 35 days at 6%.
3. \$1250 for 93 da. at 3%.
4. \$2750 for 63 da. at $4\frac{1}{2}$%.
5. \$1400 for 33 da. at $3\frac{1}{2}$%.
6. \$1820 for 105 da. at 5%.
7. \$5225 for 100 da. at 4%.

Using the table, find the exact interest on —

8. \$350 from July 12, 1906, to October 20, 1906, at 6%.

9. \$1250 from May 17 to August 2 of the same year at 3%.

10. $1500 from October 3, 1906, to April 15, 1907, at $3\frac{1}{2}\%$.

11. $6750 from July 5, 1905, to June 5, 1906, at $5\frac{1}{2}\%$.

360. Interest Problems.

Interest is an application of percentage, and in solving interest problems it will be well to remember that —

In Percentage	In Interest
The base (b)	= the principal (p);
the rate (r)	= the rate of interest (r);
the percentage (p)	= the interest (i);
the base + the percentage	= the amount.

The element of time (t), not found in percentage, is introduced as a factor in interest.

361. We have learned (Art. 249) that the percentage equals the product of the base and rate; that is (Art. 351),

$$p = b \times r.$$

We have also learned (Art. 355) that the interest for any number of years is found to be the product of the principal, the rate, and the time. Hence, if we let i = the interest, p = the principal, r = the rate, and t = the time, we may write the equation

$$i = p \times r \times t.$$

362. To find the rate.

Dividing both sides of $i = p \times r \times t$ first by t, and then by p, we have

$$\frac{i}{p \times t} = r, \text{ or } r = \frac{i}{p \times t}.$$

That is, **the rate equals the interest divided by the product of the principal and the time expressed in years.**

1. If $650 earn $78 in 2 years, what is the rate?

Applying the principle above, we may work as in (*a*) or as in (*b*):

(*a*)

$$\frac{78.00}{650 \times 2} = .06.$$

(*b*)

$$\begin{array}{r} 650 \\ 2 \\ \hline 1300 \end{array} \quad \begin{array}{r} .06 \\ \overline{)78.00} \\ 78.00 \end{array}$$

(*c*)

$$\begin{array}{r} 2)78 \\ \overline{650)39.00(.06} \\ 39\ 00 \end{array}$$

Therefore the rate is 6 %.

Observe that in the solution of this problem one of the three factors of a number is found by withdrawing the other two. In (*c*) the two factors are withdrawn separately.

Find the rate per cent at which —

2. $5670 will earn $850.50 in $2\frac{1}{2}$ years.
3. $783 will earn $164.43 in 3 yr. 6 mo.
4. $164 will earn $5.74 in 6 mo.
5. $214.50 will earn $2.86 in 4 mo.

363. To find the time.

Dividing both sides of $i = p \times r \times t$ by $p \times r$, we have

$$t = \frac{i}{p \times r}.$$

That is, **the time** (expressed in years) **equals the interest divided by the product of the principal and the rate.**

1. In how many years will $1525 earn $335.50 at 4% ?

Applying the principle above:

$$\begin{array}{r} \$1525 \\ .04 \\ \hline \$61.00 \end{array} \qquad \begin{array}{r} 5.5 \\ \overline{61)335.50} \\ 305 \\ \hline 30\ 5 \\ 30\ 5 \end{array}$$

Therefore the time is $5\frac{1}{2}$ years.

Again, we find one of the three factors of a number by withdrawing the other two.

Find the time in which —

2. $567 will earn $85.05 at 6 %.
3. $5225 will earn $783.75 at 5 %.
4. $1644 will earn $57.54 at 7 %.
5. $21.45 will earn $21.45 at 4 %.
6. Find the time in which $5016 will earn $37.62 at 6 %.

When the period is short and the answer is expected in days, we may proceed as follows:

Interest on $5016 for 60 days at 6 % = $50.16.
Interest on $5016 for 1 day = $\frac{1}{60}$ of $50.16 = $.836.
$37.62 ÷ $.836 = 45, the number of days.

Find the number of days in which —

7. $576 will earn $8.64 at 6 %.
8. $726 will earn $9.68 at 6 %.

364. To find the principal.

Dividing both sides of $i = p \times r \times t$, by $r \times t$, we have

$$p = \frac{i}{r \times t}.$$

That is, **the principal equals the interest divided by the product of the rate and time.**

1. Find the principal which will earn $51.45 in $3\frac{1}{2}$ years at 6 %.

Applying the principle above:

$$\begin{array}{r} .06 \\ 3\frac{1}{2} \\ \hline .21 \end{array} \qquad \begin{array}{r} 245 \\ 21\overline{)5145} \\ 42 \\ \hline 94 \\ 84 \\ \hline 105. \\ 105 \\ \hline \end{array}$$

Therefore the principal is $245.

Find the principal which at 6 % interest will earn —

2. $25 in 2 yr. 6 mo.
3. $37.50 in 1 yr. 3 mo.
4. $3.40 in 60 da.
5. $3.50 in 10 da.
6. $46.20 in 1 yr. 3 mo. 12 da.

PROMISSORY NOTES

365. A promise in writing to pay a stated sum of money on demand or at a specified time is called a **Promissory Note.**

The person who signs the note is called the *maker;* the person to whom it is made payable is called the *payee;* and the person who owns it is called the *holder.*

The sum of money named in a note is called the *face.*

366. The forms of notes most commonly used are "time notes" and "demand notes." One of each is given below:

(*a*)

$250.00 *Montgomery, Ala., May 2, 1906*

Thirty days after date, I promise to pay to John Rainey *or order, Two hundred fifty* $\frac{00}{100}$ *Dollars, with interest at 8%, for value received.*

L. C. Dean.

(*b*)

$400. RALEIGH, N.C., DEC. 5, 1906.

On demand I promise to pay to Wilson Jones, or bearer, four hundred dollars, with interest at 6%, for value received.

RICHARD BYRD.

1. Two other kinds of notes are used: (*a*) collateral notes, in which space is left for a description of the collateral used; (*b*) mortgage notes which are secured by real estate and recorded according to law.

2. If a note does not contain the words "with interest," it does not begin to bear interest until it becomes due. Unless otherwise specified all interest-bearing notes run at the legal rate. By contract a lower rate may be enforced. What is the legal rate in your state?

367. A person who places his name on the back of a note is called an *indorser*.

1. A note that is payable to the payee, "or bearer," may be sold and transferred by *delivery*.

2. A note payable to the order of the payee may be transferred by *indorsement*.

3. Any indorser of a note becomes liable for its payment unless he is permitted to add the words "without recourse."

4. When paid, notes are cancelled and returned to the makers.

368. In most states notes are payable at the time specified therein, but some states allow "three days of grace" after the time specified before notes are said to *mature* or to be *legally due*.

For example, a note *nominally* due Sept. 5 is *legally* due Sept. 8, and maturity is indicated thus: "Due Sept. 5/8."

1. In ascertaining the date of maturity when the *time* is given in *days*, we count forward the exact number of days from the date of the note; when the time is given in *months*, we count forward by calendar months.

2. When the time is given in months, a note dated on the last day of a "long" month falls due on the last day of the proper month. Thus, a note dated Jan. 31, 1900, is due Feb. 28, if for one month, and April 30, if for three months.

3. Days of grace have been abolished in all except the following: Alabama, Arkansas, Indiana, Kansas, Kentucky, Louisiana, Michigan, Minnesota, Mississippi, Missouri, Nebraska, Nevada, North Carolina, South Carolina, South Dakota, Texas, Wyoming, Indian Territory, New Mexico, and Oklahoma.

WRITTEN EXERCISES

369. 1. Write a note for $275, payable on demand to Geo. C. Rodgers, or order, with interest at 6%; Walter Newman, maker.

2. Write a note for $1000, due three months after date, payable to William Gibson, or order, interest at 6%; F. G. Mauzy, maker.

3. Write a note for $530, payable to Chas. F. Loomis, due 30 days from date, with interest at 6%, and signed by O. A. Bird. Indorse to W. B. Smith.

Make out, payable to the order of the payees, the following notes, and find the amount of each when due. Date when written:

	Maker	Payee	Face	Rate	Time
4.	C. M. Brown,	J. M. Chilton,	$300	4%	3 mo.
5.	R. E. Burks,	E. C. Cramer,	$500	6%	6 mo.
6.	T. A. Borce,	P. D. Curtis,	$650	5½%	1 yr. 3 mo.
7.	J. B. Adams,	J. K. Kelly,	$425	5%	1 yr. 9 mo.

Make the following entries on the notes:

8. Indorse in blank, using the payee's name, the note required in Example 4.

9. Indorse, as if sold by the payee to John V. Smith, the note required in Example 5.

10. Indicate that the note required in Example 6 was paid at maturity.

PARTIAL PAYMENTS

370. A **Partial Payment** is a payment of a part of a note or other obligation bearing simple interest.

The amount and date of payment are usually written or entered upon the back of a note.

371. The legal method of computing interest on notes when partial payments have been made is known as the *United States Rule*. It is as follows:

1. *From the amount of the principal, computed to the time when the payment or the sum of the payments equals or exceeds the interest due, subtract such payment or the sum of the payments.*

2. *Treat the remainder as a new principal, and proceed as before.*

WRITTEN EXERCISES

372. 1. A note for $600, bearing interest at 6%, and dated July 10, 1906, had the following payments entered upon it:

Jan. 25, 1907, $10; May 20, 1907, $100; Oct. 2, 1907, $350. What was due Dec. 20, 1907?

Principal	$600.00
Interest to May 20, 1907 (10⅓ mo.)	31.00
Amount	$631.00
First payment + second payment	110.00
Balance due = 2d principal	$521.00
Interest to Oct. 2, 1907 (4⅔ mo.)	11.46
Amount	$532.46
Third payment	350.00
Balance due = 3d principal	$182.46
Interest to Dec. 20, 1907 (2⅔ mo.)	2.37
Amount due Dec. 20, 1907	$184.83

The interest from July 10, 1906, to Jan. 25, 1907, is $19.50, which is more than the payment; hence, by the rule, it cannot be added to the

principal. The reason for this is that if the interest were now added and the $10 deducted, the new principal would be $609.50, and the borrower would be paying interest on $9.50 too much. We therefore compute interest to the time of the *second* payment, which together with the *first* payment is more than sufficient to pay accrued interest.

2. On a note for $1000, bearing interest at 6%, dated Sept. 15, 1905, are noted the following payments: Dec. 24, 1905, $25; Jan. 18, 1906, $250; June 12, 1906, $200. What was due Sept. 15, 1906?

3. A note for $450, dated March 21, 1906, had the following payments credited on it: April 20, 1906, $105; Oct. 5, 1906, $28; Nov. 15, 1906, $225. At 6%, what was due Dec. 25, 1906?

4. A note for $1200, dated Sept. 10, 1907, and bearing interest at 5%, had credits as follows: March 4, 1908, $100; Aug. 27, 1908, $20; Nov. 9, 1908, $250. How much was due Dec. 27, 1908?

5. A man bought a house and lot for $1500. He paid $300 cash, and gave his note for the balance bearing interest at 8%. If he paid $300 on this note every 6 months, what sum settled the note $2\frac{1}{2}$ years from its date?

6. Write a note for $2500, dated Nov. 1, 1906, payable on demand to Frank Brown or order, with interest at 6%; Frank Crum maker. Write on the back of the note the following payments: Dec. 27, 1906, $180; May 15, 1907, $600; June 21, 1907, $450. Find how much was due Dec. 21, 1907.

COMPOUND INTEREST

373. Interest found by adding interest, when due, to the principal and using the amount as a new principal is called **Compound Interest.**

1. When the interest is added to the principal at the

end of each year, the interest is said to be compounded *annually;* when it is added every six months, it is said to be compounded *semiannually;* and when it is added every three months, it is said to be compounded *quarterly.*

2. Savings banks usually allow compound interest to depositors; otherwise it is not in general use.

WRITTEN EXERCISES

374. 1. Find the compound interest for 3 years on $400 at 4%, compounding at the end of each year.

$400 = 1st principal.
.04
16.00 = 1st year's interest.
400
$416.00 = 1st amount = 2d principal.
.04
16.64 = 2d year's interest.
416
$432.64 = 2d amount = 3d principal.
.04
17.3056 = 3d year's interest.
432.64
$449.95 = the amount. (Why count extra cent?)
400
$49.95 = the compound interest.

The simple interest for the same time and rate is $48; the compound interest is greater, because it includes interest on interest.

2. Find the compound interest for 2 years on $50 at 5%, compounding semiannually.

When the rate is 5% per annum, it is $2\frac{1}{2}$% for 6 mo. In this problem the interest is compounded four times at $2\frac{1}{2}$%.

3. Find the compound interest on $800 for 3 yr. 6 mo. $2\frac{1}{4}$ da. at 6%.

The amount at compound interest for 3 yr. = $952.81
Interest on $952.81 for 6 mo. 24 da. = 32.40
Amount for 3 yr. 6 mo. 24 da. = $985.21

Find the interest compounded annually on —

4. $510 for 4 years at 5%.

5. $2500 for 2 years 8 months 15 days at 4%.

Find the interest compounded semiannually on —

6. $650 for 3 years at 4%.

7. $1000 for $2\frac{1}{2}$ years at 3%.

375. In banks and commercial houses, where many calculations are made, interest tables are used. They are especially valuable in calculating compound interest. A compound interest table is shown on the next page. The number of interest periods is given in the first column; in the other columns the amounts of $1 at the several rates per period are given.

1. Find the amount of $1750 for 6 years, interest compounded semiannually, at 5% per annum.

In 6 years there are 12 half years and the compounding is done 12 times at $2\frac{1}{2}$%.

In the "$2\frac{1}{2}$%" column of the table on the next page opposite 12 periods we find $1.3449. $1750 \times \$1.3449 = \$2353.58$, the amount.

Find amounts, interest compounded annually, of —

2. $2250 at 5% for 10 years.

3. $6755 at 6% for 9 years.

4. $55 at 4% for 15 years.

Find amounts, interest compounded semiannually, of —

5. $865 at 6% per annum for 4 years.

6. $750 at 4% per annum for 5 years.

7. Find the amount, interest compounded quarterly, of $800 at 4% per annum for 5 years.

8. A teacher deposits $300 in a savings bank that pays 4% per annum and compounds the interest quarterly. How much will she have to her credit at the end of three years?

TABLE

Showing the amount of $1 compounded for —

PERIODS	1 PER CENT	1½ PER CENT	2 PER CENT	2½ PER CENT	3 PER CENT
1	1.0100	1.0150	1.0200	1.0250	1.0300
2	1.0201	1.0302	1.0404	1.0506	1.0609
3	1.0303	1.0457	1.0612	1.0769	1.0927
4	1.0406	1.0614	1.0824	1.1038	1.1255
5	1.0510	1.0773	1.1041	1.1314	1.1593
6	1.0615	1.0934	1.1262	1.1597	1.1941
7	1.0721	1.1098	1.1487	1.1887	1.2299
8	1.0829	1.1265	1.1717	1.2184	1.2668
9	1.0937	1.1434	1.1951	1.2489	1.3048
10	1.1046	1.1605	1.2190	1.2801	1.3439
11	1.1157	1.1779	1.2434	1.3121	1.3842
12	1.1268	1.1956	1.2682	1.3449	1.4258
13	1.1381	1.2136	1.2936	1.3785	1.4685
14	1.1495	1.2318	1.3195	1.4130	1.5126
15	1.1610	1.2502	1.3459	1.4483	1.5580
16	1.1726	1.2690	1.3728	1.4845	1.6047
17	1.1843	1.2880	1.4002	1.5216	1.6528
18	1.1961	1.3073	1.4282	1.5597	1.7024
19	1.2081	1.3270	1.4568	1.5987	1.7535
20	1.2202	1.3469	1.4859	1.6386	1.8061

PERIODS	3½ PER CENT	4 PER CENT	4½ PER CENT	5 PER CENT	6 PER CENT
1	1.0350	1.0400	1.0450	1.0500	1.0600
2	1.0712	1.0816	1.0920	1.1025	1.1236
3	1.1087	1.1249	1.1412	1.1576	1.1910
4	1.1475	1.1699	1.1925	1.2155	1.2625
5	1.1877	1.2167	1.2462	1.2763	1.3382
6	1.2293	1.2653	1.3023	1.3401	1.4185
7	1.2723	1.3159	1.3609	1.4071	1.5036
8	1.3168	1.3686	1.4221	1.4775	1.5938
9	1.3629	1.4233	1.4861	1.5513	1.6895
10	1.4106	1.4802	1.5530	1.6289	1.7908
11	1.4600	1.5395	1.6229	1.7103	1.8983
12	1.5111	1.6010	1.6959	1.7959	2.0122
13	1.5640	1.6651	1.7722	1.8856	2.1329
14	1.6187	1.7317	1.8519	1.9799	2.2609
15	1.6753	1.8009	1.9353	2.0789	2.3966
16	1.7340	1.8730	2.0224	2.1829	2.5404
17	1.7947	1.9479	2.1134	2.2920	2.6928
18	1.8575	2.0258	2.2085	2.4066	2.8543
19	1.9225	2.1068	2.3079	2.5270	3.0256
20	1.9898	2.1911	2.4117	2.6533	3.2071

RELATED PROBLEMS

376. The picture shows the tracks of three great railway systems crossing each other at different grades. The following examples give some idea of the magnitude of the railroad interests of this country.

1. A recent report of the Interstate Commerce Commission showed a total of 214,478 miles of railway in the United States. The gross earnings for one year were $2,073,144,348, and the operating expenses were $1,383,597,578. What were the average gross earnings per mile? What was the per cent of operating expenses to gross earnings?

2. If 69.9% of the gross earnings came from freight traffic, 27.6% from passenger traffic, and the remainder from other sources, find the earnings credited to each.

3. The western division of the A. B. & C. railway has 665 miles of single track. If the ties were laid two feet from center to center, how many ties were used in its construction? At 45¢ each, what did they cost?

4. If this system is laid with steel rails that average 85 lb. to the yard, how many tons of rails are used? At $30 per ton, what are they worth?

5. During a recent year the railroads of this country bought 165,455 freight cars, 2551 passenger cars, and 5491 locomotives. The locomotives averaged $14,000 each, the passenger cars 50% as much, and the freight cars $14\frac{2}{7}\%$ as much as the passenger cars. How much did the roads expend for rolling stock that year?

6. In a recent year there were 1,296,000 railroad employees in this country. Of this number $3\frac{1}{10}\%$ were conductors, $4\frac{1}{10}\%$ engineers, and $4\frac{1}{4}\%$ firemen. The average daily earnings of conductors were $3.50; of engineers $4.10; and of firemen $2.35. What were the total earnings of conductors, engineers, and firemen that year (365 da.)?

7. In a recent year the railroads carried 719,650,000 passengers, of whom 15,112,650 rode in Pullman cars. What per cent of the passengers rode in Pullmans?

8. In 1904 the length of the railways of the world aggregated 534,000 miles. There were 213,600 miles in the United States that year. What per cent of the total mileage was that of the United States?

9. The earnings of the A. & B. R.R. were —

	1905	1906
From freight traffic	$858,517.63	$904,092.53
From passenger traffic	520,426.26	544,615.81
From other sources	143,388.49	149,480.47

(*a*) Find the gross earnings for each year. (*b*) Find the increase in gross earnings in 1906 over those of 1905. (*c*) Find the per cent of increase of gross earnings. (*d*) Find the per cent of increase of earnings from freight traffic. From passenger traffic.

BANK PRACTICE

377. The ordinary business of a bank is —

(1) To receive deposits;

(2) To make loans;

(3) To pay debts in distant places. (See Art. 389.)

378. Receiving Deposits.

In opening an account with a bank, it is first necessary to make a *deposit*. Each deposit corresponds with a *deposit slip* giving the amount and description of the money deposited. A common form of deposit slip is shown at the right.

DEPOSITED IN THE

First National Bank

OF BIRMINGHAM, ALA.

By Fred Hoyt

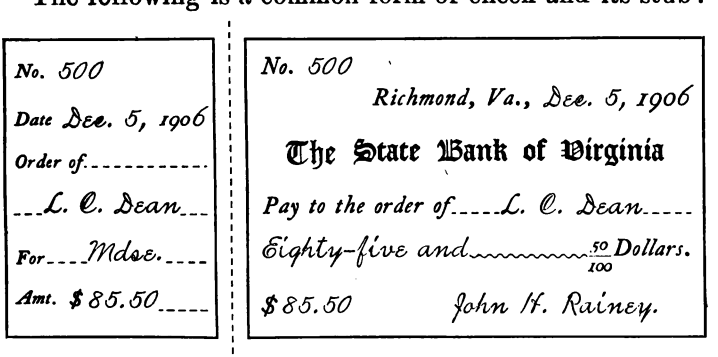
Feb. 25, 1906

	Dollars	Cents
Bills	65	
Gold	5	00
Silver	30	40
Checks	8	50
	10	75
Total	119	65

379. After opening an account, a depositor may draw *checks*, whose sum should not exceed the amount to his credit.

Each transaction has its proper record. The bank has its own system of bookkeeping, and the depositor keeps a record of his checks and deposits in a check book.

The following is a common form of check and its stub:



No. 500
Date Dec. 5, 1906
Order of.....
L. C. Dean
For Mdse.
Amt. $85.50

No. 500
Richmond, Va., Dec. 5, 1906
The State Bank of Virginia
Pay to the order of L. C. Dean
Eighty-five and 50/100 Dollars.
$85.50 John H. Rainey.

The teacher should procure from some bank the forms here introduced and show the transactions as they are actually made.

1. Make a slip showing the deposit of $550 in the Commercial National Bank of Nashville, Tenn.; then make checks to cover the following bills, and determine your balance:

Archer Bros., groceries	$28.59
Babbitt & Co., set of furniture . . .	175.00
Clement & Smith, dry goods . . .	56.75
Bell Book Co., books and stationery . .	21.20

2. John West made deposits as follows:

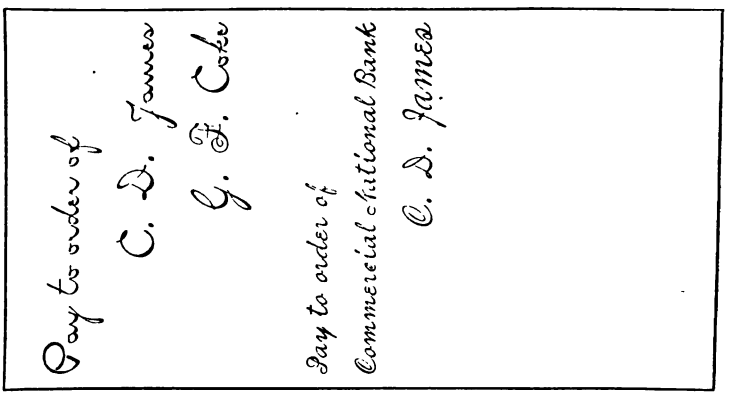
March 18, 1906, ...	$170
June 29, "	648

He drew checks as follows: March 20, $65.20 to G. F. Coke; April 15, $29.65 to E. P. Mason; May 26, $57.90 to W. B. Dabney; July 21, $125 to B. Caldwell. Make checks, and find his balance.

3. What balance had I left from a deposit of $1000 after checking out $137, $165.40, $289.50, $47.65, and $106.75?

380. Checks must be indorsed before they can be cashed or transferred to another.

For example, if John West paid $65.20 to G. F. Coke by check, and Mr. Coke transferred the check to C. D. James, who deposited it in the Commercial National Bank, the back of the check would appear as in the following cut:



When a check is presented at the bank on which it was drawn, it is paid and so stamped.

1. Show the writing on the back of a check which is made payable to A. C. Babcock and indorsed by him to D. Armstead, who deposited it in the National Traders Bank of Birmingham.

2. Make a check payable to W. T. Poindexter for $25.56. Indorse on the back the form which would be used by Mr. Poindexter in transferring the check to F. J. Turner, and then show the indorsement necessary to deposit it to Mr. Turner's credit in the Farmers National Bank of Atlanta.

381. Making Loans.

If a man wishes to borrow money from a bank, he makes out a note that the bank will accept, and has it *discounted.* In such transactions the most direct way is to make this note payable to the bank.

If an indorser is required, the note is made payable to some acceptable person who is willing to indorse it. The following is a common form of such note :

$150 *Richmond, Va., March 15, 1906*

Sixty days after date I promise to pay to the order of Walter Conrad

One Hundred Fifty Dollars

at the Bank of Richmond. *Value received.*

Due *Edward Leake.*

If the *legal rate* is 6%, and the bank discounts this note, it deducts 1% (6% per annum for 60 days) and pays (lends) Edward Leake the balance ($148.50). When the note falls due Edward Leake pays $150. In effect, he borrows $148.50, and pays $1.50 interest in advance for its use for 60 days. Does he pay more or less than 6% per annum on the sum borrowed ?

382. Interest paid in advance on the *maturity value* of a note is called **Bank Discount.** If a note does not bear interest, its face is the maturity value.

The sum paid by a bank for or on a note is called the *proceeds.*

383. The *term of discount* is the time for which the bank computes discount — the period for which it charges interest. It begins on the day of discount, and ends on the *day of maturity.*

1. Some bankers include both of these days in the term of discount.

2. In states in which days of grace have not been abolished, the term of discount is increased by three days.

3. Banks do no business on Sundays or legal holidays; hence notes falling due on such days are payable on the preceding or succeeding day. Find out when notes falling due on these days are payable in your state.

384. In finding the day of maturity, banks count forward by months or by days as the note specifies.

The term of discount is usually found in days. In counting time by days, use the table given in Art. 358.

WRITTEN EXERCISES

385. 1. What are the proceeds of a 90-day note for $1575, discounted at 7 %?

Discount = int. on $1575 for 90 days at 7 % = $27.56.
Proceeds = $1575 − $27.56 = $1547.44.

2. Find the discount and proceeds of a 60-day note for $279, discounted at 6 %.

3. Find proceeds of a 30-day note for $4578, discounted at 6 %.

4. What is the discount on a 30-day note for $1500, discounted at 6 %?

5. Find the discount on a note of $29 for 4 months, at 8 %.

Find the discount on each of the following notes:

6. $580 for 60 days at 6 %.

7. $1675 for 90 days at 6 %.

8. $205 for 93 days at 6 %.

9.

$500. RICHMOND, VA., Dec. 31, 1906.

Two months after date, I promise to pay to H. E. Gum, or order, five hundred dollars, value received.

D. E. STONE.

Discounted Dec. 31, at 6 %. Find the proceeds.

Date of maturity = Feb. 28, 1907 ; term of discount = 59 days.

10.

$3000. NEW YORK, June 30, 1906.

Sixty days after date, I promise to pay to F. J. Kerr, or order, $3000, value received.

T. S. MILLS.

Discounted June 30, at 6 %. Find the proceeds.

11.

$600. RALEIGH, N.C., Sept. 15, 1907.

Sixty days after date, I promise to pay to the order of William Day six hundred dollars, value received.

C. W. BROWN.

Discounted Sept. 15, at 6 %. Find the proceeds.

12. Make out a 90-day note for $1000, dated to-day, payable to Hugh Gordon's order at your bank, and discounted at the legal rate. Then find the proceeds.

386. Hitherto we have been considering notes given primarily for the purpose of obtaining loans. But banks also make a business of discounting (buying) notes given by one person to another in ordinary business transactions.

For example, if John Ray buys a lot on credit from Richard May for $2000 and gives his note for the purchase price, payable in 60 days with interest at 6 %, the latter may indorse the note and sell it to a bank for the maturity value less the discount.

When such notes are not discounted on the day of making, the discount is reckoned on the maturity value from the day of discount to the day of maturity.

WRITTEN EXERCISES

387. 1. A gives his note to B for $400, dated March 1, 1907, payable in 90 days with interest at 6%. On March 25, B discounts the note at 6%. Find the proceeds.

Day of maturity = May 30, 1907.
Maturity value = $400 + $6 (int. for 90 da.) = $406.
Term of discount = 66 da. (March 25 to May 30).
Discount = interest on $406 for 66 da. = $4.47.
Proceeds = $406 − $4.47 = $401.53.
In states where both days are included, the term of discount would be 67 days; when grace is allowed, it would be three days more.

Find proceeds of notes described as follows:

	Face	Date	Due	Rate of Interest	Discounted	Rate of Discount
2.	$600	June 1	Aug. 1	6%	July 2	6%
3.	$450	Aug. 15	Nov. 13	8%	Sept. 14	8%
4.	$225	Dec. 1	Jan. 30	5%	Dec. 31	6%

Find the proceeds of the following notes, which bear no interest, if discounted at 6% on June 1, 1906:

5. $245 dated May 20, 1906, payable in 55 days.

6. $1265 dated March 2, 1906, payable in 150 days.

7. $565.20 dated April 30, 1906, payable in 90 days.

8. $179.38 dated March 25, 1906, payable in 90 days.

9. For what sum is a 60-day note given, if a bank discounting it at 6% gives the maker $148.50?

Discount on $1 for 60 days at 6% = $.01.
Proceeds of $1 note = $1 − $.01 = $.99.
Entire proceeds of note = $148.50.
There must be $1 in the face of the note for every $.99 of proceeds.
Hence, $148.50 ÷ $.99 = 150, number of dollars in face of note.

EXCHANGE

388. If a man owes money to some one living at a distant place, he can pay the debt in several ways without actually sending the money. He can send a *check*, a *draft*, a *postal money order*, or an *express money order*.

389. The system of paying debts at distant places by means of checks, money orders, or drafts is called **Exchange.**

If John Brown, of Dallas, owes Richard Byrd, of Nashville, $50, the former may send his check for the amount to the latter, who indorses it and has it cashed. The bank receiving it collects the sum named from the bank on which it is drawn, and may charge a small amount for the trouble of collection.

390. A **Postal Money Order** is an order drawn by one postmaster on another, directing him to pay a specified sum of money to the order of the person named therein. The extra cost of a money order varies according to the amount of the order:

Orders for sums not exceeding $2.50	3 cents.
Over $2.50 and not exceeding $5	5 cents.
Over $5 and not exceeding $10	8 cents.
Over $10 and not exceeding $20	10 cents.
Over $20 and not exceeding $30	12 cents.
Over $30 and not exceeding $40	15 cents.
Over $40 and not exceeding $50	18 cents.
Over $50 and not exceeding $60	20 cents.
Over $60 and not exceeding $75	25 cents.
Over $75 and not exceeding $100	30 cents.

391. An **Express Money Order** is substantially the same as a postal money order. The same fees are charged, and the business is similarly conducted, express companies having offices in all parts of the country.

State cost of money orders for:

1. $25.25.	4. $20.	7. $30.40.
2. $ 7.25.	5. $47.	8. $75.75.
3. $17.50.	6. $60.	9. $99.95.

392. A **Bank Draft** is the same as a check except that it is drawn by one bank on another. The following is a common form:

The Citizens National Bank *No. 51*

Raleigh, N.C., June 15, 1907

Pay to the order of Olin Phelps $ 500.00

Five Hundred Dollars.

Jos. G. Brown, **President.**

The National Park Bank,
New York.

To enable them to sell drafts banks keep money on deposit with banks in New York and other leading cities.

A draft on a New York bank — New York exchange — is usually good in any part of the country. Hence, if you wish to pay money to a person in Charleston, you may buy New York exchange for the sum desired and send it to him. On receipt of the draft he deposits it with his local bank, or sells it to his bank. In due time it reaches the bank on which it was drawn, when it is paid.

393. In addition to the face of the draft, banks may charge either a nominal sum to cover expense, or a certain per cent of the face of the draft. On large sums it is usually $\frac{1}{10}\%$; sometimes it is a stipulated sum per $1000. Some banks agree to furnish exchange free of charge to large depositors.

394. Commercial Drafts are used extensively for the collection of debts. The following is a common form:

$ 500. *Nashville, Tenn., July 15, 1906*

At sight *Pay to the order of* *The First National Bank of Nashville*

Five Hundred $\frac{00}{100}$ *Dollars.*

Carl Hunter.

To Robert Hitt,
Clarksville, Tenn.

The party desiring to collect a debt from another makes a draft on him for the amount, and deposits it with his bank for collection. This bank sends the draft to a bank at the home of the debtor, which collects the money and returns it to the forwarding bank; or, failing to collect, returns the draft. For this service banks usually make a small charge.

395. Foreign Exchange is subject to the same general principles as exchange between the different cities of this country, differing chiefly as to currency and manner of making quotations.

Foreign Drafts are usually called **Bills of Exchange.**

The rate of foreign exchange varies, and is regularly quoted in the large daily newspapers. Exchange for sight drafts was quoted in New York, on May 5, 1906, as follows:

On London at $4.85 to the pound, meaning that a draft on London for £1 was worth $4.85 in our currency.

On Paris, at 5.18 francs for $1, meaning that $1 would buy a draft on Paris for 5.18 francs. It is sometimes quoted at so many cents to the franc.

On Berlin, at 4 marks for $.95, meaning that $.95 would buy a draft on Berlin for 4 marks. It is sometimes quoted at so many cents to the mark.

396. In English money —

12 pence (*d.*) = 1 shilling (*s.*);
20 shillings = 1 pound (£).

The value of £1, as fixed by the U. S. Treasury, is $4.8665 in our money. The value of a penny is about 2¢; that of a shilling, about 25¢.

WRITTEN EXERCISES

397. **1.** What will a draft for $350 cost, if the banker's charge for exchange is $\frac{1}{10}$ % on its face?

The charge = $\frac{1}{10}$ % of $350 = $.35.
Total cost = $350 + $.35 = $350.35.

2. What will a draft for $2500 cost, if the charge for exchange is 50¢ per $1000?

3. What must be paid in New Orleans for a draft on Boston for $3500, the charge for exchange being $1.25 per $1000? Write the draft, dating to-day and filling in suitable names.

4. What is the cost of a draft on St. Louis for $2000, if the charge for exchange is 75¢ per thousand?

5. A man owes the following amounts for goods: \$24, \$45, \$75, and \$100. What will money orders to cover all these amounts cost, including fees?

6. A merchant owes different jobbers in other cities the following amounts: \$350, \$200, and \$680. What will drafts to cover all these amounts cost, at a charge of $\frac{1}{10}$ % on the faces for exchange?

7. C. D. Key owes Smith & Co. \$750. They draw on him for the money, and the banks charge $\frac{1}{10}$ % for collection. Write the draft, and find the proceeds. What kind of a draft is this?

8. Find the cost in New York of a sight draft on London for £15 12*s*., exchange at \$4.87.

£15 12*s*. = £15.6.
15.6 × \$4.87 = \$75.972.
∴ the draft will cost \$75.97.

9. Find the cost of a 60-day draft on London for £30 10*s*., exchange at \$4.83.

10. How large a sight draft on Paris can be bought for \$1000, exchange at 5.21?

11. What is the cost of a sight draft on Berlin for 600 marks, the rate of exchange being \$.945 for 4 marks?

12. What is the cost of a sight draft on Paris for 500 francs, exchange at 5.21?

13. A merchant buys in London a bill of goods amounting to £80 10*s*. What will a sight draft for this amount cost, exchange at \$4.87$\frac{1}{4}$?

14. A Mobile firm shipped to Liverpool 100 bales of cotton, averaging 500 lb. each, which netted 6.37*d*. per pound. The proceeds were invested in New York exchange at face value (\$4.8665). What was the face of the bill of exchange on New York?

REVIEW WORK

ORAL EXERCISES

398. **1.** A college professor receives a salary of $2000 a year, and spends 75% of it. How much does he save?

2. If a man can saw .4 of a cord of wood in a day, how long will it take him to saw $\frac{4}{5}$ of a cord?

3. If 3 pecks of potatoes last a family 6 weeks, how long will 4 bushels last them at the same rate?

4. Desiring to buy eggs, a lady found that by getting a reduction of 5¢ a dozen she would save 50¢ on the purchase. How many dozen did she want?

5. James spent $\frac{1}{4}$ of his money for a history, and $\frac{1}{5}$ of it for a reader. What per cent of the original sum had he left?

6. What number increased by its half and its fourth equals 175?

7. If Marvin earns $\frac{3}{4}$ of a dollar in 1 day, how much will he earn in $\frac{2}{3}$ of a day?

8. One half of the money in Elizabeth's purse is in dimes, $\frac{1}{4}$ in nickels, and the remainder, 10¢, in pennies. What sum has she in her purse?

9. Phillips has $1 more than James, and both have $21. How much has each?

10. Mr. Brown paid $30 for the use of $500 for a year. What was the rate of interest?

11. How many years will it take $200 to earn $60, at 6% per annum?

12. Mr. Jones borrowed money at 6 %, and found that his interest for a year was $15. What was the sum borrowed?

13. At $\frac{1}{5}$ of a dollar a yard, how many yards of ribbon can be bought for $5?

$1 buys 5 yd.

14. When asked how many shells he had, Kirkman replied: "If I had twice as many as I have and 10 more, I should have 50." How many shells had he?

$50 - 10 = 40 =$ twice his number of shells.

15. A book agent (canvasser) sold books to the amount of $200, on which his commission was $80. What was the rate of his commission?

16. One eighth of a yard is what per cent of $\frac{1}{2}$ of a yard?

17. What is 50 % of 50 cents? Of $5?

18. At $7.72 a pound, what will 3 oz. of silver cost?

19. What is the number whose half exceeds its fourth by 25?

20. At $2.50 a cord, what is the value of a pile of wood 8 ft. by 8 ft. by 4 ft.?

21. If my age is doubled, and 10 yr. subtracted from the result, the remainder will be 22 yr. What is my age?

22. A boy sold papers at the rate of 2 for 5 cents. If they cost 1 cent each, how much did he gain on 100 papers?

23. The length of a room is twice its width. Its perimeter is 60 ft. What is its width? Its length?

24. After spending 50 % of his money, and 50 % of the remainder, Owen had $5 left. How much had he at first?

25. A starts on a journey, and walks 2 miles an hour for 5 hours. Then B starts out to overtake him, walking 3 miles an hour. In how many hours will B overtake A?

WRITTEN EXERCISES

399. 1. The equipment of a certain railroad consists of 46 locomotives, 56 passenger cars, 2231 freight cars, and 13 road-service cars. What is the average number of cars to each locomotive?

2. A Rochester oil lamp burns 1 gal. of oil every 10 hours. If it is lighted 1200 hr. in a year, how much will the oil cost at 15¢ a gallon?

3. A 16-candle power jet burns 5 ft. of gas per hour. If it is lighted 1200 hours in a year, what is the gas bill, at $1.10 a thousand feet?

4. The greatest known ocean depth is in the mid-Pacific, where soundings have measured 5269 fathoms. How many feet deep is this ocean? How many miles deep is it? (1 fathom = 6 ft.)

5. The greatest depth of the Atlantic Ocean yet measured is 4662 fathoms. How many feet deep is this ocean? How many miles?

6. A farmer raised 1000 bu. of wheat. How many days did it require to market it, if he made four trips a day and hauled 3000 lb. to the load?

7. For spraying fruit trees I use 1 lb. of Paris green to every 150 gal. of water. Three gallons of the mixture are used to each tree. If the Paris green costs 22¢ a pound, what is the cost of the material for spraying an orchard of 250 trees?

8. A "Star route" mail carrier has carried the mail on a 46-mile route for 18 consecutive years. If he went out one day and came in the next, making on an average 310 trips over this route each year, what is the total distance traveled by him during the eighteen years?

9. The earnings of a railroad in 1906 were \$1,714,702, an increase of \$21,161; the expenses were \$1,312,385, an increase of \$33,921. The net earnings were \$402,317, a decrease of \$12,759. Find earnings, expenses, and net earnings for the year 1905.

10. The rate on a telegram of 10 words between two of our cities is 25¢. At 2¢ for each additional word, what is the cost of an 18-word message? What is the cost of a 20-word message?

11. The rate on a cablegram from Washington to the Philippine Islands was \$1.75 for each word, including the address and signature. At this rate find the cost of sending the following cablegram:

JOHN RAY,
Manila.
Arrived well.
ROBERT.

12. In a weekly periodical there are 4 columns to a page, each 13 in. long. At \$1.25 an inch for each insertion, what is the weekly income from 6 pages of advertising matter?

13. A farmer has 30 sheep in his flock. If each sheep yields $6\frac{1}{2}$ lb. of wool, how much is the clip worth at 35¢ a pound?

14. A pay roll was met by the following: 44 five-dollar bills, 47 two-dollar bills, 15 one-dollar bills, 14 half-dollars, 4 quarters, 3 dimes, and 6 coppers. How much money was paid out?

15. Find the interest at 6% on \$1750 for 2 yr. 3 mo. 20 da.

16. A consulting engineer makes plans and specifications for water improvements for a city. If the cost of

construction is $27,500 and the engineer's fees are 5% of the cost of construction, what will be the total cost of the improvements?

17. Out of 95 operations, a surgeon gave complete relief to 80% of his patients. How many patients were not cured?

18. A daily ration for dairy cows is made up as follows: corn silage, 30 lb.; cut sheaf oats, 6 lb.; mixed meadow hay, 10 lb.; wheat bran, 4 lb.; linseed meal, 2 lb. What is the total number of pounds in this ration? What per cent of the whole ration is the corn silage?

19. Another daily ration for dairy cows is as follows: corn silage, 32 lb.; clover silage, 22 lb.; clover and timothy hay mixed, 5 lb.; wheat bran, 6 lb.; ground oats, 4 lb.; cotton-seed meal, 3 lb. What per cent of the whole ration is the corn silage? The wheat bran is what per cent of the whole?

20. James finds that a good charge for his 12-bore gun consists of 3 drams of powder and 1 ounce of shot. On this basis, how many shells can he load with 3 lb. of powder? How many pounds of shot will he need for these shells? (1 Avoirdupois dram $= \frac{1}{16}$ oz.)

21. How many shells will a pound of powder load, allowing 3 drams to the charge? $3\frac{1}{2}$ drams? (Omit fractions in results.)

22. How many shells will a pound of shot load, using $1\frac{1}{8}$ oz. to the charge? $1\frac{1}{4}$ oz.? (Omit fractions in results.)

23. A merchant's books show sales during the month of October amounting to $3000. If one half of the sales were at a profit of 25% on the cost, and the other half at a profit of 20%, find the cost of the goods sold during the month.

24. A certain ore yields 6% of nickel. If the nickel is worth 40¢ a pound after smelting, what is the nickel from 20 tons of ore worth?

25. Find the cost of 2700 lineal feet of half-round at $1.15 a hundred feet.

26. How many board feet in a beam 30 feet long and 8 in. by 8 in.?

27. Find the contents in bushels of a bin 43 ft. by 10 ft. by 5 ft.

28. Make a bill for the following, and find the amount: 7 qt. of milk @ 5¢; 3 lb. butter @ 20¢; 2 doz. eggs @ 22¢; 5 lb. coffee @ 20¢; ¼ lb. tea @ 90¢; 10 lb. sugar @ 5½¢; 8 lb. lard @ 9¾¢; 2¾ lb. cheese @ 20¢; 10½ lb. ham @ 14¢.

29. Find the cost of the following bill of lumber: 10 pieces for sills 2″ × 8″, 12 ft. long, @ $15; 100 pieces for studding 2″ × 4″, 16 ft. long, @ $15; 25 rafters 2″ × 4″, 10 ft. long, @ $15; 3000 ft. sheathing, @ $15; 3000 shingles, @ $2.25.

30. In a certain year 827,748,000 passengers were carried on the railways and steamships of the United States. If the railways carried 676,270,116, what per cent of the whole number was carried by the steamships?

31. How many steel rails must be ordered for laying a double track 36 miles, if each rail is 33 feet long?

32. How many tons will these rails weigh if 1 yard weighs 100 lb.? What will they cost at $28 a ton?

33. A farmer fenced a field 110 rd. by 49 rd., placing his posts 1 rd. apart. He paid 12¢ each for posts, and 30¢ per rod (of fence) for wire and staples. If the cost of construction was 40¢ a rod, what was the total cost of his fence? Make a diagram.

34. In a day A can saw a pile of wood 4 ft. × 16 ft. × 3 ft., B can saw a pile 4 ft. × 15 ft. × 4 ft., and C can saw a pile 4 ft. × 13 ft. × 4 ft. How many cords can they all saw in $16\frac{1}{2}$ days? (128 cu. ft. = 1 cord.)

35. A stairway contains 18 steps, each 8 in. high and 12 in. wide. How many yards of carpet will be required to furnish a strip for the stairs?

36. A has 60 logs to be delivered at a mill. B offers to haul for him 4 loads of 3 logs each for \$3.50 a day; while C proposes to haul 5 loads of 2 logs each for \$3 a day. Which offer is the lower? How much lower?

37. If a barrel of flour (196 lb.) costs \$4.90, at what price per pound must a dealer sell it in order to gain 20%?

38. If a ton of hard coal yields 182 lb. of carbon, how much carbon will 3500 lb. of such coal yield?

39. A traveling salesman received a guaranteed salary of \$1200 a year, and in addition 5% commission on all sales over \$100,000. At the end of the year it was found that he had sold \$125,000 worth of goods. How much did he receive for the year's services?

40. A stenographer made a record of 31,950 words in $7\frac{1}{2}$ hours. How many words did she average to the minute?

41. The weight of an unbound book is 12 oz. If a spruce tree makes 400 lb. of paper, how many such trees will be required for an edition of 40,000 of these books?

42. A building lot has a frontage of 72 feet. Its area is 14,400 sq. ft. What is its depth?

43. What is the area of a parallelogram whose base is 606 ft. and altitude $66\frac{2}{3}$ feet?

44. For \$75 a grocer bought 100 bunches of bananas, averaging 6 doz. to the bunch, and sold them at 20¢ a dozen. What per cent did he gain?

45. A mechanic worked 312 days during the year at $3 a day. If he saved $140.40, what per cent of his earnings did he save?

46. A dealer bought 20 dozen pairs of Irish point lace curtains for $3.50 a pair. He sold one half of them at a loss of 6%, and the remainder at a profit of 20%. How much did he gain?

47. Find the per cent gained on the following goods:

(*a*) Carpeting bought at 60¢ a yard, sold at 72¢ a yard;

(*b*) A suit of furniture bought at $69, sold at $92;

(*c*) Chairs bought at $13.50, sold at $16.74.

48. A furniture house advertises a cut of $12\frac{1}{2}$% on all of its goods. Find the selling price (1) of an oak sideboard marked $46; (2) of a Morris chair marked $16.

49. If a commission merchant remits $2640 to a customer, after deducting 4%, what was the amount of the sale?

50. Find the difference between the simple interest and the compound interest on $1320 for 3 years at 6%.

51. Make out a 60-day note for $875, dated to-day, payable to the order of Stanley Dexter at some bank in your vicinity, and discount it at 6%. Find the proceeds.

52. What will a Roanoke merchant pay for a draft on New York for $1620 at $\frac{1}{10}$% above its face?

53. In 2 yr. 6 mo. $240 amounted to $267. What was the rate?

54. What principal will earn $94.50 in $3\frac{1}{2}$ yr., at 6%?

55. A note of $650 dated Jan. 25, 1906, interest at 6%, has indorsed upon it the following record of payments:

May 15, 1906, $35; Sept. 5, 1906, $67.50; Jan. 20, 1907, $8.25; Mar. 5, 1907, $150. Find the amount due after the last payment.

INSURANCE

400. A contract by which a company agrees to indemnify the party insured against loss in the event of certain misfortunes is called an **insurance** contract.

401. The written contract of insurance is called a *policy*. It contains a promise to pay a specified sum in the event of certain contingencies. This sum is the *face* of the policy.

Persons engaged in insurance as a business are often called *underwriters*.

402. The sum paid for insurance is called the **premium.** It is sometimes computed at a given sum for each $100 or $1000 of insurance, and sometimes at a certain per cent of the face of the policy.

403. Fire Insurance.

Insurance against loss by fire is called **fire insurance.**

Premiums are based on a yearly per cent of the face of the policy.

Marine insurance, tornado insurance, etc., follow the same principle.

WRITTEN EXERCISES

404. 1. What is the annual premium for insuring a building for $3500 against loss by fire, if the premium is $1.30 per $100?

$3500 ÷ $100 = 35, number of hundred dollars.
35 × $1.30 = $45.50, the premium.

2. What will it cost to insure a building for $5000, at $1\frac{1}{4}\%$?

3. What will it cost to insure a store for $4000, at 1.2%?

4. A building worth $15,000 is insured for $\frac{3}{4}$ of its value, at $2\frac{1}{2}\%$. What is the premium?

5. What will it cost to insure a house for $2000 for five years, if the rate per year is $.50 on $100, and the house is insured for the term for three yearly premiums paid in advance?

6. What is the premium for insuring a house for a year for $8000 at $1.25 per $100, and the contents for $3750 at $1.50 per $100?

7. If a policy for $4500 costs $90, what is the rate per cent?

8. If it costs $36 to insure a house at $1\frac{1}{2}\%$, what is the face of the policy?

9. A merchant insured his stock, for $\frac{3}{4}$ of its value, at $2\frac{1}{2}\%$. The premium was $313.50. What was the value of his stock?

10. A merchant insured his store which was worth $15,000, for $\frac{3}{4}$ of its value, at 2%. If it was destroyed by fire, what was his loss including the premium?

11. A contractor shipped 1,000,000 feet of yellow pine from Gulfport, Miss., to Panama, valued at $10 a thousand. If insured at $\frac{3}{4}\%$, what was the premium?

12. A southern cotton mill with an automatic fire apparatus gets a rate of $\frac{5}{8}\%$ on its plant. If valued at $100,000 and insured for $\frac{3}{4}$ of its value, what is the annual premium?

13. By paying two annual premiums in advance, this mill gets a policy for 3 years. What is the average cost of insurance a year?

405. Life Insurance.

In **life insurance**, premiums are always stated at a given sum on each $1000 on a yearly basis. They vary with the age of the insured, being larger for older persons.

In consideration of the premiums paid, the insurance company agrees to pay a fixed sum to the person named in the policy (the *beneficiary*) at the death of the insured, or at a stipulated time.

406. Insurance companies offer various forms of policies. The more common forms are the *Ordinary Life*, *Limited Life*, and *Endowment* policies.

In the ordinary life the insured agrees to pay a certain premium annually for life. In the limited life the premiums are payable for a stated number of years, but the face of the policy is not paid until the death of the insured. At death premiums cease and the policy matures. In endowment policies premiums cease at the end of the endowment period, and the face of the policy is paid to the insured if living. If the insured dies before the end of the endowment period, the premiums cease and the face of the policy is paid to his beneficiary.

An *Accident* policy is a policy in which the company agrees to pay the person insured, or his beneficiary, certain sums for accidental injuries or death from accident.

Other forms of policies are modifications of the foregoing types.

WRITTEN EXERCISES

407. 1. What is the premium on a $3000 policy at $21.49 on $1000?

2. A man 28 years old insured his life for $3000 on the ordinary life plan, at the rate of $23.44 on $1000. What is the annual premium?

3. A man 35 years old takes out a policy for $10,000 on the 20-payment plan, at $38.34 per $1000. What is the annual premium? How much are the premiums for the 20 years?

4. At the age of 21 the annual premium on a 15-payment life is $36 per $1000. What would be the total amount of premiums paid by the insured on $15,000 in 15 years?

5. What is the annual premium on a 20-year endowment policy for $5000 at $48.48 per $1000? How much are the premiums for the 20 years?

6. A man 40 years old takes out an ordinary life policy for $7000, paying $33.01 per $1000. Should he die after paying 15 premiums, how much does his estate receive above what he paid in premiums?

7. At the age of 21 a man took a $2500 ordinary life policy at $19.12 per $1000, and a $3000 15-payment life, at $36 per $1000. How much did the face value of the two policies exceed the premiums paid if the insured died after having paid ten premiums?

8. A man insured his life for $5000, at a yearly premium of $32 per $1000. In annual settlements the company returned to him an average of 20% of his premiums. What was the net cost of his policy after 15 premiums had been paid?

9. A commercial traveler, whose salary was $125 a month, paid $2 for a thousand-dollar accident policy that allowed $10 a week for total disability due to accident. As a result of a wreck, he was disabled for 5 weeks, and lost 1 month's salary. What was his net loss of income?

TAXES

408. Charges imposed by law upon persons or property for public purposes are called **Taxes.**

The annual yield of taxes is called *revenue.*

409. State and Local Taxes.

Taxes levied by a state are known as **State Taxes**; those levied by the various minor civil divisions, as **Local Taxes.** The latter include county, city, and township taxes.

The property to be taxed is valued by officers chosen for the purpose. The value placed upon property by them is called the *assessed value.*

410. State and local taxes are reckoned at *a certain per cent* of the assessed value of property. The *rate of taxation* is usually expressed as a certain number of mills on the dollar, or as a certain number of cents on the $100. Thus, a tax of $3\frac{1}{2}$ mills on $1 is the same as a tax of 35 cents on $100.

1. In addition to the tax on property, some states levy a *poll* (head) *tax* annually on each male citizen over 21 years of age. This sum varies from $1 to $2 a year.

2. Privilege or license taxes are also levied on persons engaged in certain occupations and professions.

3. The collector of taxes usually receives a commission on the sums collected.

4. If taxes are not paid when due, the law usually requires the delinquent to pay a certain per cent of the amount of his taxes in addition to that originally levied against him.

WRITTEN EXERCISES

411. **1.** The council of a town with property assessed at $750,000 estimates the annual expenses at $5000. It expects to realize $1250 from licenses, and $500 from fines. What must be the tax rate if the levy includes an addition of 10% to the estimate for contingencies?

$5000 + 10% of $5000 = $5500, amount to be raised.
$5500 − $1750 = $3750, to be levied on property.
The tax on $750,000 = $3750.
The rate of taxation = $3750 ÷ $750,000 = .005, or ½%. (Art. 314.)

2. What is A's town tax at the above rate, if his real estate is assessed at $3000 and his personal property at $750?

$3000 + $750 = $3750 = A's taxable property.
½% of $3750 = $18.75, A's tax.

3 B pays a poll tax of $1.50, and a state tax of 60 cents on the $100. If he owns 250 acres of land assessed at $40 an acre, and personal property worth $2000, what are his state taxes?

4. A man paid 1½% in taxes (state, county, and city) on a house and lot assessed at $3000. What were the taxes?

5. Mr. Green lives in a state that levies state taxes as follows: for schools, 3 mills on the dollar; for pensions, 1 mill on the dollar; for general purposes, 2½ mills on the dollar. His real estate is assessed at $5000, and his personal property at $1400. Find his taxes for each purpose. For all the purposes stated.

6. Mr. Green's state taxes become *delinquent* if not paid by January 1. After that date a collector's fee of 50¢ and a penalty of 5% on the taxes are added. If Mr. Green left his taxes unpaid until March following, what amount had he to pay then?

7. Mr. Green's county taxes, on the same assessment, were for general purposes 35¢ on $100, and for roads 15¢ on $100. What was the county rate? What were his county taxes?

8. If the city tax rate is $\frac{1}{2}\%$ on the state assessment, what are Mr. Green's city taxes, including a license tax of $10?

9. A tax collector receives a commission of 2% on the first $10,000 collected, $1\frac{1}{2}\%$ on all over $10,000 up to $20,000, and 1% on all over $20,000. What are his commissions on collections amounting to $25,000?

10. What sum must be assessed to raise $5096 for school purposes, and pay 2% commission for collecting?

98% of the tax = $5096. (Art. 320.)

11. Find out the tax rate and poll tax (if any) in the town, city, or county in which you live, and compute Mr. Smith's taxes, if he pays on $3750 worth of property and pays a poll tax.

412. National Taxes.

Taxes levied by the general government upon goods imported from other countries are called **Customs** or **Duties.**

Duties are collected at custom houses situated at ports of entry.

413. When the duty is a certain per cent of the cost of imported goods it is called an *ad valorem duty.* In computing ad valorem duties, the invoice price is first converted into our currency to the nearest dollar.

When the duty is a fixed charge on goods without regard to their cost, it is called a *specific duty.*

Thus, $10 a ton and 50¢ a pound are specific duties, while 20% or 30% of the cost are ad valorem duties.

Some goods are subject to both ad valorem and specific duties.

A schedule of the rates of duties on imports is called a *tariff.*

414. Revenue derived from the sale of licenses to manufacture or sell certain articles, as tobacco, whisky, etc., and from *stamp taxes* on such articles, is called **Internal Revenue.**

The expenses of the general government are met by customs revenue, internal revenue, proceeds from the sale of public lands, and income from other sources.

WRITTEN EXERCISES

415. 1. If the duty on cheese is 6¢ a pound, what is the duty on a shipment of 750 pounds?

2. If the duty on toilet soap is 15¢ a pound, what is the duty on $62\frac{1}{2}$ pounds?

3. What is the duty on 25 sets of Dickens's works that cost $20 a set if the duty is 25% ad valorem?

4. A dealer imported 500 rugs, each 3 ft. by 6 ft., which cost $2.50 apiece. The duty on such rugs is 10¢ a square foot and 40% ad valorem. What was the total duty?

5. What is the duty on 240 pieces of plate glass, each 18 in. by 42 in., at 8¢ per square foot?

6. What is the duty on 500 yards of 27-inch velvet carpeting that cost $1.50 a yard, if the duty is 60¢ per square yard and 40% ad valorem?

7. What is the duty on an oil painting appraised at $1750, if the duty is 20%?

8. If the duty on silk dresses and on diamonds is 60% ad valorem, what is the duty on $500 worth of silk dresses and $2500 worth of diamonds?

9. A dealer imported 10 doz. Swiss watches that cost $40 each. If the duty was 40% ad valorem, what was the duty on the consignment? What did each watch cost including the duty?

RELATED PROBLEMS

416. 1. A town levied a tax of $\frac{3}{4}\%$ on real estate assessed at $750,000 and on personal property assessed at $300,000. How much net revenue did the levy bring, if the cost of collection was 5%?

2. A town has an assessed valuation of $1,250,000. What must be the tax on each hundred dollars to raise $5625 for the support of its high school?

3. On January 1, the cash on hand in a city treasury amounted to $125,222.59, and the collections since that time have amounted to $133,838.56. The warrants paid to date amount to $211,624.76. What cash balance remains? How much interest would it earn if left in bank for 10 days at 2% per annum?

4. In the same city the following amounts were collected in October, 1906, from the sources indicated: Markets, $516.65; scales, $45.05; Assembly Hall, $40; licenses, $640.83; pound fines, $43.95; police fines, $518.79; levy of 1905, $2219.01; sewer assessment, $19.90; sidewalk assessment, $23.79; miscellaneous, $389.08. What were the month's collections? If the cost of making these collections averaged $2\frac{1}{2}\%$, what was the total net amount received by the city during the month?

5. The amount appropriated in this city at the beginning of the fiscal year for current expenses and improvements was $247,134.88. Of this amount 75% has been expended. What is the balance of the appropriation yet unexpended?

6. If this city's total levy was $270,000, and the rate of taxation was $1\frac{1}{2}$ mills on the dollar, what was the assessed value of the real and personal property of the city?

STOCKS AND BONDS

417. When a number of persons engage in business together, they usually form themselves into an association called a **Corporation.** Corporations operate under *charters* issued in accordance with the laws of the states in which they are organized.

418. The sum of money contributed by the members to inaugurate and carry on the business is called the **Capital Stock.** The capital stock is divided into *shares* of a specified value (usually $100 each), and is issued in the form of *certificates of stock* stating how many shares the person named therein owns, and the face value of the same. The owners of certificates of stock are called *stockholders.*

419. If the corporation earns more than its expenses, part or all of the excess is divided among the stockholders as **Dividends.** The dividends are expressed as a certain per cent of the capital stock, or sometimes as a certain number of dollars a share. If a company declares a dividend of 10 %, the owner of a $100 share will get $10.

420. The price at which a stock sells in the market is called its **Market Value.**

When a stock sells for more than its face value, it is said to be *above par;* when it sells for less than its face value, it is said to be *below par;* when it sells at its face value, it is said to be *at par.*

Thus, if a $100 share of stock sells for $105, the stock is 5% *above par;* if it sells for $97, the stock is 3% *below par.*

421. A written obligation under seal securing the payment of a sum of money at a specified time is called a **Bond.**

Bonds are issued by corporations for the purpose of effecting loans. They are also issued by local, state, and national governments for the same purpose.

422. Bonds bear interest on their face value at a fixed rate; hence their market price does not affect the interest they yield. The income from stocks is variable, as it depends upon the prosperity of the business.

423. Persons who buy and sell stocks and bonds for others are called *brokers*, and their commission is called *brokerage.*

The brokerage is usually $\frac{1}{8}\%$ of the par value, and is charged for buying and for selling.

424. The market values of stocks as given in the daily newspapers are called *stock quotations.*

For example, a quotation of 115 means that a $100 share or bond is selling for $115. In this case the seller receives $115 − $\frac{1}{4}$, or $114\frac{7}{8}$ for each share; and the buyer pays $115 + $\frac{1}{8}$, or $115\frac{1}{8}$ for each share.

425. The following are the stock quotations in a newspaper of recent date:

American Sugar	$147\frac{5}{8}$	Norfolk & Western	$92\frac{1}{4}$
American Tobacco	$106\frac{1}{4}$	Seaboard Air Line	$30\frac{1}{2}$
Baltimore & Ohio	$116\frac{1}{4}$	Southern Railway	$42\frac{1}{4}$
Chesapeake & Ohio	$60\frac{3}{4}$	Western Union	$93\frac{1}{2}$

The following are bond quotations on the same date:

U. S. 3's registered	$102\frac{1}{4}$	American Tobacco 6's	$116\frac{3}{4}$
U. S. 3's coupon	$102\frac{1}{2}$	Chesapeake & Ohio 4's	$108\frac{1}{2}$
U. S. new 4's reg.	$129\frac{1}{4}$	Northern Pacific 4's	$105\frac{1}{2}$
U. S. new 4's coupon	$130\frac{1}{4}$	Louisville & Nashville 4's	103

Unless otherwise noted, the par value of stocks in the following problems is $100 per share, and brokerage is reckoned at $\frac{1}{8}\%$.

WRITTEN EXERCISES

426. 1. What is the cost of 50 shares of stock quoted at $103\frac{3}{8}$, brokerage $\frac{1}{8}\%$?

1 share costs $\$103\frac{3}{8} + \$\frac{1}{8}$, or $\$103\frac{1}{2}$.
50 shares cost $50 \times \$103\frac{1}{2}$, or $\$5175$.

2. A man buys 40 shares of Western Union at 92, and sells it at $95\frac{1}{4}$. What is his gain?

$40 \times \$92\frac{1}{8} = \$3685$, cost.
$40 \times \$95\frac{1}{8} = \$3805$, net selling price.
$\$3805 - \$3685 = \$120$, gain.

3. How much must be paid for 120 shares of Tennessee Coal & Iron stock at $118\frac{1}{8}$, brokerage $\frac{1}{4}\%$?

4. What will 60 shares of American Sugar stock cost at $141\frac{1}{4}$?

5. A man bought 80 shares of Baltimore & Ohio stock at 80 and sold it at the quotation given on page 308. What did he gain?

6. Find the cost of 20 shares of Chesapeake & Ohio, 60 shares of Seaboard, 40 shares of Norfolk & Western, and 30 shares of Southern Railway at prices quoted.

7. How many shares of railroad stock at $160\frac{1}{2}$ can be bought for $38,550?

1 share cost $\$160\frac{1}{2} + \$\frac{1}{8} = \$160\frac{5}{8}$.
$\therefore$ the number of shares $= \$38,550 \div \$160\frac{5}{8} = 240$.

8. How many shares of Atlantic Coast Line stock can be bought for $6445 at 161?

9. When Northern Pacific is quoted at $198\frac{3}{4}$, how many shares can be bought for $31,820?

10. What is the income from 84 shares of bank stock that pays an 8% dividend?

The par value of 84 shares $= \$8400$.
8% of $8400 $= \$672$.

11. What income will a man receive from \$3075 invested in railroad stock at $102\frac{3}{8}$, if the stock pays a 4% dividend annually?

1 share costs $\$102\frac{3}{8} + \$\frac{1}{8} = \$102\frac{1}{2}$.
The number of shares $= \$3075 \div \$102\frac{1}{2} = 30$.
$\therefore$ the income $= 30 \times \$4 = \$120$.

12. What annual income will a man receive if he invests \$ 23,100 in bank stock which pays a 6% annual dividend, if he pays 110 for the stock? (No brokerage.)

13. If I invest \$15,675 in United States 4% bonds at $104\frac{1}{2}$, what does the government pay me annually?

14. If I buy a 4% bond at $79\frac{7}{8}$, what is the rate of income on my investment, without considering the length of time the bonds run?

Each \$80 invested yields \$4.
$\therefore$ the rate of income $= \frac{4}{80}$, or 5%.

15. What amount of 4% bonds must I buy in order that they may yield an income of \$1200 a year?

16. What will the bonds required in problem 15 cost if bought at $79\frac{7}{8}$?

17. A southern "industrial" stock pays \$15 a share each year in dividends. What price must be paid for one share in order to yield 6% on the investment? (No brokerage.)

18. A bank with a capital stock of \$150,000 has an expense account of \$8000. Its gross earnings are \$20,000 a year. What per cent dividend can it pay?

19. A railroad company has a capital stock of \$1,000,000, and a bonded indebtedness of \$3,000,000. What must be its net earnings to enable it to pay a 4% dividend on its capital stock and 6% interest on its bonds after setting aside \$10,000 as a surplus fund?

RATIO AND PROPORTION

RATIO

427. You have already learned (Art. 141) that the relation of one number to another of the same kind, expressed by the quotient of the first divided by the second, is called **Ratio.**

1. How does 3 ft. compare with 12 ft.? With 15 ft.?
2. What is the relation of 4 to 8? Of 5 lb. to 15 lb.?
3. What is the ratio of 3 to 6? Of 4 qt. to 9 qt.?
4. What is necessary before the ratio of 3 ft. to 8 yd. can be determined?
5. Why can there be no ratio between 3 hats and 3 dollars?

428. The *sign of ratio* is the colon (:). Thus, the ratio of 5 to 8 is written 5 : 8; or it may be written as a fraction, $\frac{5}{8}$.

1. What is the ratio of \$5 to \$10?

$$\$5 : \$10 = \frac{\$5}{\$10} = \frac{5}{10} \text{ or } \frac{1}{2}.$$

2. What is the ratio of 3 ft. to 12 ft.? Of 6 to 4?

429. The first of the two numbers compared is called the **Antecedent**; the second, the **Consequent.** Thus, in the ratio of 3 : 7, 3 is the antecedent and 7 the consequent.

1. How do you find the ratio of one number to another?
2. Why is the ratio always an abstract number?

ORAL EXERCISES

430. What is the ratio of —

1. 6 to 8?
2. 12 to 36?
3. 7 to 42?
4. 9 to 27?
5. 28 to 4?
6. 4 to 28?
7. 17 to 68?
8. 68 to 17?
9. \$4 to \$6?
10. 10 in. to 30 in.?

WRITTEN EXERCISES

431. 1. Find the ratio of $\frac{2}{3}$ to $\frac{3}{4}$.

$\frac{2}{3} \div \frac{3}{4} = \frac{8}{12} \div \frac{9}{12} = \frac{8}{9}$, or $\frac{2}{3} \div \frac{3}{4} = \frac{2}{3} \times \frac{4}{3} = \frac{8}{9}$.
By either method the ratio is found to be $\frac{8}{9}$.

Find the ratio of —

2. $\frac{2}{3}$ to $\frac{4}{5}$.
3. $\frac{3}{5}$ to $\frac{1}{4}$.
4. $\frac{5}{6}$ to $\frac{8}{9}$.
5. $\frac{8}{9}$ to $\frac{5}{6}$.
6. 2 to $\frac{1}{6}$.
7. 2.7 to 3.6.
8. 2.5 to 25.
9. .25 qt. to .25 gal.

10. In a recent year, the total enrollment of pupils in the common schools of the United States was 16,256,000 out of a population of 81,280,000. What was the ratio of the school enrollment to the population in that year?

11. The same year the average daily attendance was 11,379,200. What was the ratio of attendance to population?

12. If the earth's diameter is 7925.6 miles and its circumference 24,899.06 miles, what is the ratio of the earth's circumference to its diameter? Compare your result with the ratio of the circumference of a circle to the diameter, as found in Art. 175.

PROPORTION

432. 1. What is the ratio of 3 to 6? Of 5 to 10? Are these ratios equal to each other?

2. If they are equal, we may write the equation

$$3 : 6 = 5 : 10, \text{ or } \tfrac{3}{6} = \tfrac{5}{10}.$$

433. The expression of the equality of two equal ratios is called a **Proportion.**

Thus, $2 : 4 = 3 : 6$; and $\frac{2}{4} = \frac{4}{10}$ are proportions.

The first is the usual form of a proportion, and it is commonly read "2 is to 4 as 3 is to 6"; sometimes as "the ratio of 2 to 4 *equals* the ratio of 3 to 6." The double colon (: :) is sometimes written instead of the sign of equality, but this does not affect the reading of the proportion.

1. Write three proportions from the following ratios:

$$1 : 2, \quad 3 : 4, \quad 5 : 6, \quad 5 : 10, \quad 10 : 12, \quad 18 : 24.$$

2. Try to select equal ratios from the following, and write a proportion: $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{8}$, $\frac{5}{6}$.

3. If you see an expression in the form of a proportion, how may you test its accuracy?

4. State the missing numbers in the following:

$3 : 12 = 4 : (\ )$. $\qquad (\ ) : 6 = 3 : 9$.

$7 : 42 = (\ ) : 54$. $\qquad 8 : 16 = 5 : (\ )$.

$8 : (\ ) = 6 : 3$. $\qquad 10 : 12 = (\ ) : 60$.

434. Since the two ratios of a proportion are equal, and each may be expressed as a fraction, it follows that the principles that apply to the solution of equations (Art. 345) and to fractions apply also to proportions.

435. The first and fourth numbers of a proportion are called the **extremes**, and the second and third the **means.**

Name the extremes and the means in the proportion

$$3 : 5 = 21 : 35.$$

436. I. Consider the proportion $3 : 8 = 15 : 40$. Writing it as an equation, we have

$$\frac{3}{8} = \frac{15}{40}.$$

Multiplying each side by 8,

$$3 = \frac{15 \times 8}{40}.$$

Next multiplying each side by 40,

$$3 \times 40 = 15 \times 8.$$

That is, **the product of the extremes equals the product of the means.**

This gives another simple test of a proportion. What is it? Write three proportions, and apply the second test to each.

For convenience we may write x in place of the missing or unknown number.

II. Find the value of x in the proportion

$$x : 8 = 9 : 12.$$

This proportion is true for a particular value of x, which may be found by placing the product of the extremes equal to the product of the means and solving; thus,

$$12x = 8 \times 9.$$
$$x = \frac{8 \times 9}{12} = 6.$$

That is, **one extreme equals the product of the means divided by the other extreme.**

III. Find the value of x in the proportion

$$12 : 10 = x : 5,$$

and show that in any proportion *one mean equals the product of the extremes divided by the other mean.*

These principles enable us to find the missing or unknown number no matter where it occurs in a proportion.

Find the value of x in each of the following:

1. $x : 60 = 3 : 12$.
2. $x : 2 = 9 : 18$.
3. $16 : x = 18 : 27$.
4. $2 : x = 6 : 72$.
5. $16 : 10 = 40 : x$.
6. $18 : 12 = x : 15$.
7. $9 : 42 = x : 14$.
8. $\frac{2}{5} : 2 = 5 : x$.
9. $19 : 32 = 38 : x$.
10. $7.5 : 2.5 = x : 13$.

437. All the problems that follow may be solved by methods already given, but the use of proportion is sometimes convenient.

Note carefully the following problem and solution:

If 6 hats cost \$18, how much will 9 hats cost at the same rate?

(Hats) (Hats) (\$) (\$)

(*a*) 6 : 9 = 18 : x

(*b*) 9 : 6 = x : 18

Since only *like numbers* can be compared in a ratio, we place the two given numbers that are alike in the first ratio in either order, 6 : 9 or 9 : 6. For the second ratio we compare the number of dollars that 6 hats cost with the required number that 9 hats will cost. Now we do not know the value of x, but we do know from the problem that it is greater than 18, since 9 hats cost more than 6 hats.

Hence, in (*a*), where the antecedent in the first ratio is less than the consequent, the less of the two remaining numbers (18) must be the antecedent of the second ratio; otherwise the ratios would not be equal. Similarly in (*b*), the greater of the remaining numbers must be the antecedent of the second ratio. Either (*a*) or (*b*) gives

$$6x = 9 \times 18$$

$$\therefore x = \frac{9 \times 18}{6} = 27.$$

$$\begin{array}{r} 18 \\ \times\ 9 \\ \hline 6)\overline{162} \\ 27 \end{array}$$

That is, 18 hats cost \$ 27.

Note that the placing of 6 as the first antecedent fixed the place of each of the other numbers in the proportion (*a*); so also of 9 in (*b*).

438. The following suggestions will be found helpful in stating and solving problems by proportion:

1. *Let x = the required number.*

2. *For the first ratio compare the two given numbers that are alike.*

3. *For the second compare the remaining given number with x, making the antecedent greater or less according to the arrangement of the first ratio.*

4. *Find the value of x in the proportion thus formed.*

WRITTEN EXERCISES

439. Solve the following problems by proportion:

1. If 2 tons of hay cost $22, what will 11 tons cost at the same rate?

$2 : 11 = 22 : x$

$$x = \frac{11 \times 22}{2} = 121.$$

Therefore 11 tons cost $121.

Is the value of x greater or less than 22? Why? Why is x placed last in the second ratio? How is the calculation shortened?

2. If 4 men can build a wall in 15 days, in how many days can 10 men build it, if all work at the same rate?

$4 : 10 :: x : 15$

$$x = \frac{4 \times 15}{10} = 6.$$

It is evident in this case that x is less than 15, since 10 men can do the work in less time than 4 men. Hence the proportion is as stated.

3. If the interest on $250 is $15, what will be the interest on $800 at the same rate for the same time?

$250 : 800 :: 15 : x$

$$x = \frac{800 \times 15}{250} = 48.$$

Therefore the interest on $800 is $48.

4. A merchant pays \$10.80 for 24 yd. of cloth. At this rate, what will 40 yd. cost?

5. If a train runs 135 miles in 3 hours, how far will it run in 5 hours at the same rate?

6. If 28 bu. of wheat make 6 barrels of flour, how many bushels will be required to make a car load of 150 barrels?

7. A farmer raised 115 bu. of wheat on 5 acres. How many bushels would he raise on 18 acres at the same rate?

8. In 20 lb. of potatoes there are 4.2 lb. of solid matter. How many pounds of solid matter are there in a bushel (56 lb.) of potatoes?

9. If it takes 5 bu. of oats to seed $2\frac{1}{2}$ acres, how many bushels are required to seed $8\frac{1}{2}$ acres?

10. A dealer buys hats at \$33 a dozen. At this rate, what will 50 hats cost?

11. A tree 80 ft. high casts a shadow 100 ft. long. How long a shadow will a tree 65 ft. high cast at the same time?

12. Desiring to know the height of the Washington monument, I set a five-foot pole in a vertical position at its base and found that the pole cast a shadow 3.5 ft. long, when that of the monument was 388.5 ft. long. What is the height of the monument?

13. If a boy skates 100 yd. in 10 seconds, how far can he skate in 3 minutes at the same rate?

14. A teacher earns \$140 in $3\frac{1}{2}$ months. At this rate, how many months will it take her to earn \$360?

15. A tenant paid \$125 rent in 5 months. At that rate, how much rent did he pay in a year?

16. If a 4-cent loaf of bread weighs 8 oz. when flour is worth \$6 a barrel, what should a 5-cent loaf weigh when flour sells at the same price?

17. If 4 loads of hay feed 6 horses 8 weeks, how long will the same quantity feed 8 horses?

18. If \$600 yield \$144 interest in 4 years, how much interest will \$450 yield in the same time at the same rate?

19. If \$600 at 6% for 4 years amount to \$744, what will \$450 amount to at the same rate for the same time?

20. If 75 pupils use 5 boxes of crayons in 3 months, how many months will 15 boxes last them?

21. If an automobile goes 70 miles in $3\frac{1}{2}$ hours, how far will it go in 9 hours at the same rate?

22. A does $\frac{2}{3}$ as much work in a day as B. If A receives \$3 a day, what should B receive?

23. A man can build a fence in 16 da. by working 10 hr. a day. In how many 8-hour days could he build it?

24. If it takes 9 bricks to lay 2 square feet of pavement, how many bricks will be required to lay a pavement 6 feet by 110 yards?

440. Partitive Proportion.

The separation of a number into parts having a given ratio is sometimes called **Partitive Proportion.** The problems that follow give some practical applications of this phase of proportion.

WRITTEN EXERCISES

441. 1. Divide \$1000 between A and B so that A's part shall be to B's as 3 to 5.

A receives \$3 and B \$5 out of every \$8 that both receive. Therefore A receives $\frac{3}{8}$ of \$1000, or \$375, and B receives $\frac{5}{8}$ of \$1000, or \$625.

2. Divide \$300 into two parts that have the same ratio as $\frac{1}{2}$ to $\frac{1}{3}$.

$\frac{1}{2} = \frac{3}{6}$, and $\frac{1}{3} = \frac{2}{6}$. Hence $\frac{1}{2} : \frac{1}{3} = 3 : 2$.

3. The ages of A and B are to each other as 3 to 4, and the sum of their ages is 56 years. What is the age of each?

4. The sum of two numbers is 450. What are the numbers, if they are in the ratio of 3 to 7?

5. Divide 270 in three parts that shall be to each other as 2, 3, and 5.

6. A granolithic walk was laid in front of the suburban properties of Smith, Brown, Jones, and Robertson, at a cost of $780, which sum was assessed against them according to their frontage. If they owned 150 ft., 250 ft., 350 ft., and 450 ft., respectively, what was the assessment of each?

7. A and B rented a pasture for $693 for the season. If A grazed 35 cattle and B 42 cattle, what was each man's part of the rent?

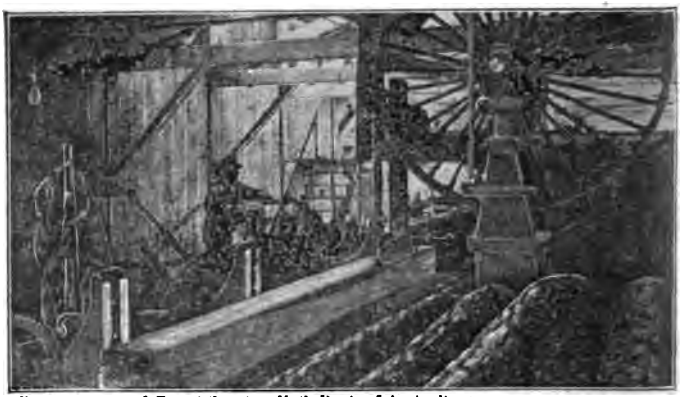
8. A, B, and C went into business together. A put in $1000, B $1500, and C $3500. They gained the first year $2400. What was each man's share of the gain?

$1000 + $1500 + $3500 = $6000, the capital.
A furnished $\frac{1000}{6000}$, or $\frac{1}{6}$ of the capital.
B furnished $\frac{1500}{6000}$, or $\frac{1}{4}$ of the capital.
and C furnished $\frac{3500}{6000}$, or $\frac{7}{12}$ of the capital.
Therefore, A's share of the gain = $\frac{1}{6}$ of $2400, or $400,
B's share of the gain = $\frac{1}{4}$ of $2400, or $600,
and C's share of the gain = $\frac{7}{12}$ of $2400, or $1400.

9. Three men engaged in business with a joint capital of $8450, of which the first put in $1250, the second $2700, and the third the remainder. They gained $1859. What was the share of each?

10. A and B entered into partnership with a capital of $7500, of which A furnished $1250 and B the remainder. They gained $1500. What per cent of the profit should each receive? How much should each receive?

RELATED PROBLEMS



By permission of Forest Service, U. S Dept. of Agriculture.

442. 1. This picture represents a large sawmill in a lumber district. The picture on the next page shows a log loader at work in the woods. If this mill cuts 200,000 board feet in a day, how many feet will it cut in a month (26 da.)? What will a month's output bring at $10 per M on the yards?

2. How many houses can be built with a month's output of this mill, if 10,000 ft. is required for each house?

3. The stand of yellow pine in Mississippi is estimated at 8000 ft. per acre. If that state has 5,860,000 acres in yellow pine, what is the value of the lumber that these forests will produce at $10 per M?

4. If the stumpage value of timber in a certain section is $3.25 per thousand feet, and the cost of logging and manufacture is $8 per thousand feet, what is the per cent of profit to the lumber manufacturer who sells his product at $15 per M? What is the total profit on 24,000 acres of this timber, if it averages 4000 ft. per acre?

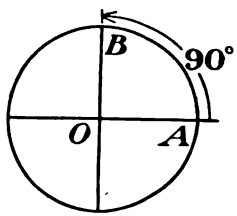
5. In 1905 the value of the lumber products in the Southern states was $250,000,000. In 1880 the lumber products of the rest of the United States were worth $194,000,000. By what per cent did the value of Southern lumber products in 1905 exceed that of the rest of the country in 1880?

6. A timber belt extending through Idaho, Washington, and Oregon is estimated to contain 8,000,000 acres of available timber, with an average of 10,000 ft. per acre. If 9,000,000,000 feet of this is white pine, what per cent of the total is white pine?

7. The output of a Maine paper mill is 180 tons of paper a day. If the average spruce tree supplies wood pulp for 500 lb. of paper, how many days will the timber from 225,360 spruce trees supply pulp for this mill?

8. In a recent year the output of the lumber mills of this country was 35,084,166,000 ft. At $16.25 per M, what was its value? If $33\frac{1}{3}$ % of the value was profit to the lumber manufacturers, what was their profit?

LONGITUDE AND TIME



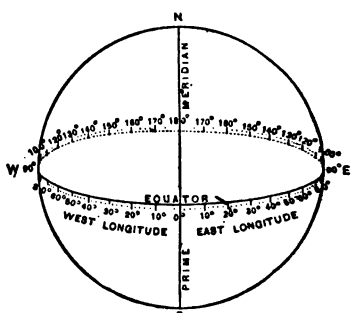
443. The circumference of any circle is measured in **degrees.** The whole circumference is divided into 360 degrees.

1. How many degrees are there in $\frac{1}{2}$ of a circumference? In $\frac{1}{4}$ of a circumference?

2. How many degrees are there in any line (circle) passing around the earth on its surface?

444. In measuring circumferences, each degree is divided in 60 equal parts, called **minutes,** and each minute into 60 equal parts, called **seconds.**

In this measure, degrees, minutes, and seconds are written with symbols. Thus, 3 degrees, 25 minutes, and 20 seconds is written 3° 25′ 20″.



445. A line on the earth's surface running north and south from pole to pole is called a **Meridian.** All places on the same meridian have their midday, or noon, at the same time; that is, when the sun is directly above that meridian.

446. The distance in degrees east and west from a given meridian is called **Longitude.**

The given meridian from which longitude is generally

reckoned is called the **Prime Meridian.** It passes through the Royal Observatory at Greenwich, England. From this meridian longitude is reckoned east 180° and west 180°.

447. Places are located on a map, or on the earth's surface, by giving their distance in degrees from the equator north or south, and from the prime meridian east or west; that is, by giving their latitude and longitude.

1. In how many hours does the earth rotate (turn about on its axis) once?

2. If a city in the rotation passes through 360° in 24 hours, through how many degrees does it pass in one hour? ($\frac{1}{24}$ of 360° = 15°.)

448. A train standing on a sidetrack appears to move in the opposite direction when we pass it rapidly. In like manner the sun appears to move westward as the earth turns eastward. In reality, however, the earth makes a complete rotation in 24 hours, turning every place on its surface through 360 degrees in that time. Hence we may deduce the following:

(*a*)

360° of longitude correspond to 24 hours of time.
1° of longitude corresponds to $\frac{1}{360}$ of 24 hr., or $\frac{1}{15}$ hr., or 4 min.
1′ of longitude corresponds to $\frac{1}{60}$ of 4 min., or $\frac{1}{15}$ min., or 4 sec.
1″ of longitude corresponds to $\frac{1}{60}$ of 4 sec., or $\frac{1}{15}$ sec.

(*b*)

24 hours of time correspond to 360° of longitude.
1 hour of time corresponds to $\frac{1}{24}$ of 360°, or 15°.
1 minute of time corresponds to $\frac{1}{60}$ of 15°, or 15′.
1 second of time corresponds to $\frac{1}{60}$ of 15′, or 15″.

From the foregoing it will be seen that when it is noon at any given place it is 1 P.M. 15° east of that place, and 11 A.M. 15° west of that place. Hence clocks show *later* (faster) time at places east of a given place, and *earlier* (slower) time at places west of a given place. For example, clocks in New York show later time than clocks in Chicago.

449. The following short list of cities with their longitude is given for reference:

Austin	97° 44′ 0″ W.	Paris	2° 20′ 15″ E.
Chicago	87° 30′ 42″ W.	Richmond	77° 26′ 0″ W.
Denver	104° 59′ 33″ W.	San Francisco	122° 25′ 42″ W.
Manila	120° 58′ 0″ E.	St. Louis	90° 15′ 15″ W.
New Orleans	90° 3′ 28″ W.	St. Petersburg	30° 19′ 0″ E.
New York	73° 58′ 25½″ W.	Washington	77° 3′ 55½″ W.

450. To find the difference in time, when the difference in longitude is given.

WRITTEN EXERCISES

1. The difference in longitude between two cities is 78° 3′ 30″. What is the difference in time?

(*a*)

78° corresponds to 78 × $\frac{1}{15}$ hr., or 5$\frac{1}{5}$ hr., or 5 hr. 12 min. (Art. 448, (*a*).)

3′ corresponds to 3 × $\frac{1}{15}$ min., or $\frac{1}{5}$ min., or 12 sec.

30″ corresponds to 30 × $\frac{1}{15}$ sec., or 2 sec.

5 hr. 12 min. + 12 sec. + 2 sec. = 5 hr. 12 min. 14 sec., the difference in time.

(*b*)

15)78° 3′ 30″
5 12 14

Since the numbers expressing the difference in time between two places are $\frac{1}{15}$ of those expressing the difference in longitude between the same places, we may divide as at the left, and interpret the result as 5 hr. 12 min. 14 sec.

2. Find the difference in time between New York and Austin.

Longitude of Austin 97° 44′ 0″
Longitude of New York 73° 58′ 25½″
Subtracting, we have 23° 45′ 34½″, difference in longitude.

The difference in time is readily found as in problem 1.

If two places are both east or both west of the prime meridian, the difference in longitude is found by subtracting as above; but if one is east and the other west of the prime meridian, the difference is found by adding.

Find the difference in time between —

3. New Orleans and Denver.

4. New York and St. Louis.

5. Chicago and New York.

6. Washington and Paris.

7. If a cablegram is filed in London (on prime meridian) at 7 A.M., what time does it reach New York, if no allowance is made for loss of time in transmission?

8. An ocean liner leaving this country passes a signal ship in 75° W. longitude. Eight days are required to reach London. How must the clocks be changed each day if the rate of traveling is uniform?

9. A traveler sails from London with his watch set by Greenwich time. When he reaches Boston in longitude 71° 3′ 30″ W., is his watch faster or slower than the time of Boston, and how much?

10. If the polls open in a general election at sunrise at every place, how long may voting begin in New York before it begins in San Francisco?

451. To find the difference in longitude, when the difference in time is given.

WRITTEN EXERCISES

1. The difference in time between two cities is 3 hr. 8 min. 45 sec. What is their difference in longitude?

hr.	min.	sec.
3	8	45
		15
47	11	15

Since there are 15 times as many degrees, minutes, and seconds of longitude between two places as there are hours, minutes, and seconds of time, we may multiply as at the left, and interpret the result as 47° 11′ 15″. (See Art. 448, (*b*).)

Find the difference in longitude between two places whose difference in time is —

2. 1 hr. 47 min. 23 sec.
3. 2 hr. 20 min. 16 sec.
4. 6 hr. 45 min. 58 sec.
5. 5 hr. 17 min. 39 sec.

6. What is the difference in longitude between two observatories whose difference in time is 2 hr. 12 min. 28 sec.?

7. The difference in time between St. Louis and San Francisco is 2 hr. 8 min. 41.8 sec. What is their difference in longitude?

8. At 9 A.M. a telegram is sent from a city whose longitude is 77° 3′ W. to another city. If the message is received at 9:30 A.M., what is the longitude of the second city?

9. A vessel carries a chronometer that keeps Greenwich time. The sun crosses the vessel's meridian at 10:30 A.M. by the chronometer. What is the ship's longitude at that hour?

STANDARD TIME

452. The time which we have been considering is known as *local time*. In 1883 the railroad companies of this country adopted the time of certain meridians *as convenient standards* by which to run their trains. This railroad time is called **Standard Time**.

Most civilized countries have now adopted a system of standard time, and it is generally used for all practical purposes.

453. According to this system the United States is divided into four time sections, each about 15° wide.

As shown by the map, the dividing lines are somewhat irregular, passing through leading railway termini.

1. The first (eastern) section lies on both sides of the 75th meridian, and its time is called *Eastern Time.*

2. The second section lies on both sides of the 90th meridian. The time in this section is called *Central Time.*

3. The third section lies on both sides of the 105th meridian. Its time is called *Mountain Time.*

4. The fourth section lies on both sides of the 120th meridian. Its time is called *Pacific Time.*

5. Name one or more large cities near each of these time meridians. Name one or two near each dividing line between the sections.

454. Standard time is uniform within each of these sections, and the time of each section differs from that next to it by exactly one hour. Thus, at 12 noon in New York City (eastern time), it is 11 A.M. in Chicago (central time); 10 A.M. in Denver (mountain time); and 9 A.M. in San Francisco (Pacific time).

ORAL EXERCISES

455. 1. A man who goes from Richmond, Va., to Denver will find his watch how much too fast?

2. When it is 10 A.M. at New York, what is the time in St. Louis? In New Orleans? In Salt Lake City? In Charleston?

3. In going from Washington to Memphis, a train is scheduled to arrive at a station at 9 A.M., to stop 20 minutes for breakfast, and to leave at 8:20 A.M. Can you explain this? Name two cities where this might happen.

WRITTEN EXERCISES

456. 1. The longitude of Richmond is 77° 26′ 0″ W. What is the difference between the local and the standard time?

Longitude of Richmond,	77° 26′ 0″
Longitude of its time meridian,	75°
	15) 2° 26′ 0″
	9′ 44″

The difference in longitude between the *time* meridian and the Richmond meridian is readily found, and the numbers representing the difference in time are $\frac{1}{15}$ as great. Hence, standard time at Richmond is 9 min. 44 sec. faster than local time.

2. What is the difference between the standard and the local time of New York? Which is later, and how much?

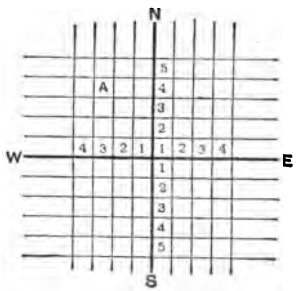
3. What is the difference between the local and standard time of Washington? Of Chicago? Of San Francisco?

4. When it is 12 M., local time at St. Louis what is the standard time at Denver?

5. Is standard time in Austin fast or slow as compared with local time? How much?

RELATED PROBLEMS

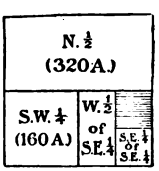
457. Many of the more recently settled states are divided into *townships* 6 miles square. Townships are located with reference to a north and south line called the *principal meridian* and an east and west line called a *base line*. The north and south rows of townships are called *ranges*. For example, in the first diagram, *A* is the fourth township north of the base line, in the third range west of the meridian, and is designated T. 4 N., R. 3 W.



6	5	4	3	2	1
7	8	9	10	11	12
18	17	16	15	14	13
19	20	21	22	23	24
30	29	28	27	26	25
31	32	33	34	35	36

A township is divided into *sections*, each 1 mile square, and these are numbered as in the second diagram.

Each section is then subdivided into half sections and quarter sections, as shown in the third diagram. For example, if this diagram is taken to represent section 15 of the fourth township north, in the third range west, the shaded part would be designated N. E. $\frac{1}{4}$ of S. E. $\frac{1}{4}$ of Sec. 15, T. 4 N., R. 3 W.



1. Draw a diagram of the following: N. E. $\frac{1}{4}$ of Sec. 10, T. 1 S., R. 2 E. Find the area.

2. Draw a plan of a township and locate the following: W. $\frac{1}{2}$ of N. E. $\frac{1}{4}$ of Sec. 8, T. 2 N., R. 2 W. Also find the area.

3. Find the area of this farm: S. $\frac{1}{2}$ of S. E. $\frac{1}{4}$ of Sec. 20, T. 2 S., R. 3 W. (Draw a diagram of a township and locate the farm.)

4. Mr. Wright owns the S. W. $\frac{1}{4}$ of the N. E. $\frac{1}{4}$ and the S. E. $\frac{1}{4}$ of the N. W. $\frac{1}{4}$ of a section of land. How many acres does he own? (Show by a diagram the location of his farm in the section.)

5. A man owns the E. $\frac{1}{2}$ and the N. E. $\frac{1}{4}$ of the S. W. $\frac{1}{4}$ of Sec. 1, T. 1 N., R. 18 W. How many acres does he own? How many rods of fence are needed to inclose his farm?

6. A Dakota wheat farm occupies sections 21, 22, 26, and 27 in a certain township. The owner raised 20 bu. of wheat to the acre and sold it at 70¢ a bushel. What did he receive for the crop? What is the distance around this farm?

7. Mr. Hull owns the S. E. $\frac{1}{4}$ of the N. W. $\frac{1}{4}$, the S. W. $\frac{1}{4}$ of the N. E. $\frac{1}{4}$, and the W. $\frac{1}{2}$ of the S. E. $\frac{1}{4}$ of Sec. 6 in a township. What is his farm worth at $75 an acre?

8. The area of the state of Kansas is 82,080 square miles. The area of this state is equal to the area of how many townships? How many farms of 160 acres each would this state make?

9. In some states the government sets apart the 16*th* section in each township for the education of the children of the township. One township sold the 16th section at $10 an acre, and lent the proceeds to the state at 6%. What did this township receive annually from its "Sixteenth Section Fund"?

10. Another township reserves the S. E. $\frac{1}{4}$ of the S. E. $\frac{1}{4}$ of its 16th section for a schoolhouse and a residence for a teacher, and rents the remainder at $5 an acre. What is the annual rent?

THE METRIC SYSTEM

458. The **Metric System** is a system of weights and measures, so named because its prime unit of linear measure is the meter, approximately one ten-millionth of the distance from the equator to the pole. To be exact, it is the length of a standard meter preserved in Paris, a copy of which is in our own national capital.

459. The metric system derives its value from two characteristics; viz.,

1. A uniform (decimal) scale.

2. The interrelation of the denominations of length, weight, and capacity. Thus, a cubic deçimeter measures the cubical contents of 1 liter (lētĕr) of water which weighs a kilogram.

460. Since its uniform scale is a decimal scale, one table is the model for all, and problems involve only such difficulties as those that are met with in decimal fractions. (See Art. 266.)

461. The names of the higher and lower units are formed by attaching certain prefixes to the names of the principal units. These prefixes with their meanings are as follows:

GREEK		LATIN	
dĕka, *meaning*	10	deçi, *meaning*	.1
hĕkto, *meaning*	100	çenti, *meaning*	.01
kĭlo, *meaning*	1000	mĭlli, *meaning*	.001
mўria, *meaning*	10000		

462. TABLE OF LENGTH

1 myriameter (Mm.)	=	10,000 meters.
1 **kilometer** (Km.)	=	1000 meters.
1 hektometer (Hm.)	=	100 meters.
1 dekameter (Dm.)	=	10 meters.
Meter (m.)		
1 decimeter (dm.)	=	.1 of a meter.
1 **centimeter** (cm.)	=	.01 of a meter.
1 **millimeter** (mm.)	=	.001 of a meter.

This table may be arranged thus:

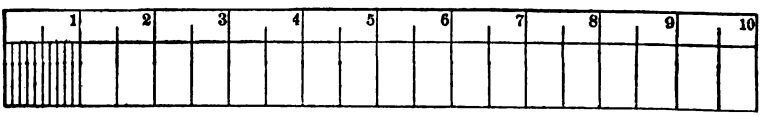
10 **millimeters**	= 1	**centimeter.**
10 **centimeters**	= 1	decimeter.
10 decimeters	= 1	**meter.**
10 **meters**	= 1	dekameter.
10 dekameters	= 1	hektometer.
10 hektometers	= 1	**kilometer.**
10 **kilometers**	= 1	myriameter.

1. Note that abbreviations of subdivisions of metric units begin with small letters; those of multiples with capitals.

2. In U. S. money we seldom speak of anything except dollars and cents; so in the metric system only the units printed in heavy type are commonly used.

3. A length given in one unit may be changed to another by simply moving the decimal point the required number of places. Thus, 75 dm. = 7.5 m., and 75 cm. = .75 m.

463. The annexed scale shows the decimeter divided into centimeters, and one of the latter into millimeters.



ORAL EXERCISES

464. 1. How many millimeters in 1 meter? What does the prefix *milli* mean?

2. How many centimeters in 1 meter? What does *centi* mean?

3. How many decimeters in 1 meter? What does *deci* mean?

4. How many meters in a dekameter? What does *deka* mean?

5. How many meters in a hektometer? What does *hekto* mean?

6. How many meters in a kilometer? What does *kilo* mean?

7. How many millimeters in a kilometer?

8. How many centimeters in a hektometer?

9. How many decimeters in a dekameter?

10. What part of a kilometer is a meter? A millimeter?

WRITTEN EXERCISES

465. 1. Reduce 5 Dm. 2 m. 6 dm. to millimeters, and compare with the changes from one order to another in decimals.

5 Dm. = 50,000 mm.
2 m. = 2,000 mm.
6 dm. = 600 mm.
52,600 mm.

2. Reduce 15 Km. 3 Dm. to meters.

Compare with the foregoing process, reduce mentally, and write the result. Indicate a missing order by 0, as in decimals. (Art. 216.)

Reduce:

3. 57 m. to kilometers.

4. 82.5 cm. to meters.

5. .0098 Km. to meters and decimeters.

6. 9,870,540 mm. to centimeters, decimeters, dekameters, hektometers, and kilometers.

466. TABLE OF SQUARE MEASURE

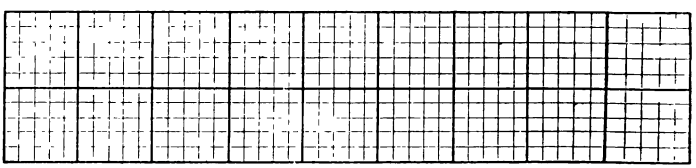
100 square millimeters (qmm.) = 1 **square centimeter.**
100 **square centimeters** (qcm.) = 1 square decimeter.
100 square decimeters (qdm.) = 1 **square meter.**
100 **square meters** (qm.) = 1 square dekameter.
100 square dekameters (qDm.) = 1 **square hektometer.**
100 **square hektometers** (qHm.) = 1 **square kilometer.**

The square dekameter is also called an *are* (*är*). Hence, we have another table of square measure as follows:

100 **centares** = 1 **are.**
100 **ares** = 1 **hektare.**

This table is used in land measure, such as areas of farms, and coördinately with the other square units.

In the figure the rulings show square centimeters and smaller squares. How many square millimeters in each of the smaller squares?



ORAL EXERCISES

467. 1. How many square meters in a centare?

2. How many square meters in a hektare?

3. How many ares in a rectangle 12 m. long and 60 m. wide?

4. What part of a square meter is a square centimeter?

5. A rug is 4 m. long and 3 m. wide. How much floor space will it cover?

WRITTEN EXERCISES

468. 1. Find the area of a triangular meadow whose base is 125 m. and altitude 157 m.

$125 \times 1\frac{57}{2} = 12\frac{825}{2}$ qm. = 98.125 ares.

2. Find the area of a circle whose radius is 14 cm.

3. Find the area of a field in the shape of a parallelogram whose base is 187 m. and altitude 96 m.

4. A hall is 60 m. long and 23.5 m. wide. How much will it cost to carpet the floor at $1.50 a square meter?

5. At $1.25 an are, what is the cost of a farm in the shape of a triangle whose base is 1500 m. and altitude 600 m.?

6. Find the base of a triangle which has an area of 321.25 qm. and altitude 12.5 m.

469. TABLE OF CUBIC MEASURE

1000 cubic millimeters (cmm.) = 1 **cubic centimeter.**
1000 **cubic centimeters** (ccm.) = 1 cubic decimeter.
1000 cubic decimeters (cdm.) = 1 **cubic meter** (cu. m.).

When used in measuring wood the cubic meter is called a *stēre* (*st.*).

WRITTEN EXERCISES

470. 1. A pile of wood is 6.5 m. long by 2.75 m. wide and 3 m. high. How many steres are there in the pile?

2. Find the cubical contents of a rectangular solid whose dimensions are 10 m., 4 m., and 1.25 m.

3. The rectangular base of a basin is 3 m. long and 2.5 m. wide. If the depth of the basin is 5.5 m., how long would it take to fill it with a pipe which delivers 20 cdm. each second?

471. TABLE OF CAPACITY

10 milliliters (ml.)	= 1 **centiliter** (cl.).
10 **centiliters**	= 1 deciliter (dl.).
10 deciliters	= 1 **liter** (l.).
10 **liters**	= 1 dekaliter (Dl.).
10 dekaliters	= 1 **hektoliter** (Hl.).
10 **hektoliters**	= 1 kiloliter (Kl.).

A cube whose three dimensions are each one decimeter contains a liter.

472. TABLE OF WEIGHT

10 **milligrams** (mg.)	= 1 centigram (cg.).
10 centigrams	= 1 decigram (dg.).
10 decigrams	= 1 **gram** (g.).
10 **grams**	= 1 dekagram (Dg.).
10 dekagrams	= 1 hektogram (Hg.).
10 hektograms	= 1 **kilogram** (Kg.).
1000 **kilograms**	= 1 **metric ton** (T.).

A kilogram is the weight of a liter of pure water at its maximum density. It is sometimes called a *kilo*.

ORAL EXERCISES

473. 1. How many cubic centimeters of water will weigh one kilogram? One gram?

2. How many cubic meters of water will weigh a metric ton?

3. What is the weight of a centiliter of metal that weighs 7 times as much as an equal quantity of water?

WRITTEN EXERCISES

474. 1. Find the number of liters in a rectangular tank 8 m. by 8 m. by 16 m.

Compare with Exercise 2, Art. 299. Which is simpler?

2. If wheat is .77 as heavy as water, find the number of kilograms of wheat a rectangular bin 5 m. long, 4 m. wide, and 2 m. deep will hold.

3. Find the capacity in hektoliters and the weight in metric tons of the water in a reservoir 40 m. long, 26.6 m. wide, and 52 dm. deep.

475. The pupil can transfer from units of one system to those of the other by using the following :

TABLE OF EQUIVALENTS

1 meter	= 39.37 inches.
1 kilometer	= .6213 miles.
1 square inch	= 6.452 square centimeters.
1 acre	= 40.47 ares.
1 liter	= 1.0567 liquid quarts.
1 gram	= 15.432 grains.
1 kilogram	= 2.2 pounds (Av.).

For approximate estimates we may allow 1.1 yd. to 1 m., .6 mi. to 1 Km., and 1 qt. to 1 liter. Use these values in oral exercises.

ORAL EXERCISES

476. **1.** I bought 8 meters of silk for a skirt in the Palais Royal of Paris. How many yards did I buy ?

2. Eleven yards will make a silk dress. How much must I buy in a Parisian shop ?

3. How many liters of milk would supply for one week a family which uses a half gallon daily ?

4. If two cities are 150 kilometers apart, what is the distance between them in miles ?

WRITTEN EXERCISES

477. **1.** The distance from one city to another is 420 Km. Express this distance in miles.

2. The Eiffel Tower in Paris is 300 m. high. How many feet high is it ?

3. How many pounds will a person weigh who weighs 85 kilograms ?

POWERS AND SQUARE ROOT

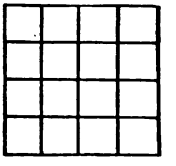
478. 1. What is the product of 2 multiplied by 2, or used twice as a factor?

2. How many times must 3 be used as a factor to make 9? To make 27? ($3 \times 3 = 9$; $3 \times 3 \times 3 = 27$.)

3. If $5 \times 5 \times 5 = 125$, how many factors of 125 are shown? How do they compare with each other?

479. The product of equal factors is called a **Power.**

480. The product of two equal factors is called the **second power.** Thus, since $3 \times 3 = 9$, 9 is the second power of 3.



The second power of a number is also called its **square,** because the area of a square is expressed by the product of two equal numbers that measure its length and breadth. (Art. 162.)

1. What is the area of a square 12 inches on a side? What is it called? (Art. 160.)

2. What is the second power or square of 12?

3. What is the second power or square of 7? Of 8? Of 9? Of 11?

481. The product of three equal factors is called the **third power.** Thus, since $5 \times 5 \times 5 = 125$, 125 is the third power of three.

The third power of a number is also called its **cube,** because the volume of a cube is the product of the three equal numbers that measure its length, breadth, and thickness. (Art. 188.)

1. What is the volume of a cube 12 inches each way, and what is it called? (Art. 185.)

2. What is the third power or cube of 12?

3. What is the third power or cube of 4? Of 6?

482. To show how many times a factor enters into a power, a small figure is placed to the right of the factor and slightly higher. Thus, 2^2 is read "2 square" or "2 to the second power," and means 2×2; 2^3 is read "2 cube," or "2 to the third power," and means $2 \times 2 \times 2$.

Finding any power of a number is merely a case of multiplication, viz., finding the product of equal factors.

1. Write 3×3 as a power, and give its value.

$$3 \times 3 = 3^2 = 9.$$

Write the following as powers, and give their values:

2. 5×5.	5. 15×15.	8. $3 \times 3 \times 3$.
3. 9×9.	6. 20×20.	9. $5 \times 5 \times 5$.
4. 10×10.	7. 25×25.	10. $10 \times 10 \times 10$.

Find the value of —

11. 23^2.	14. 144^2.	17. 12^3.	20. $(\frac{2}{3})^2$.
12. 45^2.	15. 125^2.	18. 26^3.	21. $(\frac{3}{5})^2$.
13. 3.5^2.	16. 240^2.	19. 3.8^3.	22. $(\frac{3}{8})^2$.

483. Since $43 = 40 + 3$, the square of 43 may be found as follows:

(a)	(b)
43	43
43	43
$9 = 3^2$	129
$120 = 40 \times 3$	172
$120 = 40 \times 3$	1849
$1600 = 40^2$	
$1849 = 40^2 + 2\ (40 \times 3) + 3^2$	

That is, $43^2 = (40 + 3)^2 = 40^2 + 2\ (40 \times 3) + 3^2 = 1600 + 240 + 9 = 1849$.

Representing the number of *tens* by t and the number of *ones* by o, we may write

$$(t + o)^2 = t^2 + 2\ t \times o + o^2.$$

Hence, **the square of a number consisting of tens and ones equals the square of the tens, plus twice the tens multiplied by the ones, plus the square of the ones.**

Note that in (*a*) on the preceding page the parts of the square are clearly shown, while in (*b*) and in the final result, 1849, they are not seen.

Square the following, and point out the several parts of each square as in (*a*):

1. 11.	4. 15.	7. 18.
2. 13.	5. 16.	8. 19.
3. 14.	6. 17.	9. 21.

SQUARE ROOT

484. 1. Separate 9 into two equal factors. Name one of them.

2. Separate each of the following into two equal factors: 16, 25, 36, 49.

485. A number that can be separated into 2 equal factors is called a **perfect square.** One of the two equal factors into which a number can be separated is called the **Square Root** of the number.

The square root of a number is indicated by the symbol $\sqrt{\ }$. Thus, $\sqrt{25}$ calls for the separation of 25 into two equal factors, or for the square root of 25.

486. Finding the square root of a number is the reverse of finding the square of a number. Just as finding a square is a special case in multiplication, finding a square root is a special case in division.

1. To find the square root of 81 is to find a number that will divide 81 and give itself for a quotient.

2. What number will divide 144 and give itself for the quotient. $\sqrt{144} =$

487. The square roots of small numbers that are perfect squares are readily found by inspection.

Give the values of the following by inspection:

1. $\sqrt{100}$.	3. $\sqrt{169}$.	5. $\sqrt{1600}$.
2. $\sqrt{121}$.	4. $\sqrt{400}$.	6. $\sqrt{3600}$.

488. By Factoring.

The square roots of perfect squares are sometimes conveniently found by first separating them into factors that are perfect squares.

1. Find the square root of 6561 by factoring.

Since $6561 = 9 \times 9 \times 81$,

$\sqrt{6561} = \sqrt{9 \times 9 \times 81}$.

$\therefore \sqrt{6561} = \sqrt{81 \times 81} = \sqrt{81^2} = 81$.

PROOF. $81^2 = 6561$.

$$\begin{array}{r} 9\overline{)6561} \\ 9\overline{)729} \\ 81 \end{array}$$

2. Find the square root of 2916.

Since $2916 = 4 \times 9 \times 81$,

$\sqrt{2916} = \sqrt{4 \times 9 \times 81} = 2 \times 3 \times 9 = 54$.

$\therefore \sqrt{2916} = 54$.

$$\begin{array}{r} 4\overline{)2916} \\ 9\overline{)729} \\ 81 \end{array}$$

Observe that the square root of a number equals the product of the square roots of its factors.

By factoring find the values of the following:

3. $\sqrt{625}$.	6. $\sqrt{1764}$.	9. $\sqrt{3025}$.
4. $\sqrt{1296}$.	7. $\sqrt{3136}$.	10. $\sqrt{9216}$.
5. $\sqrt{2401}$.	8. $\sqrt{5184}$.	11. $\sqrt{11664}$.

489. General Method.

Since $(t+o)^2 = t^2 + 2\,t \times o + o^2$ (Art. 483), it is clear that the square root of $t^2 + 2\,t \times o + o^2$ must be $t + o$. Hence, to discover a method of finding the square root of such expressions, we must reverse the process of squaring $t + o$.

By inspecting the square and its square root, it is seen that:

1. The first part of the square, t^2, is the square of the first part of the root. By reversing the process we see that the first part of the root is the square root of the first part of the square. This gives t for the first part of the root. $t = \sqrt{t^2}$.

2. The second part of the square contains twice the product of the two parts of the root. Hence, if the second part of the square, $2\,t \times o$, is divided by twice the first part of the root, $2\,t$, as a trial divisor, the quotient will be the second part of the root. $\frac{2\,t \times o}{2\,t} = o$, second part of the root.

3. Therefore, if we add to $2\,t$ the second part of the root, o, and multiply the sum (or complete divisor) by o, the product equals the remaining parts of the square after t^2 has been subtracted. This is seen to be true, since $(2\,t + o)o = 2\,t \times o + o^2$.

The relation between this square and its square root is more clearly shown in the following arrangement of the work:

$$\begin{array}{lll} & \underline{t} & +\,o = \text{Root} \\ \text{Square} & = t^2 + 2\,t \times o + o^2 & \\ & \underline{t^2} & \\ \text{Trial Divisor} & = 2\,t & \big| \; 2\,t \times o + o^2 \\ & \underline{+\,o} & \big| \\ \text{Complete Divisor} & = 2\,t + o & \big| \; 2\,t \times o + o^2 \end{array}$$

Compare the preceding solution with the following:

$(34)^2 = (30 + 4)^2 = 900 + 2 \times 30 \times 4 + 16.$

		t	$+o$	
		30	$+4$	= Root
Square	=	$900 + 240 + 16$		
		900		
Trial Divisor	$= 2 \times 30 = 60$		$240 + 16$	
	4			
Complete Divisor	$= 64$		$240 + 16$	

490. $1^2 = 1$, and $9^2 = 81$. Hence the square of any integer expressed by one figure is a number of one or two figures. $10^2 = 100$, and $99^2 = 9801$. Hence the square of any integral number expressed by two figures is a number of three or four figures.

Stated inversely, the square root of a number expressed by one or two figures is a number of one figure; that of a number expressed by three or four figures is a number of two figures; and so on.

491. If an integral square is separated into groups of two figures each, beginning at the right, the number of groups will be equal to the number of figures in the root.

Finding the square root of numbers usually differs from the illustrative solution in Art. 489, mainly because the parts of the square are not readily seen. The preceding article enables us to partially separate them.

Compare the solutions in Art. 489 with the following:

Find the square root of 2025.

(*a*)

		$40+5$	= Root
Square	=	20	25
		16	00
Trial Divisor	$= 2 \times 40 = 80$	4	25
	5		
Complete Divisor	$= 85$	4	25

(*b*)

20	25	45	
16			
85	4	25	
	4	25	

Separating the number into groups, as shown in (*a*) and (*b*), we see that the root will contain 2 figures, tens and ones.

The greatest square in 2000 is 1600, and its square root is 40, as shown in the root.

$2025 - 1600 = 425$, the remaining parts of the square. In other respects, this solution is like those given in Art. 489.

In practice unnecessary ciphers are omitted, and the work is conveniently arranged as in (*b*). We write only the 8 as a trial divisor, but consider it 80 in dividing. When found, we merely annex the next root figure, 5, to it as shown in heavy type.

492. We may apply the method given above to numbers of more than two groups of figures, by always regarding *the part of the root already found as so many tens* with respect to the next figure of the root.

Find the square root of 25,921.

(*a*)

	2 59 21	161
	1	
$2 \times 10 = 20$	1 59	
$20 + 6 = 26$	1 56	
$2 \times 160 =$	320 | 3 21	
$320 + 1 =$	321 | 3 21	

(*b*)

	2 59 21	161
	1	
26	1 59	
	1 56	
321	3 21	
	3 21	

PROOF. $161^2 = 25{,}921$.

The first trial divisor is found as it would be if we were finding the square root of 259 ; that is, $t = 10$, and $2t = 20$.

Note that $159 \div 20 = 7$; but $27 \times 7 = 189$, which exceeds 159. This is due to the fact that 159 contains 2 parts of the square, $2t \times o$, and o^2 (Art. 489). We therefore write and use 6 for o. Usually, however, the trial divisor gives the correct root figure, since twice the tens by the ones, $(2t \times o)$, is by far the larger part of the dividend, $2t \times o + o^2$.

The second trial divisor is 2×160, or 320.

In (*a*) the steps are shown ; in (*b*) the work appears as in practice.

493. From the preceding we may derive the following general method of finding the square root of a number :

1. *Beginning at ones, separate the number into as many groups of two figures each as possible.* (If the number is composed of an odd number of digits, consider the single digit at the left as a group.)

2. *Find the greatest square in the left-hand group, and write its root as the first figure of the required root.*

3. *Subtract the square of this partial root from the left-hand group, and to the remainder annex the next group for a dividend.*

4. *Divide this dividend by twice the partial root considered as tens. The quotient thus found* (or the quotient diminished) *will be the next figure of the root.*

5. *Add to the trial divisor the part of the root last found, thus completing the divisor. Multiply this complete divisor by the part of the root last found, subtract the product from the dividend, to the remainder annex the next group for a new dividend, and proceed as before.*

(1) A decimal is separated into groups of two figures each by beginning at the decimal point, and its root is found precisely as the root of an integer is found.

(2) The square root of a common fraction is equal to the square root of the numerator divided by the square root of the denominator; or to the square root of the equivalent decimal. Thus: $\sqrt{\frac{16}{25}} = \frac{4}{5}$; or $\sqrt{\frac{16}{25}} = \sqrt{.64} = .8$.

WRITTEN EXERCISES

494. Find the square root of each of the following, and test the results:

1. 1225.	6. 15,625.	11. .1369.	16. $\frac{169}{196}$.
2. 4624.	7. 60,516.	12. .094864.	17. $\frac{144}{289}$.
3. 1849.	8. 71,289.	13. 41.2164.	18. $\frac{225}{361}$.
4. 7396.	9. 160,801.	14. 5416.96.	19. $\frac{289}{441}$.
5. 5776.	10. 136,161.	15. 97.8121.	20. $3\frac{6}{25}$.

Find the square roots of the following:

21. 2 to three decimal places.

22. 3 to three decimal places.

23. 5 to three decimal places.

495. By the preceding methods of finding square roots the side of a square can be found if the area is given.

The measure of the side of a square is found by extracting the square root of the number representing the area.

Find the side of a square whose area is—

1. 40 acres.	4. 9216 sq. ft.	7. 3249 sq. yd.
2. 160 acres.	5. 25,921 centares.	8. 1722.25 qcm.
3. 13,225 sq. in.	6. 4489 qcm.	9. 97.6144 qm.

10. A square field contains 10 acres. How many rods are there in the length of each side?

11. A rectangular field is 60 rd. long and 15 rd. wide. What is the side of a square field of equal area?

12. A square field contains 90 acres. How many rods of fencing are required to inclose it?

13. A square hall is paved with tiles 8 inches square. If 576 tiles are used, what are the dimensions of the hall (in feet)?

14. The area of a certain square together with that of a smaller one is 1898 sq. ft. The side of the larger one is 37 ft. Find the perimeter of the smaller one.

15. The area of a square garden together with that of a walk which surrounds it is 2500 sq. yd. If the area of the garden is 1600 sq. yd., what is the width of the walk?

16. The area of Texas is 265,780 sq. mi. Find (correct to one decimal) the side of a square that has the same area.

496. The Right-angled Triangle.

1. About what angles do plants and trees usually make with level ground?

2. What angles are most frequently found in our houses and school buildings?

3. What is a right angle? What is a right-angled triangle? (Art. 169.)

497. In a right-angled triangle, the side opposite the right angle is called the **Hypotenuse**. The other sides are called **legs**.

1. Take a ruler and carefully draw a right-angled triangle with equal legs. Construct squares upon the sides, and draw dotted lines dividing these squares into triangles as shown in Fig. 1. If your drawing is accurate, each of these triangles will be equal to the original triangle. Count the triangles, and tell how the square on the hypotenuse compares with the sum of the squares on the legs.

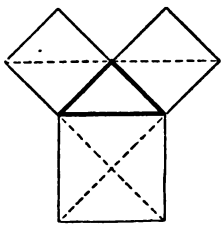


FIG. 1.

2. Again, draw a right-angled triangle whose legs are 3 inches and 4 inches. Measure and see if the hypotenuse is 5 inches.

3. Construct squares on the sides of this triangle. By dividing these into small squares, show the number of square inches in each, as in Fig. 2.

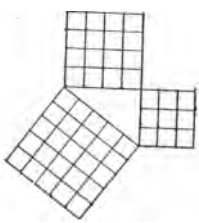


FIG. 2.

How many small squares in the square on the hypotenuse? In the square on the base? In the square on the altitude?

What conclusion do you draw from these experiments?

498. From the preceding we see that —

The square on the hypotenuse of a right-angled triangle equals the sum of the squares on the base and altitude.

1. Using initials for hypotenuse, base, and altitude, this fact may be expressed by the equation,

$$h^2 = b^2 + a^2.$$

$$\therefore h = \sqrt{b^2 + a^2}.$$

That is, *the hypotenuse equals the square root of the sum of the squares of the legs.*

2. Since $h^2 = b^2 + a^2$, we get, by subtracting a^2 from both sides,

$$h^2 - a^2 = b^2, \text{ or } b^2 = h^2 - a^2,$$
$$\therefore b = \sqrt{h^2 - a^2}.$$

3. Find the value of a from the equation $h^2 = b^2 + a^2$.

WRITTEN EXERCISES

499. 1. The base and altitude of a right-angled triangle are 6 ft. and 8 ft. respectively. Find the hypotenuse.

Using 6 for b, and 8 for a in the equation $h^2 = b^2 + a^2$, we have

$$h^2 = 36 + 64 = 100.$$
$$\therefore h = \sqrt{100} = 10.$$

Therefore, the hypotenuse is 10 feet.

TEST. Using a scale of 1 inch to the foot, draw with your ruler the base, altitude, and hypotenuse of the triangle in this problem. How long should the hypotenuse be on your drawing?

2. The hypotenuse of a right-angled triangle is 15 in. and the base 12 in. Find the altitude.

Since $h^2 = b^2 + a^2$,
$$a^2 = h^2 - b^2 = 225 - 144 = 81.$$
$\therefore a = \sqrt{81} = 9$, number inches in altitude.

PROOF. $225 = 144 + 81$.

Find the hypotenuse of each of the following right-angled triangles, and check the results:

3. Base 12 in., alt. 5 in.
4. Base 15 in., alt. 8 in.
5. Base 24 m., alt. 7 m.
6. Base 40 in., alt. 9 in.
7. Base 35 in., alt. 12 in.
8. Base 63 Km., alt. 16 Km.

9. Find the altitude of a right-angled triangle whose hypotenuse is 65 in., and base 56 in.

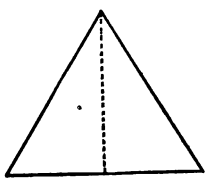
10. Find the base of a right-angled triangle whose hypotenuse is 53 ft. and altitude 28 ft.

11. Find to two decimal places the diagonal of a square which is 24 in. on a side. (Draw a diagram.)

12. A ladder 61 ft. long rests against the top of a wall which is 60 ft. high. Find the distance between the foot of the ladder and the wall.

13. How many rods will a man walk in going diagonally across a square farm containing 20 acres?

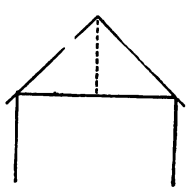
14. Find to two decimals the altitude of a triangle each side of which is 9 inches.



15. Two trees 117 ft. high and 125 ft. high respectively stand 15 feet apart on a plane. What is the distance between their tops? (Draw a diagram showing the required distance as the hypotenuse.)

16. Two men left a city at noon on the same day in automobiles. One went south at the rate of 11 miles an hour; the other west at the rate of $18\frac{2}{3}$ miles an hour. How far apart were they at 3 P.M. that afternoon?

17. If the ridge pole of a building 30 feet wide is 8 ft. above the level of the plates, how long must the rafters be cut to allow a projection of 1 ft. at the eaves?



18. A baseball diamond is 90 ft. square. Its diagonals meet in the center at right angles. How far is it from the center to the first base?

19. The outer lines of a tennis court form a rectangle 36 ft. by 78 ft. Find correct to 1 decimal the diagonal of the court.

RELATED PROBLEMS

By permission U. S. Dept. of Agriculture.

500. 1. The picture shows an orchard, which yielded the first year after being irrigated $350 an acre. If the cost of the canals which bring water to this orchard was $13.50 an acre, and the cost of preparing the land for irrigation was $15 an acre, what per cent of the value of the first year's product was the total cost of irrigation?

2. For a fee of $2 an acre an irrigation company delivers water upon a farm, and the additional cost to the owner for distribution and maintenance of ditches is $22 an acre. If his crop is worth 150% more than the total cost of irrigation, what is the value of the crop per acre?

3. If by irrigation our government should convert 1,909,000 acres of desert into fertile farm lands, how many square miles would be so converted? What would be the value of this land at $47 an acre? If the cost of irrigating averaged $13.50 an acre, how much would the value of the land exceed the cost of irrigation?

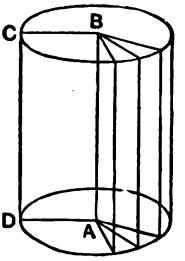
4. If this redeemed land were in the form of a square, what would be the length of each side?

THREE ROUND BODIES

501. The three round bodies here treated are of especial interest because so many familiar objects resemble them in shape.

THE CYLINDER

502. The figures represent two **Cylinders.** The bottom and top of a cylinder are equal circles, called **the bases.** Its altitude is the perpendicular distance between its bases. The curved surface which extends from base to base is called its **lateral surface.**



A perfect cylinder is made by revolving a rectangle, as $ABCD$, about one of its sides, AB, as an axis.

1. What dimension of the cylinder does the axis AB measure?

2. What dimension of the base does the side AD measure? What part of the surface does AD trace? What part of the surface does the side CD trace?

3. How do you find the area of a circle? (Art. 177.)

503. Volume of a Cylinder.

It is evident that if a cylinder were 1 inch high, it would contain as many cubic inches as there were square inches in the area of the base; and if it were 5 inches high, the volume would be 5 times as great (Art. 187). It follows that —

The volume of a cylinder is found by multiplying the measure of the area of either base by its altitude.

That is, if a = the area of the base, c = the circumference, and r = the radius, we have $a = \frac{1}{2}\ rc$ (Art. 177); or, since c is equal to $2\frac{2}{7} \times 2\ r$ (Art. 175), we may write $a = \frac{1}{2}\ r \times 2\frac{2}{7} \times 2\ r$, or $3\frac{1}{7}\ r^2$. Therefore, if h is the height,

$$\text{the volume of a cylinder} = 3\tfrac{1}{7}\ r^2 \times h, \text{ or } \pi r^2 \times h,$$

where the value $3\frac{1}{7}$, or $2\frac{2}{7}$, is represented by the Greek letter π (pī).

WRITTEN EXERCISES

504. 1. Find the volume of a cylinder the radius of whose base is 3 ft. and whose altitude is 6 ft.

$2\frac{2}{7} \times 3^2 = 19\frac{5}{7}$, number of square feet in base.
$19\frac{5}{7} \times 6 = 169\frac{5}{7}$, number of cubic feet in volume.

2. Find the volume of a cylinder having the radius of its base 2.5 in. and its altitude 20 in.

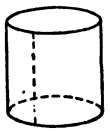
3. Find the number of cubic feet of water in one 14-foot length of a city water main 10 in. in diameter.

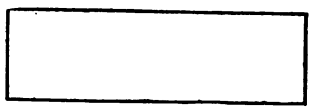
4. A cylindrical standpipe is used as a reservoir. Find its capacity in cubic feet, if its height is 70 ft. and the diameter of its base 20 ft.

5. How many gallons of oil will a cylindrical tank 25 ft. high and 21 ft. in diameter hold? (Use $7\frac{1}{2}$ gal. = 1 cu. ft.)

505. Lateral Surface of a Cylinder.

Cut a rectangular strip of paper whose width equals the altitude of some cylinder. Wrap the paper around the cylinder, and mark off the length that goes once around. Unwrap the strip and you will have a rectangle equal in area to the lateral surface.





How does the length of this strip compare with the circumference of the base? How do you find the area of the strip?

From the preceding we see that —

The area of the lateral surface of a cylinder is found by multiplying the circumference of its base by its altitude.

WRITTEN EXERCISES

506. 1. Find the area of the lateral surface of a cylinder whose altitude is 6 ft. and the radius of whose base is 3 ft.

$2 \times 3 \times 3\frac{1}{7} \times 6 = 113\frac{1}{7}$, number of square feet in area. Explain the process.

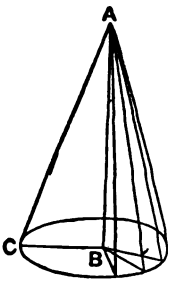
2. Find the area of the entire surface of the cylinder in problem 2, Art. 504.

3. Find how much paint will be needed for two coats on the standpipe in problem 4, Art. 504, if a gallon of paint covers 40 sq. yd. of surface with one coat.

4. A water-tube boiler has 350 tubes, each of which is 8 ft. long and 2.5 in. in diameter (inside measurement). Find in square feet the entire heating surface (interior curved surface) of the tubes.

THE CONE

507. The second round body is the **Cone**, as shown in the pictures. Can you recall any ordinary solids which are in this shape? What is the shape of a funnel? In what shape is an Indian tepee?



A cone is made by revolving a right-angled triangle, as ABC, about one of its legs, AB, as an axis.

1. Which side of the triangle is the altitude of the cone? Which is the radius of the base? What does BC trace?

2. Which side of the triangle traces the lateral surface of the cone?

3. Make the best cone you can of stiff paper or pasteboard. Make a cylinder of the same base and altitude. Fill the cylinder with sand, and find out how many times the cone can be filled with the same amount of sand.

508. Volume of a Cone.

From the preceding experiment it is found that a cone is one third of a cylinder of the same base and altitude. Therefore —

The volume of a cone is found by multiplying the measure of the area of its base by one third of its altitude.

WRITTEN EXERCISES

509. 1. Find the volume of a cone whose height is 5 in. and the area of whose base is 12 sq. in.

$$\frac{12 \times 5}{3} = 20, \text{ number of cubic inches.}$$

Find the volume of a cone whose height is —

2. 7 cm. and area of base 27 qcm.

3. 6 in. and area of base 19 sq. in.

4. 16.2 in. and area of base 85.72 sq. in.

5. 389.7 cm. and area of base 1 qm.

6. Find the volume of a cone the radius of whose base is 3 in. and whose altitude is 7 in.

If the radius of the base and the altitude of a cone are given, the area of the base must first be found. (See Ex. 1, Art. 504.)

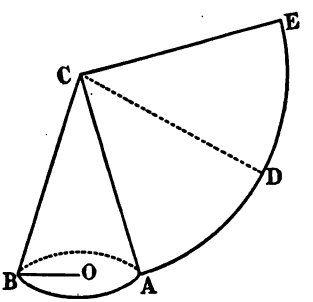
Find the volumes of cones whose dimensions are —

7. Radius of base 2.5 in., altitude 12 in.

8. Radius of base 28 cm., altitude .6 m.

510. Lateral Surface of a Cone.

Suppose the surface of the cone shown in the figure to be cut along CA, and then unrolled and flattened out. The surface will take the form of part of a circle, CAE, in which the radius CD equals the slant height CA, and the length ADE equals the circumference of the base of the cone. By regarding a circle as made up of triangles, we found that its area equals the circumference multiplied by half the radius (Art. 177). Likewise, we find that the area of part of a circle, CAE, equals the part of the circumference, ADE, multiplied by half the radius CD. It follows that —



The area of the lateral surface of a cone equals the circumference of the base multiplied by half of the slant height.

WRITTEN EXERCISES

511. **1.** Find the area of the lateral surface of a cone whose circumference at the base is 30 ft. and whose slant height is 34 ft.

2. Find the area of the lateral surface of a cone whose diameter at the base is 8 ft. and whose slant height is 12 ft.

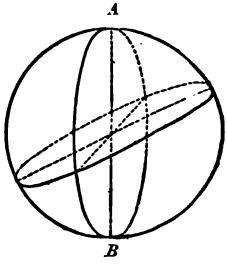
3. The conical tower on a building is 20 ft. in diameter at the base, and the slant height is 27 ft. What will be the cost of painting it at 35¢ a square yard?

4. How many square yards of canvas in a conical tent which has a slant height of 11 ft. and a circumference of 27 ft. at the base?

5. The slant height of a cone is 10 cm., and the diameter of its base is 12 cm. Find to the nearest square centimeter the entire surface (the base included) of this cone.

THE SPHERE

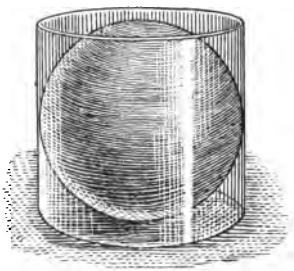
512. The third round body is a **Sphere**, as shown in the pictures. Name two or more familiar objects that resemble a sphere in shape. Name several fruits that are spherical in shape.



The figure on the right shows the sphere made by revolving a circle about a diameter AB.

513. Volume of a Sphere.

If we drop a sphere into a cylindrical cup of the same diameter and height, which is full of water, we shall find that it causes two thirds of the water to overflow. From this experiment it is seen that a sphere equals two thirds of a cylinder of the same diameter and height. And since the volume of a cylinder is $\frac{22}{7}r^2 \times h$, or $\pi r^2 \times h$ (Art. 503), where $h = 2r$, and the sphere is $\frac{2}{3}$ as large as the cylinder, therefore—



the volume of a sphere $= \frac{2}{3} \times \pi r^2 \times 2r = \frac{4}{3}\pi r^3$, or $\frac{88}{21} r^3$.

That is, **the volume of a sphere is found by multiplying the cube of the number that measures its radius by $\frac{88}{21}$.**

WRITTEN EXERCISES

514. 1. Find the volume of a sphere whose radius is 3 feet.

$\frac{88}{21} \times 3^3 = 113\frac{1}{7}$, the number of cubic feet.

2. Find the number of cubic inches in a basket ball whose diameter is 10 in.

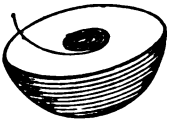
3. Find the number of cubic feet of water that a boiler in the shape of a hemisphere will hold, if the diameter of its open top is 3 ft.

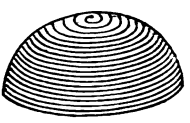
4. A hemispherical bowl has an inner radius of $10\frac{1}{2}$ in. How many gallons will it hold?

5. How many solid spheres 6 cm. in diameter can be molded from a solid metal cylinder which is 45 cm. long and 4 cm. in diameter?

515. Surface of a Sphere.

In the picture there are two *hemispheres*.





If we wind one of the curved surfaces of a hemisphere with a waxed cord, as shown in the picture, and then wind the flat surface with the same cord, we find that the curved surface takes twice as much cord as the flat surface. Hence it is seen that the curved surface of the complete sphere is four times as large as one flat surface. And since the area of one of the flat surfaces is $\frac{2}{7}^2 r^2$, or πr^2, therefore—

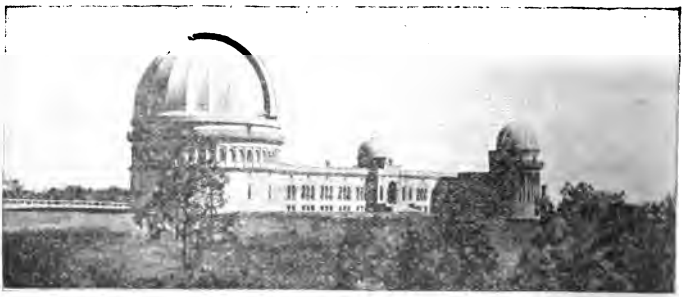
The area of the surface of a sphere is found by multiplying the square of the number that measures its radius by $\frac{8}{7}^8$.

WRITTEN EXERCISES

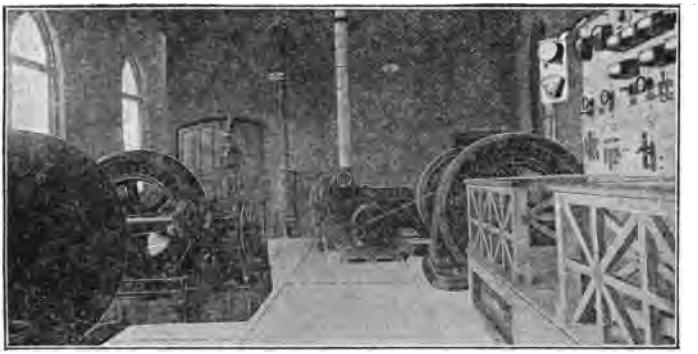
516. 1. Find the area of a sphere whose radius is 8 feet.

2. Find the number of square inches of leather needed to cover a basket ball whose diameter is 10 in.

3. Find the number of square feet of tin needed to cover the roof of an observatory in the shape of a hemisphere whose radius is $12\frac{1}{2}$ ft.



RELATED PROBLEMS



517. This picture shows a scene in a power house where electricity is generated to propel street cars, to run elevators, to turn fans, and to furnish heat and light.

1. At one of our educational institutions a central heating and lighting plant is in operation. During the month of December in a recent year 191,580 lb. of coal were consumed. What was the average daily consumption in tons? What was the cost of the coal, if the price at the mines was $1.05 a ton and freight $1.60 a ton?

2. An engineer at $50 per month of 4 weeks and two firemen at $25 per month each are required to run the plant. Find the cost of running it for 40 weeks, the average consumption of coal being 2.5 tons daily.

3. A gentleman has his residence lighted by electricity at 12¢ per kilowatt, less 10% discount. On an average he uses 20 kilowatts per month. What do his lights cost a year?

4. The cast iron wheels of an electric car are guaranteed by the makers to run 30,000 miles. How many revolutions would a wheel 33 in. in diameter make in running this distance?

5. On a certain electric road the fare is 5 cents, but 6 tickets are sold for 25 cents. Tickets for laborers and school children, however, are only $2\frac{1}{2}$ cents each. How much money did the road earn in one day, if 19,210 straight fares, 28,140 tickets, 16,210 labor tickets, and 9201 school tickets were collected?

6. An electric road is 65 mi. long and has trolley poles opposite each other every 130 ft. of its length. The tax on these poles is $2 each per year. How much tax does the road pay per year on its poles?

7. The mechanical power needed to run a dynamo is 598 horse power. The machine converts into useful electric power 75.25% of the power put upon it, and it takes 11.615 kilowatts of electric power to run one street car. How many cars can be moved with this dynamo? ($\frac{3}{4}$ no. horse power = no. of kilowatts.)

8. On a certain street railway the cars are scheduled to leave each terminus simultaneously and to run 5 minutes apart. Their average speed is 10 mi. an hour, including stops. If the line is 5 mi. long, how many cars are needed for the service?

MISCELLANEOUS PROBLEMS

518. **1.** The average age of 150 boys is 14.5 yr. The average age of 100 girls is 12.25 yr. What is the average age of all of them?

2. A bank customer's deposit at the beginning of the month was $395.60. During the month he gave checks for the following amounts: $25; $7.50; $19.25; $63.50; $75; $60.85; $23.50; $40.25. Find his balance at the end of the month. (Use method given in Art. 334.)

3. A large steamer will carry 640,000 bu. of wheat. How many car loads of wheat averaging 48,000 pounds to the car will it take to load such a steamer?

4. What number divided by 275 will give 275 for a quotient, and 10 for a remainder?

5. The production of coal in the New River district of West Virginia during October amounted to 414,650 tons, a decrease of 34,495 tons as compared with the output in September. What was the output in September? How many 50-ton cars were required to move the October output?

6. My horse consumes each year 8 times his weight in hay. If he weighs 1000 pounds, what per cent of his own weight does he consume in hay? What per cent of the hay consumed is his weight?

7. Find the number of bricks needed to build for a smokestack a solid foundation 5 ft. by 5 ft., and 12 ft. high.

8. Find the number of bushels contained in a bin 32 ft. by 16 ft. by 15 ft.

9. The estimated cost per acre of producing wheat on a Minnesota farm was as follows: seed, 87¢; plowing, 90¢; dragging, 23¢; drilling, 26¢; harvesting, 33¢; twine, 18¢; shocking, 9¢; stacking, 33¢; threshing, $1.15; machinery cost, 38¢; land rental, $1.80. What does it cost to cultivate and harvest 850 acres? If the yield is 25 bu. to the acre and wheat sells for 85¢ a bushel, what does the farmer make above the cost of production?

10. The cost of cultivating a 100-acre rice farm in Louisiana was as follows: plowing and harrowing, $4 an acre; seeding, $3.50 an acre; harvesting, threshing, and hauling the crop to mill or station, $5 an acre; levee work and other items, $7.50 an acre. The yield was 12 barrels per acre and the rice was worth $3.50 a barrel. What was the profit above the cost of production, if the straw and bran from the crop brought $500?

11. A 1-acre tea field at Pinehurst, S.C., yielded 1200 lb. of green leaf, or 300 lb. cured, to the acre. If the cost of production and preparation for market was $27\frac{1}{2}$¢ a pound (cured), and the tea sold at wholesale for 50¢ a pound, what was the profit per acre? If the retail price is $1 a pound, what per cent profit does the retailer make?

12. If a pile of wood 8 ft. by 5 ft. by 4 ft. costs $7.50, what is the cost per cord?

13. From a tank which is $\frac{3}{4}$ full 500 gal. are drawn. If it is then $\frac{2}{3}$ full, what is its capacity?

14. After spending $\frac{3}{5}$ of his money, a man paid bills of $20, $35, and $50, and then had $51 left. How much money had he at first?

15. Bruce has $2.80 and Dick has $2\frac{1}{2}$ times as much. How much must Dick give Bruce so that each may have the same amount?

16. A man had a certain sum in a bank. He drew out $\frac{3}{7}$ of this sum, and later made a deposit equal to $\frac{2}{5}$ of the sum which remained. If he then had $2240 on deposit, how much had he in the bank at first?

17. If a chickadee eats 30 female cankerworm moths per day during the 25 days which these moths crawl up trees, and the average number of eggs to each moth is 185, how many eggs of this noxious insect does a chickadee destroy in the 25 days?

18. The report of Mary Jones for the three terms of the session ending May 26, 1906, is as follows:

Subjects	Terms			Average
	Fall	Winter	Spring	
Arithmetic	85	87	91	
Spelling	95	94	99	
Geography	80	88	89	
History	90	90	99	
English	98	99	100	
Writing	75	85	95	
Term averages				

Find her average by terms; by subjects; and her general average for the session.

19. Find the sum of 175.6, $226\frac{8}{9}$, $.7\frac{7}{8}$, $10.9\frac{2}{3}$, and $114\frac{5}{6}$.

20. Two men and a boy earned $36.20. If the boy earned half as much as each man, what did each person earn?

21. It took $\frac{2}{5}$ of my money to pay a debt. After paying out $\frac{5}{9}$ of the remainder for a horse I had $240. How much had I at first?

22. In a certain meeting a motion was carried by a majority of 73 votes. The total vote both for and against the motion was 165. What was the affirmative vote?

23. Massachusetts has a population of 348 persons to the square mile of territory. Texas has an area of 265,780 sq. mi. If the entire population of the United States, estimated at 83,189,140, could be moved into Texas, what would be the ratio of the population per square mile to the population per square mile in Massachusetts?

24. Find the difference between the area of a rectangle 10 in. by 14 in. and that of a circle whose diameter is 14 in.

25. If sound at a given temperature travels at the rate of 1132 ft. per second, how far away is a gun whose flash is seen 20 seconds before its report is heard? (Give answer in miles correct to two decimals.)

26. If one man can shovel a cubic yard of earth into a car in half an hour, how long will it take 6 men to load a car that is 27 ft. long, 6 ft. wide, and $3\frac{1}{4}$ ft. deep?

27. Find the number of perches of masonry in the walls of a foundation 33 ft. by 22 ft. (outside measurement), 12 ft. high, and 2 ft. thick.

28. What is the weight in tons of the water which falls on an acre in a year, where the annual rainfall is 48 inches? (1 cu. ft. of water weighs $62\frac{1}{2}$ lb.)

29. If the annual rainfall at a place is 62 inches, how deep a pool one acre in area would the rainfall on 50 acres fill?

30. A silver dollar is 900 fine; that is, it contains 900 parts of pure silver out of 1000. It weighs $412\frac{1}{2}$ grains. How many grains of silver are there in it?

31. How many pounds (Avoirdupois) will 1000 silver dollars weigh? (7000 gr. = 1 lb.)

32. If a gold dollar weighs 25.8 grains, how many pounds will 1000 gold dollars weigh?

33. What is the value of 1 ton of silver dollars? What is the value of 1 ton of gold dollars?

34. A field 36 rd. long by 27 rd. wide is inclosed by a tight board fence 12 ft. high. Find the cost of painting the fence on one side at $4\frac{1}{2}¢$ a square yard.

35. A railway station is 90 ft. by 48 ft. The platform surrounding it is 16 ft. wide and is built of 2-inch planks. Find the value at $18 per M of the lumber used in the platform.

36. It is estimated that 26,500,000 barrels of cement were produced in the United States in the year 1904. If this cement were used in constructing a wall 18 ft. high and 6 ft. thick, which required 1 barrel of cement to each cubic yard, how long would the wall be?

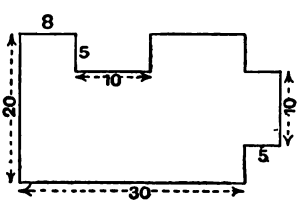
37. If a meter is equal to 39.37 inches, how many yards are there in 2000 meters?

38. If 1 kilogram equals 2.2 lb., how many kilograms are there in a ton of coal?

39. If 1 gallon equals 3.79 liters, how many liters are there in a bushel? (See Art. 298.)

40. Find the cost of repairing a mile section of a 16-foot roadbed, if it is first covered with crushed stone to an average depth of 6 in. at a cost of $1.50 a cubic yard, and then faced to a depth of 4 in. with gravel at a cost of 50¢ a cubic yard.

41. Find the area of the field whose plan is given at the right, the dimensions being marked in rods.



42. At the following prices, find the cost of paving and curbing 1 mile of street, which is 40 ft. wide from curb to curb: 35¢ a cubic yard for excavating to a depth of 2 feet, \$1.70 a square yard for crushed stone filling, sand, and concrete, and 30¢ a lineal foot for each line of curbing.

43. The bill of lumber for building a 200-ton silo was as follows: 156 pieces of studding 2″ × 12″, 28′ long, @ \$20; 6 sills 2″ × 10″, 26′ long, @ \$19; 16 sills 2″ × 10″, 16′ long, @ \$14; 30 rafters 2″ × 4″, 20′ long, @ \$16; 450 ft. sheathing, @ \$15; 5000 shingles, @ \$3; 2800 ft. drop siding, @ \$16; 4256 ft. lining, @ \$15.

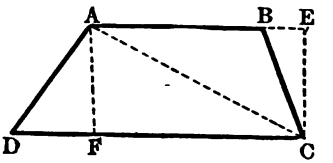
The following materials were also needed:

426 lb. tarred paper, @ 2¢; 1 barrel coal tar, \$4.50; painting 25 squares, @ 60¢; nails and hinges, \$10; cement for bottom, \$5; 18 bolts, @ 5¢. What was the total cost of lumber and other materials?

44. The circumference of one of the big trees in California 6 ft. above the ground is 90 ft. Find the diameter of the tree at the same height.

45. The length of a rectangle is 5 times its width. Its area is 2205 sq. yd. Find its dimensions. (Make a diagram.)

46. Find the area of a field in the shape of a trapezoid whose parallel sides are 30 rd. and 50 rd. and whose altitude is 20 rd.



The area of $ABCD$ = area of triangle ABC + area of triangle ACD. $AF = EC = 20$ rd.

47. Draw a trapezoid whose parallel sides are 6 in. and 9 in. and whose altitude is 8 in. Find its area.

48. A man owned the E. $\frac{1}{2}$ of the S.E. $\frac{1}{4}$ of the N.E. $\frac{1}{4}$ of a certain section of land. What was his farm worth at \$40 an acre?

49. A rectangular plat, 60 ft. long and 40 ft. wide, is enlarged 10% in each dimension. What is the per cent of increase in area?

50. For paving a hall 1920 tiles 4 in. square are required. If the hall is 8 ft. wide, what is its length?

51. The freight tonnage of the railways of the United States during a recent year amounted to 1,300,000,000 tons. Of the freight carried 51.56% consisted of products of the mines, 9.59% of agricultural products, 13.41% of manufactured products, 12.51% of lumber and forest products, 2.74% of animal products, and 10.19% of unclassified products. Find the number of tons of each kind carried by the railways that year.

52. The Jefferson Hotel Company asks for bids from firms who furnish table silver. The first firm makes a bid of \$1000, subject to a discount of 30%; the second bids \$900 with his rate of discount 25%; the third bids \$750, and discounts further at the rate of 10%. Find what the net cost to the hotel company would be in each case.

53. A furniture house makes an offer of 10% discount for cash or payment within ten days, or 5% off for payment within 30 days. Find the cost of (*a*) 1 bedstead marked \$17.50, paid in 5 days; (*b*) 1 bookcase marked \$22.60, paid in 20 days.

54. The same furniture house advertised a cut of $12\frac{1}{2}$% on all goods and maintained its discounts for cash and time payments. Find (*a*) the cash price of an oak sideboard marked \$44.80, and (*b*) price of a refrigerator marked \$24, if paid in 30 days.

55. A merchant offers 50% discount to a customer while his competitor offers 25% and 25%. Which offer is the better, and by what per cent?

56. Find the gain per cent on the following goods: (*a*) linoleum bought @ 96¢, sold at $1.08; (*b*) suite of furniture bought for $69.60, sold at $81.20; and (*c*) chairs bought at $15.30 a dozen, sold at $1.70 each.

57. A man offered to sell a house for 25% more than it cost him. He afterward sold it for $3800, which was 5% less than he first asked for it. What did the house cost him?

58. A dealer bought a stove listed at $60 at a discount of 20% from the list price, and sold it at 15% below list price. What per cent did he gain?

59. In how many years will $250 amount to $500, at 4% per annum?

60. Find the interest at 6% on $725 for 2 yr. 5 mo. 10 da.

61. At what rate per cent will $522.40 earn $78.36 in 3 yr.?

62. Find the principal which at 6% interest will earn $7.82 in 6 yr. 6 mo. 6 da.

63. Find the amount of $4850 for 5 yr., at 3%, interest compounded annually.

64. Find the amount of $1600 for 6 years, at 4% per annum, interest compounded semiannually.

65. An agent receives $714 with which to purchase goods, after deducting his commission of 2%. What was the cost of the goods? What was his commission?

102% of cost = $714. (See Art. 320.)
On what is commission reckoned?

66. If a commission merchant remits $3312 to a customer after deducting 4% commission, what was the amount of the sale?

67. An agent charged a commission of 3% for selling some property. After deducting his commission he remitted $3395. How much was the commission?

68. An agent received $2625 to invest in wheat and to pay him 5% commission on the purchase. If he paid 80¢ a bushel for wheat, how many bushels did he buy?

69. A man deposited in a savings bank $350 Jan. 1, 1906, and $500 May 1, 1906. If the bank allows 4% interest per annum on all deposits, compounded on Jan. 1 and July 1 of each year, how much did the man have to his credit on Jan. 1, 1907?

70. At noon an express train started from New York to Washington, 225 mi. distant, and ran at the average speed of 50 mi. an hour. At the same time a train left Washington for New York, and its average speed was 40 mi. an hour. At what hour did they meet? How far from New York?

71. If an Edison bulb lasts 250 nights, burning 4 hr. each night, how long would it last if burned 5 hr. a night?

72. If air consists of 21 volumes of oxygen to 79 volumes of nitrogen, how many cubic feet of oxygen are there in a room 15 ft. by 18 ft. by 12 ft.?

73. Seventy-five per cent of the pupils in a certain school study arithmetic. If six more studied this subject, 80% of the pupils would be studying arithmetic. How many pupils are there in the school?

74. If 5 times the interest on a certain sum for $2\frac{1}{2}$ yr. equals the principal, what is the rate of interest?

75. The following indorsements are made upon a note of $5000, dated July 1, 1906, bearing interest at 6%: Dec. 15, 1906, $1500; March 27, 1907, $2750. On Sept. 15, 1907, the balance of the note and interest were paid. How much was paid on that date?

76. At what time between 1 and 2 o'clock are the hands of a clock together?

In 1 hr. the minute hand moves over 60 minute-spaces, and the hour hand over 5. Hence the former gains 55 minute-spaces in that time. To gain 1 space requires $\frac{1}{55}$ of an hour. At 1 o'clock the hands are 5 spaces apart. To gain these 5 spaces will require $5 \times \frac{1}{55}$ hr. $= \frac{1}{11}$ hr., or $5\frac{5}{11}$ min. Hence the time is $5\frac{5}{11}$ min. past 1 o'clock.

77. At what time between 4 and 5 o'clock are the hour and minute hands of a watch together?

78. At what time between 1 and 2 o'clock are the hands of a clock opposite each other?

79. A and B start from the same place and travel in the same direction. A goes 7 mi. an hour and B 3 mi. an hour. If B has a start of 6 hr., in how many hours after B starts will A overtake him?

80. A can do a piece of work in 3 days, and B can do it in 5 days. How many days will it take both together to do the work?

81. A and B can do a piece of work in 6 days, and A can do it in 9 days. In how many days can B do the work?

82. A water tank is supplied by 3 pipes. The first will fill it in 2 hr., the second in 3 hr., and the third in 5 hr. How long will it take all three together to fill it?

83. The distance from A to B is 160 mi., and the average rate of passenger trains between the two places is 40 mi. an hour. If a train starts from A 15 min. late, at what rate must it run to arrive at B on time?

84. An invoice of shoes which cost \$180 is marked so as to sell at a profit of $33\frac{1}{3}\%$. Will the merchant gain or lose, and how much, if the shoes are sold at 20 % discount from the marked price?

85. A dealer bought 20 dozen pairs of shoes at \$3 a pair, and a certain number of shoes of a cheaper grade at \$2.40 a pair. If the entire cost was \$1152, how many pairs of the cheaper kind did he buy?

86. A customer bought a hat for \$3.15, which was 10% below the marked price. If the hat was marked to gain 25 %, what did it cost the dealer?

87. A bookseller bought a book whose catalogue price is \$3, at a discount of 25 %, and sold it at the catalogue price. What per cent profit did he make?

88. If peaches are bought at 4 for 3 ¢ and sold at 3 for 5 ¢, what per cent is gained?

89. A merchant sold goods at a profit of $33\frac{1}{3}\%$ on the cost price, but lost 10 % of his sales through bad accounts. What per cent did he gain? What was the cost of goods that yielded him a profit of \$5000? What was their selling price?

90. In a recent year Cuba produced 1,152,000 tons of cane sugar, and Louisiana 335,000 tons. In the same year the United States produced 209,722 tons of beet sugar. The amount of cane sugar produced in Louisiana was what per cent of that produced in Cuba? The amount of beet sugar produced in the United States was what per cent of the amount of cane sugar produced by Cuba and Louisiana together?

91. A boat going 10 mi. an hour steams for 4 hr. due east, and then for 3 hr. due south. How far is it then from its starting point?

92. An upright pole 13 meters high casts a shadow 19 meters long. How far from the top of the pole is the extremity of the shadow?

93. A wire is stretched across a road 32 ft. wide between the tops of two poles 12 ft. and 27 ft. high, respectively. Find the length of the wire.

94. A house which cost me $8000 is assessed at $6000. The rate of taxation is $1\frac{1}{2}\%$, and annual repairs amount to $46. What monthly rent must I receive in order to clear 10% on my investment?

95. A house which cost $4500 rents for $400. If taxes are 2%, insurance $\frac{3}{4}\%$, and repairs $6.25 per annum, what per cent of the cost is the net income?

96. If 5 kilograms of steel make 25,000 needles, how many centigrams will 25 needles (1 paper) weigh?

97. Find the interest on 1250 francs for $2\frac{1}{2}$ yr. at 3% per annum, and express the result in our currency. (1 franc = $.193.)

98. A pearl necklace made in Paris for an American bride was invoiced at 128,175 francs. What is its value in our currency? If insured in transit at $\frac{1}{8}\%$, what was the premium?

99. Find the cost, at $12 per M, of the following bill of framing materials for a cottage:

5 pieces, 6″ × 8″, 16 ft. long, for sills.
2 pieces, 6″ × 8″, 12 ft. long, for sills.
37 pieces, 2″ × 8″, 16 ft. long, for joists, floor, and porch.
6 pieces, 2″ × 8″, 12 ft. long, for joists.
4 pieces, 4″ × 4″, 16 ft. long, for corner studding.
150 pieces, 2″ × 4″, 16 ft. long, for studding, plates, etc.
42 pieces, 2″ × 4″, 14 ft. long, for rafters.
4 pieces, 1″ × 6″, 16 ft. long, for joist bearers.
3 pieces, 2″ × 10″, 16 ft. long, for stair horses.
150 pieces, 1″ × 3″, 16. ft. long, for roof sheathing.

100. Four boys bargained for a football, agreeing to share the cost equally. Later they took in a fifth boy, and found that this reduced the amount each had to pay by 24¢. What was the price of the football?

101. An American lady in Paris desired to purchase 10 meters of a certain fabric. At one shop it was priced to her at \$3 a meter; at another, at \$2.75 a yard. Which was the better offer, and how much?

102. The Washington monument is 555 ft. high. Find its height in meters.

103. The radius of the base of a cylindrical cistern is 6 ft. and its depth is 20 ft. How many gallons does it hold? How many barrels? ($31\frac{1}{2}$ gal. = 1 barrel.)

104. In fourth-class post offices, postmasters receive a certain per cent of the cancellations, as follows: on the first \$50 or less per quarter, 100%; on the next \$100 or less per quarter, 60%; on the next \$200 or less per quarter, 50%; on all over \$350 per quarter, 40%. Find the yearly pay of a postmaster whose cancellations were: for first quarter, \$190; for second quarter, \$250; for third quarter, \$325; and for fourth quarter, \$420.

105. What is the sum which put at interest will amount to \$621 in 2 yr. 6 mo., if money is worth 6%?

Amount of \$1 for $2\frac{1}{2}$ yr. at 6% = \$1.15; entire amount = \$621.
There must be \$1 of principal for every \$1.15 of entire amount.
∴ \$621 ÷ \$1.15 = 540, number of dollars of principal.

106. What sum must I put at interest at 6% to liquidate a debt of \$1744, due in 1 yr. 6 mo.?

107. A cyclist with 3 hr. at his disposal set out for a ride. By reason of the grade, he could make 15 mi. an hour on the outward trip, but only 12 mi. an hour on the return trip. How far could he go before turning?

108. A bank has a capital of $200,000, and deposits of $1,800,000. Its circulation is equal to its capital. If its annual expenditures are $25,000 for salaries and expenses and it earns $2\frac{1}{2}\%$ interest on capital, deposits, and circulation, how much are the profits in a year?

109. In the example just given, what amount could be placed to the surplus account, if a dividend of 8% was paid on the capital stock?

110. The average head has 105,600 strands of hair. On this basis, how long a strand will the hair of a young lady make if it grows to a length of 18 inches and each hair is placed to the end of another in a single strand?

111. Two vessels at sea exchanged time signals by wireless telegraphy. One of the vessels is in longitude 32° 30′ W., and its time is 10: 25 A.M. If the time of the other vessel is 11 A.M., what is its longitude?

112. The distance between two cities is 802 mi. Four railroads make up this distance, covering respectively 150 mi., 242 mi., 275 mi., and 135 mi. On a shipment of 26,500 lb. of apples at a through rate of 34¢ a hundred, how much would each railroad earn if they received amounts proportional to their mileage?

113. Instead of using the third railroad for 275 mi., a longer route may be used and the amount earned by these 275 mi. be divided among three railroads, which get amounts proportional to 138 mi., 175 mi., and 125 mi. Find the amount that would be apportioned to each of these three railroads.

114. Find as in examples 112 and 113 the apportionment of a first-class freight rate of 92¢ (*a*) to the four railroads; (*b*) to the three which haul by the longer line. (Calculate correct to one tenth of a cent.)

115. If 4 men can do a certain piece of work in 7 days, how many men can do three times as much work in 14 days?

116. Considering the radius of the earth as 4000 miles, what is its volume? How many square miles are there on its surface?

117. Find the volume of a cone when the radius of the base is 3 cm. and altitude 7 cm.

118. What is the longest distance in a straight line that a boy can run in his back yard, which is in the form of a rectangle 60 yd. long and 35 ft. wide?

119. A Texas oil well flowed at the rate of 15,000 barrels (52 gal. each) a day. At that rate how long would it take it to fill 6 cylindrical tanks each 42 ft. in diameter and 25 ft. high? (Use $7\frac{1}{2}$ gal. = 1 cu. ft.)

120. A race course is laid out in the form of a circle whose radius is 280 yd. How many laps will it take to make a 2-mile course?

121. Assuming a drop of water to be spherical and $\frac{1}{10}$ of an inch in diameter, how many drops will it take to fill a gallon bucket?

122. The boiler of a steam engine is 20 ft. long by 4 ft. in diameter, inside measurement. At a steam pressure of 225 lb. to the square inch, what is the total pressure upon its surface?

123. A book contains 588 pages, each 6 in. by 8 in. How many feet are there in the side of a square that is equal in area to the entire surface of the pages of this book?

124. What is the per cent of increase in pay if printers are paid as much for 8 hours' work as they were formerly paid for 9 hours' work?

125. The following table shows the material development of the South from 1890 to 1905:

	1890	1905
Capital invested in cotton mills . .	$60,000,000	$225,000,000
Cotton bales used	546,000	2,163,000
Value of cotton crop	$390,000,000	$680,000,000
Pig iron made, tons	2,600,000	3,219,600
Coal mined, tons	21,200,000	70,000,000
Petroleum, barrels	498,632	42,495,802
Lumber products, value	$90,700,000	$250,000,000
Value of manufactured products .	$917,589,000	$1,750,000,000
Value of exports	$306,000,000	$555,480,000
Farm products, value	$773,000,000	$1,750,000,000

Find the per cent of increase (correct to two decimals) in each case during the 15 years.

126. Out of a total income of $1,950,743 636 in a recent year, the railroads of the United States paid in wages and salaries of employees $780,321,415, and to owners in the shape of dividends $165,750,586. Find (*a*) the per cent of amount of income paid in wages and salaries; (*b*) the per cent of amount paid in dividends.

In finding the rate per cent in such problems, it is sufficiently accurate to divide by the number made by the first four digits only.

127. In a year when the total estimated expenditures of the national government were $746,590,000, the items of expenditure were as follows: for the civil establishment, $128,000,000; for the military establishment, $93,000,000; for the naval establishment, $118,000,000; for the Indian service, $13,000,000; for pensions, $143,000,000; for public works, $57,000,000; for interest on the public debt, $24,000,000; for postal service, $170,590,000. What per cent of the total expenditures went to each department?

ANSWERS

Page 18. — **2.** 13,908. **3.** 12,982. **4.** 16,941. **5.** 20,321.

Page 19. — **6.** $28,978. **7.** $29,823. **8.** $34,923. **9.** $254.82. **10.** $331.73. **11.** 252,516. **12.** 114,277. **13.** 47,080. **14.** 98,525. **15.** 349,798. **16.** 177,022. **17.** 245,274. **18.** 143,916. **19.** 121,609. **20.** 174,375.

Page 20. — **2.** 326,306. **3.** 360,189. **4.** 291,732. **5.** 343,731. **6.** 346,321. **7.** 4,062,362. **8.** $1675. **9.** 524 bu. **10.** 3,756,884 sq. mi. **11.** $157,800,000.

Page 21. — **12.** 87,810,000 bu. **13.** $662,700,000. **14.** $11,250,000.

Page 26. — **3.** 240. **4.** 565. **5.** 541. **6.** 261. **7.** 613. **8.** 144. **9.** 236. **10.** 2483. **11.** 1379. **12.** 6174. **13.** 638. **14.** 4296. **15.** $1770. **16.** $4490. **17.** $3745. **18.** $2460. **19.** $1879. **20.** 2658. **21.** 1224. **22.** 4295. **23.** 2399. **24.** 9118. **25.** 17,986. **26.** 13,552. **27.** 38,980. **28.** 21,393. **29.** 15,465. **30.** 389,097. **31.** 395,685. **32.** 642,500. **33.** 176,740. **34.** 2455. **35.** 8972. **36.** 19,692. **37.** 90,302. **38.** $15,149. **39.** $51,805. **40.** $3211. **41.** 2,289,360.

Page 27. — **42.** 377,870. **43.** 4,308,849. **44.** 1,892,998. **45.** $740,720. **46.** $45,873. **47.** $1,008,695. **48.** 3923. **49.** 430. **50.** 4253. **51.** 479. **53.** 284 yr. **54.** 169 yr. **55.** 8389 sq. mi. **56.** 51,935. **57.** $6750. **58.** $673.

Page 28. — **59.** 92,551,150 mi. **60.** 8354. **61.** 14,147 ft. **62.** 305,200,000 lb. **63.** 2,751,201,702. **64.** $14,572,584.

Page 34. — **2.** $4350. **3.** $17,392. **4.** $15,880. **5.** $57,869. **6.** $223,195. **7.** 184,198. **8.** 211,698. **9.** 332,423. **10.** 338,748. **11.** 543,408. **12.** 146,184. **13.** 240,678. **14.** 904,925. **15.** 1,435,320. **16.** 4,221,702.

Page 35. — **7.** 680. **8.** 1380. **9.** 2240. **10.** 3660. **11.** 1470. **12.** 2280. **13.** 2350. **14.** 5360. **15.** 5200. **16.** 5110. **17.** 7740. **18.** 17,800.

Page 36. — **22.** 6026. **23.** 9828. **24.** 7630. **25.** 20,304. **26.** 14,595. **27.** 28,268. **28.** 28,658. **29.** 33,605. **30.** 43,094. **31.** 24,852.

32. 38,090. **33.** 37,062. **34.** 207,808. **35.** 551,250. **36.** 256,760. **37.** 412,386. **38.** 567,155. **39.** 32,625. **40.** 288,435. **41.** 220,632. **42.** 194,340. **43.** 223,780. **44.** 295,008. **45.** 779,625. **46.** 1,668,371. **47.** 2,714,000. **48.** 4,260,864.

Page 37. — **49.** 146,190. **50.** 282,125. **51.** 348,150. **52.** 283,920. **53.** 197,236. **54.** 487,425. **55.** 764,946. **56.** 354,552. **57.** 947,348. **58.** $90,601. **59.** $240,450. **60.** $340,272. **61.** $500,240. **62.** $602,650. **63.** $226,395. **64.** $435,831. **65.** $239,112. **66.** $798,724. **67.** 559,125. **68** 2,881,602. **69.** 6,073,830. **70.** 1,347,584. **71.** 6,062,700. **72.** 4,907,448. **73.** 6,267,045. **74.** 1,820,280. **75.** 3,985,488. **76.** 126,164,642. **77.** 144,964,040. **78.** 475,488,306. **79.** 416,206,530. **80.** 131,446,912. **81.** 130,083,375. ART. **48.** — **2.** 5504 lb. **3.** 7200 ; 57,600. **4.** $234. **5.** 2475 mi. **6.** $2275.

Page 38. — **7.** 912 mi. **8.** 360 bales. **9.** 576. **10.** 8432. **11.** 20,736. **12.** 45,000 lb. **13.** 1,115,776 lb. **14.** $112,500. **15.** 633,600 lb. **16.** 1176 mi. **17.** $2.81. **18.** $336.

Page 39. — **19.** 76,303,380. **20.** $38.10. **21.** 7056 lb. **22.** $44.10. **23.** $23.04. **24.** $4.32. **25.** $350. **26.** $310.50. **27.** $80.50. **28.** $150. **29.** $5.40. ART. **49.** — **3.** 2100 sq. rd. **4.** 225 sq. ft.

Page 40. — **5.** 323 sq. ft. **7.** 45,000 sq. mi. **8.** 72,000 sq. ft. **10.** 1400 sq. ft. **11.** 143 sq. ft.

Page 47. — **2.** 145. **3.** 162. **4.** 176. **5.** 142. **6.** 125. **7.** 183. **8.** 126. **9.** 179. **10.** 122. **11.** 122. **12.** 113. **13.** 157. **14.** 1881. **15.** 1825. **16.** 2378. **17.** 1324. **18.** 1301. **19.** 2064. **20.** 3089. **21.** 525. **22.** 1741. **23.** 1558.

Page 48. — **26.** 445. **27.** 902. **28.** 1463. **29.** 358. **30.** 442. **31.** 1152. **32.** 538. **33.** 412. **34.** 663. **35.** 423. **36.** 514. **37.** 315. **38.** 230. **39.** 362. **40.** 114. **41.** 234. **42.** 243. **43.** 312. **44.** 314. **45.** 357. **46.** 125. **47.** 198. **48.** 233. **49.** 457. **50.** 342. **51.** 472. **52.** 2376. **53.** 534. **54.** 302. **55.** 635.

Page 49. — **57.** 1198. **58.** 673. **59.** 518. **60.** 650. **61.** 348. **62.** 617. **63.** 1064. **64.** 573. **65.** 3260. **66.** 525. **67.** 586. **68.** 439. **69.** 733. **70.** 1111. **71.** 682. **72.** 46,347. **73.** 29,814. **74.** 64,433. **75.** 155,754. **76.** 110,708. **77.** 33,178. **78.** 72,680.

Page 50. — **11.** 34. **12.** 170. **13.** 70. **14.** 66. **15.** 153. **16.** 163. **17.** 63. **18.** 80. **19.** 81. ART. **62.** — **1.** 30 doz. **2.** $122. **3.** 35 tons. **4.** 40 bu. **5.** 48 mi. **6.** 468. **7.** $12,500. **8.** 128. **9.** 18 mo. $6.

Page 51. — **11.** $38. **12.** 289. **13.** 880. **14.** 138 bbl. **15.** $26. **16.** $225. **17.** 20. **18.** 2000. **19.** 8 da. **20.** $4,000,000. **21.** 32 mi.

Page 52. — **22.** 10 da. **23.** 31. **24.** $15. **25.** 46,000 lb. **26.** $30. **27.** 18. **28.** 250. **29.** 50 da.

Page 58. — **1.** 18,757. **2.** 14,336. **3.** 36,337. **4.** 25,995. **5.** 28,850. **6.** 62,572. **7.** 71,279. **8.** 28,968. **9.** 81,759. **10.** 30,865. **11.** 88,889. **12.** 343,594. **13.** 928,985. **14.** 338,198. **15.** 188,666.

Page 59. — **16.** 15,832,642. **17.** 152,945,170. **18.** 621,092,704. **19.** 69,474,516. **20.** 369,189,354. **21.** 102,731,808. **22.** 1,386,636,986. **23.** 997,231,544. **24.** 7,851,237. **25.** 8,217,652. **26.** 830,705. **27.** 504,836. **28.** 1,640,712. **29.** 40,620. **30.** 246,428. **31.** 90,008. **32.** $44. **33.** 1140 mi.; 68,400 mi. **34.** 228 mi. **35.** 4836 times. **36.** 19,244 ft. **37.** 12 wk. **38.** 207,360 times. **40.** 20.

Page 60. — **41.** $23.75. **42.** $2661. **43.** 3300 mi. **44.** $145. **45.** 115 yr. **46.** $210. **47.** 28,491,606. **48.** 187. **49.** $14.44. **50.** 209. **52.** 6 mo.

Page 61. — **53.** 586,930. **54.** 35 mi. **55.** $5950. **56.** $450. **57.** $47. **58.** 9 hr. **59.** 90 bbl. **60.** $300. **61.** $2985.70.

Page 62. — **1.** 4,046,000,000 bu.; $474,000,000; 39,500,000 acres. **2.** $383,000,000; 460,000,000 bu.; 388,000,000 bu.; 244,000,000 bu.; 232,000,000 bu.; 208,000,000 bu.

Page 63. — **3.** $42,700,000; 2 times. **4.** $421,189,000; 3,444,200 bales. **5.** $179,289,000. **6.** $30,733,500; $3,444,000.

Page 64. — **7.** Cattle, 660,000; sheep, 520,000; swine, 280,000; horses, 210,000; mules, 36,000. **8.** $1,664,064,000. **9.** $95,800,000. **10.** $762,000,000; 1,540,000.

Page 68. — **3.** 2, 3, 7. **4.** 2, 2, 2, 3, 3. **5.** 2, 2, 2, 3, 5. **6.** 2, 2, 2, 2, 3, 3. **7.** 2, 2, 7, 11. **8.** 3, 3, 5, 5. **9.** 3, 3, 5, 7. **10.** 2, 2, 107. **11.** 2, 11, 83. **12.** 2, 2, 2, 2, 2, 3, 11. **13.** 2, 3, 5, 7, 11. **14.** 2, 3, 3, 3, 89. **15.** 2, 877. **16.** 3, 5, 7, 13. **17.** 5, 7, 131. **18.** 2, 2, 7, 97. **19.** 2, 7, 11, 13. **20.** 2, 2, 2, 197. **21.** 8. **22.** 7. **23.** 288. **24.** 105. **25.** 25. **26.** 22.

Page 69. — **2.** 120. **3.** 864. **4.** 770. **5.** 120. **6.** 480. **7.** 360. **8.** 420. **9.** 280. **10.** 48. **11.** 15,015.

Page 75. — **1.** 131.6. **2.** 177. **3.** 194.3. **4.** 206.86. **5.** $201.54.

Page 76. — **6.** 53.23. **7.** 65.55. **8.** 55.56. **9.** 42.7. **10.** $44.8.

Page 78. — **2.** .56. **3.** 1.392. **4.** .072. **5.** 1.5. **6.** 3.25. **7.** 5.616. **8.** .074664. **9.** .00153. **10.** .00001. **11.** $37.50. **12.** $.345. **13.** 151.9. **14.** 18.55. **15.** $10.50. **16.** .8463. **17.** 417.04. **18.** $1.458. **19.** 12.8125. **20.** 1. **21.** .875. **22.** .040764. **23.** 100. **24.** .01. **25.** 27.5. **26.** 2.75. **27.** .0275. **28.** 320.05. **29.** 780.6. **30.** $945.75. **31.** $243.75. **32.** 768.5. **33.** 181.3. **34.** 11.61. **35.** $81.81. **36.** 58.225. **37.** 18.07775. **38.** $446.62. **39.** $4329.595. **40.** 53.26854. **41.** $267.2904. **42.** $3.12576. **43.** 157.6506.

Page 80. — **3.** 2.5. **4.** 25. **5.** 140. **6.** 2.6. **7.** .5. **8.** 35.6. **9.** 2.2. **10.** .15. **11.** 9.6. **12.** 42.3. **13.** .325. **14.** 48.1. **15.** 76. **16.** 720. **17.** 1.7. **18.** 30. **19.** 3.8. **20.** 7.4. **21.** 32.5. **22.** 79.3. **23.** 1800. **24.** 4500. **25.** 43.7. **26.** 26.8. **27.** 6.73. **28.** 63.7. **29.** $1.17. **30.** $1.65. **31.** $1.02. **32.** $1.62. **33.** 21. **34.** $.06. **35.** $2.61. **36.** $8.

Page 81. — **37.** $1.65. **38.** $.40. **39.** $275. **40.** $.035. **41.** $.24. **42.** $.075. **43.** $108. **44.** $1.20. **45.** 8 tons. **46.** $34.50. **47.** 13,016 lb.; $2733.36. **48.** $6.60.

Page 82. — **1.** 11,700 bu. **2.** $5265. **3.** $2997. **4.** 655,200 lb. **5.** $1441.44. **6.** 20. **7.** $3240.

Page 89. — **2.** $\frac{3}{4}, \frac{2}{3}, \frac{3}{5}$. **3.** $\frac{4}{5}, \frac{7}{8}, \frac{4}{3}$. **4.** $\frac{9}{11}, \frac{8}{11}, \frac{11}{13}$. **5.** $\frac{2}{3}, \frac{3}{10}, \frac{14}{15}$. **6.** $\frac{2}{3}, \frac{2}{3}, \frac{7}{9}$. **7.** $\frac{3}{4}, \frac{2}{3}, \frac{2}{3}$. **8.** $\frac{3}{8}, \frac{5}{8}, \frac{3}{5}$. **9.** $\frac{19}{31}, \frac{19}{19}, \frac{11}{13}$. **10.** $\frac{7}{9}, \frac{9}{10}, \frac{7}{9}$. **11.** $\frac{1}{12}, \frac{27}{41}, \frac{2}{3}$. **12.** $\frac{4}{5}, \frac{2}{9}, \frac{4}{9}$. **13.** $\frac{12}{13}, \frac{5}{8}, \frac{11}{13}$. ART. **118.** — **4.** $\frac{63}{4}$. **7.** $\frac{109}{8}$. **10.** $\frac{503}{20}$. **11.** $\frac{344}{9}$. **12.** $\frac{173}{7}$. **15.** $\frac{765}{22}$. **16.** $\frac{5003}{40}$. **17.** $\frac{2000}{3}$.

Page 90. — **18.** $9\frac{2}{3}$. **19.** $2\frac{13}{25}$. **20.** $15\frac{4}{5}$. **21.** $8\frac{3}{8}$. **22.** $7\frac{10}{11}$. **23.** 28. **24.** $19\frac{3}{4}$. **25.** $5\frac{1}{11}$. **26.** 25. **27.** $20\frac{41}{42}$. **28.** $11\frac{3}{16}$. **29.** 35. **30.** $24\frac{4}{5}$. **31.** 25. **32.** $16\frac{8}{13}$. ART. **122.** — **2.** $\frac{54}{72}, \frac{45}{72}, \frac{56}{72}$. **3.** $\frac{40}{60}, \frac{45}{60}, \frac{48}{60}$. **4.** $\frac{4}{14}, \frac{12}{13}, \frac{15}{18}$. **5.** $\frac{56}{84}, \frac{63}{84}, \frac{60}{84}$. **6.** $\frac{6}{24}, \frac{9}{24}, \frac{14}{24}$. **7.** $\frac{45}{60}, \frac{10}{60}, \frac{16}{60}$. **8.** $\frac{28}{32}, \frac{6}{32}, \frac{5}{32}$. **9.** $\frac{22}{30}, \frac{15}{30}, \frac{24}{30}$. **10.** $\frac{80}{210}, \frac{56}{210}, \frac{147}{210}$. **11.** $\frac{9}{60}, \frac{18}{60}, \frac{16}{60}$. **12.** $\frac{42}{72}, \frac{64}{72}, \frac{45}{72}, \frac{12}{72}$. **13.** $\frac{72}{120}, \frac{105}{120}, \frac{66}{120}, \frac{16}{120}$.

Page 92. — **4.** $1\frac{1}{3}$. **5.** $\frac{7}{10}$. **6.** $\frac{23}{30}$. **7.** $1\frac{3}{10}$. **8.** $\frac{23}{30}$. **9.** $\frac{3}{10}$. **10.** $1\frac{5}{12}$. **11.** $\frac{13}{20}$. **12.** $1\frac{1}{12}$. **13.** $\frac{11}{20}$. **14.** $\frac{1}{10}$. **15.** $\frac{4}{15}$. **16.** $\frac{3}{14}$. **17.** $\frac{7}{12}$. **18.** $\frac{4}{21}$. **19.** $4\frac{3}{4}$. **20.** $9\frac{3}{8}$. **21.** $9\frac{13}{20}$. **22.** $8\frac{1}{2}$.

Page 93. — **23.** $11\frac{3}{8}$. **24.** $14\frac{23}{24}$. **25.** $17\frac{1}{2}$. **26.** $28\frac{23}{60}$. **27.** $21\frac{1}{4}$. **28.** $30\frac{5}{8}$. **29.** $5\frac{1}{4}$. **30.** $2\frac{7}{12}$. **31.** $2\frac{11}{15}$. **32.** $3\frac{4}{15}$. **33.** $4\frac{1}{5}$. **34.** $4\frac{5}{8}$. **35.** $15\frac{5}{12}$. **36.** $11\frac{1}{8}$. **37.** $14\frac{7}{12}$. **38.** $37\frac{7}{12}$. **39.** $11\frac{1}{40}$. **40.** $8\frac{9}{16}$. **41.** $16\frac{2}{15}$. **42.** $377\frac{8}{9}$. **43.** $121\frac{9}{14}$. **44.** $17\frac{1}{27}$. **45.** $11\frac{1}{48}$. **46.** $1\frac{7}{24}$. **47.** $11\frac{333}{850}$. **48.** $1\frac{5}{7}$. **49.** $\frac{1}{3}$. **50.** $1\frac{3}{7}$. **51.** $\frac{8}{21}$. **52.** $64\frac{43}{60}$. **53.** $20\frac{17}{20}$. **54.** $2\frac{5}{8}$ in. **55.** $22\frac{1}{24}$ mi.

Page 94. — **56.** $17\frac{1}{12}$ yr. **57.** $416\frac{5}{12}$ rd. **58.** $1\frac{17}{20}$. **59.** 275 bu.

Page 95, ART. **127.** — **1.** $3\frac{1}{3}$. **2.** $4\frac{2}{3}$. **3.** $3\frac{1}{3}$. **4.** $1\frac{1}{2}$. **5.** $6\frac{2}{3}$. **6.** $7\frac{1}{2}$. **7.** $16\frac{16}{17}$. **8.** $10\frac{1}{5}$. **9.** $9\frac{1}{2}$. **10.** $5\frac{23}{32}$. **11.** 14. **12.** $2\frac{2}{7}$.

Page 96. — **1.** $\frac{2}{3}$. **2.** $\frac{3}{8}$. **3.** $\frac{5}{8}$. **4.** $\frac{5}{12}$. **5.** $\frac{5}{16}$. **6.** $\frac{5}{24}$.

Page 97. — **7.** $\frac{2}{7}$. **8.** $\frac{3}{10}$. **9.** $\frac{2}{5}$. **10.** $\frac{15}{128}$. **11.** $\frac{5}{16}$. **12.** $\frac{21}{50}$. **13.** $\frac{3}{4}$. **14.** $\frac{3}{8}$. **15.** $\frac{7}{20}$. **16.** $\frac{1}{4}$. **17.** $\frac{1}{24}$. **18.** $\frac{5}{8}$. **20.** 44. **21.** 18. **22.** 403. **23.** $32\frac{2}{5}$. **24.** $88\frac{1}{2}$. **25.** 480. **26.** 225. **27.** 322. **28.** 312. **29.** 645. **30.** 175. **31.** 1400. **32.** 400. **33.** 100. **34.** 1125. ART. **133.** — **2.** $16\frac{31}{80}$. **3.** $60\frac{1}{2}$. **4.** $130\frac{1}{35}$. **5.** $139\frac{1}{3}$. **6.** $119\frac{5}{8}$. **7.** $1702\frac{109}{192}$.

Page 98. — **8.** $\frac{1}{2}$. **9.** $\frac{1}{3}$. **10.** $\frac{5}{12}$. **11.** $\frac{7}{80}$. **12.** $\frac{8}{15}$. **13.** $\frac{1}{8}$. **14.** $\frac{33}{80}$. **15.** $\frac{1}{3}$. **16.** 220. **17.** 354. **18.** $245\frac{5}{8}$. **19.** $143\frac{37}{40}$. **20.** $\frac{1}{15}$. **21.** $1\frac{1}{4}$. **22.** $\frac{14}{45}$. **23.** $2\frac{37}{84}$. **24.** $3\frac{1}{2}$. **25.** $243\frac{3}{4}$. **26.** 154 lb. **27.** $269\frac{9}{16}$. **28.** $11.25. **29.** $10.50. **30.** $16. **31.** $1.38. **32.** 83¢. **33.** $6\frac{7}{8}$. **34.** $25. **35.** $19. **36.** 1 mi. **37.** $4.60.

Page 101. — **2.** 32. **3.** 28. **4.** 15. **5.** 20. **6.** 8. **7.** $9\frac{3}{5}$. **8.** $16\frac{2}{3}$. **9.** $9\frac{1}{7}$. **10.** $5\frac{1}{3}$. **12.** $\frac{2}{3}$. **13.** $1\frac{1}{2}$. **14.** 4. **15.** 6. **16.** $\frac{3}{4}$. **17.** $2\frac{2}{3}$. **18.** $\frac{1}{2}$. **19.** $\frac{5}{8}$. **20.** $\frac{5}{8}$. **21.** $1\frac{1}{4}$. **22.** $\frac{1}{2}$. **23.** $\frac{3}{8}$. **24.** $\frac{1}{2}$. **25.** $1\frac{73}{96}$. **26.** $\frac{17}{44}$. **27.** 64. **28.** $5\frac{13}{15}$. **29.** $7\frac{1}{4}$. **30.** $\frac{3}{10}$. **31.** $\frac{3}{22}$. **32.** $\frac{7}{8}$. **33.** 12 da. **34.** $1\frac{1}{4}$. **35.** 60. **36.** $12. **37.** $15\frac{3}{4}$ mi. **38.** $\frac{1}{10}$. **39.** $123\frac{3}{4}$ bu.

Page 103. — **1.** $62\frac{5}{8}$. **2.** $81\frac{5}{6}$. **3.** $66\frac{1}{2}$. **4.** $3\frac{1}{2}$. **5.** 40. **6.** $429\frac{14}{15}$. **7.** $205\frac{5}{9}$. **8.** $225\frac{1}{15}$. **9.** $741\frac{7}{18}$. **10.** $77\frac{3}{10}$. **11.** $53\frac{1}{10}$. **12.** $26\frac{1}{20}$. **13.** $7\frac{1}{6}$. **14.** $394\frac{11}{15}$. **15.** $24\frac{1}{3}$. **16.** $35\frac{1}{12}$. **17.** $36\frac{3}{16}$. **18.** $37\frac{2}{3}$.

Page 104. — **19.** 504. **20.** 112. **21.** 180. **22.** 775. **23.** 1125. **24.** 372. **25.** 4095. **26.** 1776. **27.** $58\frac{4}{15}$. **28.** $282\frac{4}{9}$. **29.** $622\frac{2}{9}$. **30.** $7845\frac{8}{20}$. **31.** $3796\frac{7}{18}$. **32.** $6422\frac{1}{16}$. **33.** $4362\frac{2}{9}$. **34.** $1\frac{3}{32}$. **35.** $\frac{51}{88}$. **36.** $\frac{75}{512}$. **37.** $\frac{1}{32}$. **38.** $\frac{43}{72}$. **39.** $\frac{203}{225}$. **40.** $\frac{2}{15}$. **41.** $\frac{47}{56}$. **42.** $3\frac{23}{35}$. **43.** $4\frac{13}{48}$. **44.** $17\frac{118}{145}$. **45.** $9\frac{8}{350}$. **46.** $\frac{15}{16}$ lb. **47.** 1 ft. **48.** 19. **49.** $10\frac{7}{8}$ ft. **50.** 15 mi. **51.** 200. **52.** 5. **53.** $1\frac{3}{4}$.

Page 105. — **54.** 29 mi. **55.** $997.50. **56.** $106.25. **57.** 188 sq. in. **58.** 32¢. **59.** 80. **60.** $52. ART. **140.** — **1.** $10. **2.** $9.60. **3.** $18.19.

Page 106. — **4.** $2\frac{3}{4}$ in. **5.** 60¢. **6.** $7.45. **7.** $17.31. **8.** 5.12. **9.** $1.14. **10.** $6.90.

Page 110. — **2.** $30.

Page 111. — **3.** $115.20. **4.** $100. **7.** $72. **8.** $1600. **9.** $14. **10.** 75¢.

Page 112. — **12.** $2. **13.** $4. **14.** $16. **18.** 16. **19.** 50. **20.** 40. **21.** $32. **22.** 50 mi.

Page 113. — **24.** 10,800 mi. **25.** 2 min. **26.** 20 ft. **28.** $40,000. **29.** 350.

Page 114. — **31.** 45. **32.** $62.04. **33.** $15. **34.** 15. **35.** 70. **36.** 27½. **37.** 15. **38.** 20¢; $1. **39.** $1.28. **40.** $1.20. **41.** $.30. **42.** 60¢. **43.** $.55. **44.** $1.50. **45.** $2.50.

Page 115. — **46.** $1.08. **47.** $5. **48.** $6. **49.** 15. **50.** 15 lb. **51.** 600 bu. **52.** ⅛ lb. **53.** 5 lb. **54.** 42. **55.** $19.25. **56.** $20. **57.** 66 bu. **58.** ¼ lb. **59.** 49⅓ tons.

Page 116. — **60.** 156; 3; $208. **61.** 9 lb. **62.** 15¢. **63.** $3.48. **64.** 16. **65.** 60¢. **66.** 15 mi. **67.** 85,000. **68.** $14. **69.** $16; $832. **70.** 43 ft.

Page 117. — **71.** 10½ hr. **72.** 47½ ft. **73.** $11.30. **74.** $1.19.

Page 118. — **1.** $860. **2.** $225. **3.** $635. **4.** $381.25.

Page 119. — **5.** $2525. **6.** $1820. **7.** $1158.75. **8.** $2916. **9.** $712,032,264.96. **10.** $27.43. **11.** $74.25.

Page 123. — **2.** 153 in. **3.** 204 in. **4.** 660 rd. **5.** 260 in. **6.** 2640 ft. **7.** 15,840 in. **9.** 3 mi. **10.** 2 mi. **11.** 2½ mi. **12.** 33 rd. 5½ ft. **13.** 22 yd. 2 ft. **14.** 10 yd. 2 ft. 1 in. **15.** 99 in. **16.** 160 rd. **17.** 234 in. **18.** 45,056 ft. **19.** 8000 rd. **20.** 20 ft. 10 in. **21.** 11,220 ft. **22.** 8 yd. **23.** 654 in. **24.** ¼ yd. **25.** ⅓ yd. **26.** 150 yd.

Page 124. — **27.** $6.40. **28.** 6.

Page 126. — **1.** 154 sq. in. **2.** 3456 sq. in. **3.** 9169 sq. in. **4.** 3312 sq. ft. **5.** 840 sq. rd. **6.** 3896 sq. in. **7.** 800 A.

Page 127. — **8.** 6 sq. ft. 36 sq. in. **9.** 133 sq. yd. 3 sq. ft. **10.** 1 sq. yd. 1 sq. ft. 60 sq. in. **11.** 12 A. 80 sq. rd. **12.** 1 sq. mi. 360 A. **13.** 486 sq. in. **14.** $\frac{1}{200}$ A. **15.** 288 sq. ft. **16.** 8 sq. rd. **17.** 4 sq. yd. **18.** $250. **19.** 4840 sq. rd.; 48,400 sq. yd. **21.** 46 bu.

Page 128. — **1.** 48 sq. ft. **2.** 720 sq. ft. **3.** 1728 sq. ft. **4.** 16 sq. ft. **5.** 210 sq. rd. **6.** 250 sq. ft. **8.** 6$\frac{1}{18}$ sq. yd.

Page 129. — **1.** 72 sq. ft. **2.** 133 sq. ft. **3.** 675 sq. rd. **4.** 76½ sq. ft.

Page 130. — **5.** 384 sq. in. **6.** 200. **7.** 32 ft.

Page 131. — **1.** 22 in.; 44 in.; 66 in. **2.** 50$\frac{2}{7}$ ft.; 176 ft.; 22 ft. **3.** 1 in.; 14 in. **4.** 21 ft.; 4.9 ft.

Page 132. — **2.** 18,634 sq. in. **3.** 26,026 sq. ft. **4.** 1$\frac{3}{11}$ sq. yd. **5.** $1540. **6.** 138,600 sq. ft. **7.** 346.5 sq. ft.

Page 135. — **1.** 3456 cu. in. **2.** 139,968 cu. in. **3.** 135 cu. ft. **4.** 88,128 cu. in. **5.** 110,592 cu. in. **6.** 5 cu. yd. **7.** $\frac{1}{15}$ cu. yd. **8.** 18 cu. ft. **9.** 1296 cu. in. **10.** 2970 lb. **11.** 1250 cu. ft. **12.** 63,000 cu. ft.

Page 137. — **1.** 46 pt. **2.** $2\frac{1}{2}$ qt. **3.** 66 qt. **4.** 6 qt. **5.** 131 pt. **6.** 6 pt. **7.** 58 pk. **8.** 6 qt. **9.** 308 gal. 3 qt. **10.** 37 bu. 2 pk. **11.** 57 bu. 3 pk. **12.** 205 pk. 5 qt. **13.** 18 pk. 7 qt. 1 pt. **14.** $\frac{5}{8}$ gal. **15.** $\frac{1}{8}$ gal. **16.** $6\frac{1}{4}¢$. **17.** 12,000 bu.

Page 140. — **1.** 88,000 lb. **2.** 5600 oz. **3.** 1475 lb. **4.** 142,400 oz. **5.** 11,800 oz. **6.** 1000 oz. **7.** 30 lb. **8.** 38 lb. 2 oz. **9.** 50 cwt. 25 lb. **10.** 62.5 lb. **11.** 2 T. 7 cwt. 11 lb. **12.** \$7.50. **13.** \$20.25. **14.** \$26.40. **15.** 450 lb.

Page 142. — **1.** 222 da. **3.** 98 da.; 14 wk. **4.** 13. **5.** 351,000. **6.** 10. **7.** 69 da. **8.** \$49.50.

Page 143. — **2.** 12 yd. 1 ft. 9 in. **3.** 25 bu. 2 pk. **4.** 13 mi. 19 rd. **5.** 161 sq. yd. 6 sq. ft. **6.** 10 gal. 1 qt. **7.** 36 lb. 3 oz. **8.** 14 yr. 2 mo. 6 da. **9.** 43 hr. 59 sec.

Page 144. — **2.** 2 mi. 315 rd. **3.** 9 da. 22 hr. **4.** 29 A. 140 sq. rd. **5.** 12 bu. 3 pk. **6.** 264 lb. 10 oz. **7.** 25 ft. 5 in. **8.** 31 gal. 2 qt. **9.** 2 yr. 2 mo. 19 da. **10.** 6 yr. 1 mo. 22 da.

Page 145. — **2.** 26 yd. 2 in. **3.** 256 hr. **4.** 40 bu. 2 pk. **5.** 9 gal. **6.** 164 sq. yd. **7.** 383 cu. yd. 9 cu. ft. **8.** 218 rd. 3 ft. 8 in. **9.** 11 bu. 1 pk. **10.** 1084 lb. 8 oz. ART. **206.** — **2.** 1 T. 9 cwt. **3.** 2 ft. 11 in. **4.** 5 da. 10 hr. 24 min. **5.** 7. **6.** 16 ft. 4 in. **7.** 15.

Page 146. — **1.** 55,440 in. **2.** $37\frac{1}{2}$ gal. **3.** 4 bu. 5 qt. 1 pt. **4.** 6540 sq. ft. **5.** 1,209,600 cu. in. **6.** 3 T. 425 lb. **7.** 1 hr. 50 min. **8.** $\frac{5}{288}$ yd. **9.** 129 lb. 6 oz. **10.** 4 yd. 1 ft. 5 in. **11.** 4. **12.** 1350. **13.** \$5.40. **14.** 112. **15.** $3\frac{3}{8}$ bu. **16.** 10 da.

Page 147. — **17.** 15 sq. ft.; 180 cu. ft. **18.** 7. **19.** 10 A.; \$120. **20.** 16,500 ft.; 1000 lb. **21.** 10 A.; 160 rd. **22.** $2\frac{1}{4}$ T. **23.** 360 times. **24.** $41\frac{1}{7}$ T. **25.** 600 bu. **26.** 380 lb. **27.** 683.58 cu. in.

Page 148. — **1.** 60,000 lb. **2.** 5. **3.** 40 bales. **4.** \$300. **5.** \$2100. **6.** \$1180; \$23.60. **7.** 20 T.; \$250.

Page 149. — **8.** \$440,400,000. **9.** 85 lb. **10.** 738 T. **11.** 143,880 bales. **12.** 3,970,000 bales.

Page 150. — **13.** \$2,412,800. **14.** 130 lb. **15.** 1,520,000 lb.; 12,160 spindles. **16.** \$5,550,000. **17.** \$5,456,995.50. **18.** 702,800 bales. **19.** 87,000 lb.

Page 154. — **2.** $\frac{7}{10}$. **5.** $\frac{13}{20}$. **7.** $\frac{21}{50}$. **8.** $\frac{19}{50}$. **9.** $\frac{11}{20}$. **10.** $\frac{3}{8}$. **11.** $\frac{17}{40}$. **12.** $\frac{13}{40}$. **13.** $\frac{5}{8}$. **14.** $\frac{1}{80}$. **15.** $\frac{8}{80}$. **16.** $\frac{7}{8}$. **17.** $\frac{283}{400}$.

Page 155. — **5.** .375. **6.** .75. **7.** .83$\frac{1}{3}$. **8.** .15. **9.** .36. **10.** .45. **11.** .88. **12.** .55. **13.** .41$\frac{2}{3}$. **14.** .33$\frac{1}{3}$. **15.** .875. **16.** .91$\frac{2}{3}$. **17.** .58$\frac{1}{3}$. **18.** .72. **19.** .4. **20.** .65. **21.** .075. **22.** .06.

Page 156. — **3.** $284.19. **4.** 78.707. **5.** $81.32. **6.** .964. **7.** $13.59. **8.** 14.32. **9.** $7.68. **10.** .189. **11.** .0038. **12.** $1.92. **13.** 1.27. **14.** $4.16. **15.** .2526. **16.** .2475.

Page 157. — **17.** 480.91. **18.** 74.298. **19.** 199.042. **20.** 69.284. **21.** 9.213. **22.** 13.09. **23.** 1.451. **24.** 9.839. **25.** 37.904. **26.** 2.096. **27.** 7.574. **28.** 91.755. **29.** $2.74. **30.** 16.8 mi. **31.** $874.75. **32.** 141.25 bu. **33.** 374.25 gal. **34.** $350.50. **35.** $425.

Page 158. — **1.** 17.1. **2.** 13. **3.** 39.6. **4.** 13.3. **5.** 36. **6.** .72. **7.** 1.44. **8.** 2.25. **9.** 2.38. **10.** .72. **11.** 3.72. **12.** 21.75. **13.** 47.11. **14.** 7.68. **15.** 25.3.

Page 159. — **16.** 8.1. **17.** 8.4. **18.** 9.3. **19.** 12.9. **20.** 1.18. **21.** 4.5. **22.** 12.9. **23.** 6.4. **24.** 7.2. **25.** 6.7. **26.** 4.41. **27.** 2.42.

Page 160. — **1.** 1563.25. **2.** .09702. **3.** 367.5. **4.** .1989. **5.** 6.8276. **6.** 1.50156. **7.** 21.892. **8.** $51.20. **9.** 6.94768. **10.** .00009494. **11.** 2.1. **12.** 2500. **13.** 9210. **14.** 2180. **15.** 780. **16.** .6. **17.** 50. **18.** 68.6. **19.** 2.68. **20.** .0000012.

Page 161. — **21.** 3056.8. **22.** 33.92. **23.** 1.8. **24.** .6. **25.** 51.564. **26.** 5.72 +. **27.** 21.22 +. **28.** 1415.625. **29.** 1869.875. **30.** 2.6. **31.** 33.875 bu.; 101.625 bu. **32.** $37.80. **33.** $550. **34.** $45. **35.** $15.645. **36.** 32 barrels. **37.** 504 lb. **38.** 15 T. **39.** 24 bu. **40.** 51,940 lb.

Page 162. — **41.** $21.50. **42.** $296.50. **43.** $82.02. **44.** $21.52. **45.** $48. **46.** $86.30; $29.30.

Page 164. — **1.** $42.16. **2.** $10.28. **3.** $31.28. **4.** $4.10. **5.** $47.66. **6.** $3.05.

Page 165. — **7.** $22.12. **8.** $46.30. ART. **239.** — **1.** $249. **2.** $386,584,000. **3.** 127,600,000 bu. **4.** 6.3 bu. **5.** 244,169,600 lb.

Page 166. — **6.** $861,107,400. **7.** $55,566,000. **8.** 365,000 T.; 18,250. **9.** $28,226,520; 672,060. **10.** $1514.70. **11.** 11,295,000 lb. **12.** 9,000,000; $101,250.

Page 170. — **2.** $36. **3.** $90. **4.** 200 bu. **5.** $87. **6.** $900. **7.** $80. **8.** $10. **9.** $22.50. **10.** $72. **11.** 60 A. **12.** $24.90. **13.** 44. **14.** 12 yr. **15.** $8.75. **16.** 15 mi. **18.** $\frac{3}{4}$; 75%. **19.** $200. **20.** $40.

Page 171. — **21.** 18¢. **22.** $2. **23.** $157.50. **24.** $100. **25.** $1950. **26.** $54. **27.** 100 bu. **28.** $13.50. **29.** 40 pupils. **30.** $1875. **31.** $214.20. **32.** 15¢. **33.** $450.

Page 172. — **34.** $1.98. **35.** $8. ART. **254.** — **2.** $60; $690. **3.** $123.50.

Page 173. —**4.** $240. **5.** $12. **6.** $25.30. **7.** $7.50. **8.** $81. **9.** $427.50. **10.** $126. **11.** $21. **12.** $45. **13.** $25.20. **14.** $23.94. **15.** $5.85.

Page 174. —**1.** $9. **2.** $13.75. **3.** $15; $36.

Page 175. —**4.** $56.25. **5.** $13.50. **6.** $7.50. **7.** $58.50. **8.** $2.43. **9.** $22. ART. **261.** —**1.** $42. **2.** $63. **3.** $56. **4.** $87.50. **5.** $10.50. **6.** $5.25. **7.** $56. **8.** $70. **9.** $31.50. **10.** $3.50. **11.** $3.50. **12.** $10.50. **14.** $24. **15.** $43.20. **16.** $1.60. **17.** $7.20.

Page 176. —**18.** $43.20. **19.** $39. **20.** $95. **21.** $154.80. **22.** $45. **23.** $52.50. **24.** $70. **25.** $41.25. **26.** $17.50. **27.** $75. **28.** $239.40. **29.** $42.30. **30.** $145.10. **31.** $216. **32.** $181.60. **33.** $535. **34.** $322.50. **35.** $255.375. **36.** $390. **37.** $357. **38.** $253.75. **39.** $337,500.

Page 177. —**1.** 900 bu.; $693. **2.** 936 bu. **3.** 5¢. **4.** $2,012,976. **5.** 32 bu. **6.** $7,201,316.76.

Page 182. —**1.** $302.35. **2.** $295.17. **3.** 305.31. **4.** $333.32. **5.** $281. **6.** 3299.68. **7.** 2524.64. **8.** $3302.51. **9.** 461.44. **10.** $1355.79. **11.** $3170.45. **12.** $3256.15 **13.** $499.70. **14.** 42.85. **15.** $189.72. **16.** 117.6.

Page 183. —**17.** 419.75. **18.** $288.80. **19.** 51.91. **20.** $465.30. **21.** $1088.81. **22.** 377.7. **23.** $7099.05. **24.** 2971.55. **25.** $8\frac{7}{12}$. **26.** $1\frac{9}{16}$. **27.** $37\frac{14}{15}$. **28.** $2\frac{1}{8}$. **29.** $1\frac{13}{24}$. **30.** $17\frac{3}{4}$. **31.** $21\frac{17}{24}$. **32.** $50\frac{17}{18}$. **33.** $57\frac{41}{48}$. **34.** $127\frac{23}{24}$. **38.** 13,860. **39.** 21 yd. 2 ft. 11 in.

Page 184. —**40.** 25 gal. 1 qt. 1 pt. **41.** 36 bu. 2. pk. **42.** 26 ft. 9 in. **43.** 13 yr. 4 mo. **44.** 38 hr. 14 min. 50 sec. **45.** 343 rd. 1 yd. **46.** 207 bu. 3 pk. 3 qt. **47.** 2 bu. 2 pk. **48.** 4 gal. 3 qt. **49.** 2 lb. 11 oz. **50.** 2 ft. 5 in. **51.** 1 yd. 1 ft. 10 in. **52.** 1 T. 1998 lb. **53.** 7 hr. 45 min. **54.** 5 yr. 1 mo. 22 da. **55.** 4 yr. 8 mo. 25 da.

Page 185. —**59.** 2595.6 mi. **60.** 242 bu. 1 pk. **61.** 15 yr. 3 mo. 22 da. **62.** 32 lb. 8 oz. **63.** 7 gal. 2 qt. 1 pt. **64.** $6\frac{1}{2}$ yd. **65.** $21,084,000.

Page 187. —**1.** 17,340. **2.** 80,115. **3.** 5,410,920. **4.** 1,582,350. **5.** 3,413,100. **6.** 1,821,864. **7.** 8,428,875. **8.** 3,513,588. **9.** $33,840. **10.** $215,852. **11.** $87,958. **12.** $512,784. **13.** $90,120. **14.** $380,992.20. **15.** $263,691.60. **16.** $747,032.25. **17.** 31 yd. 9 in. **18.** 100 hr. **19.** 28 bu.

Page 188. —**20.** 376 ft. **21.** 364 lb. 6 oz. **22.** 76 gal. 2 qt. **23.** 224.9. **24.** 1.3125. **25.** 2.95. **26.** .059. **27.** 1.5729. **28.** 36.8449.

29. 5.15625. **30.** .583. **31.** 4.3248. **32.** $\frac{1}{2}$. **33.** $\frac{21}{40}$. **34.** $\frac{35}{144}$. **35.** $\frac{55}{288}$. **36.** $1\frac{43}{192}$. **37.** $\frac{9}{64}$. **38.** 2301. **39.** 4784. **40.** $4218\frac{3}{4}$. **41.** $3.64. **42.** 75¢. **43.** $1.20. **44.** $156. **45.** 29 bu. 1 pk.; $65.81. **46.** $13.50. **47.** 5280 ft. **48.** $21.60.

Page 191. — **1.** 215. **2.** 159. **3.** 318. **4.** 143. **5.** 752. **6.** 33. **7.** 682. **8.** 371. **9.** 2144. **10.** 1239. **11.** 367. **12.** $1337. **13.** $376. **14.** 26. **15.** 32. **16.** 2 T. 7 cwt. 36 lb. **17.** 2 ft. 10 in. **18.** 4 da. 7 hr. **19.** 3 bu. 2 pk. **20.** 12 gal. 2 qt. **21.** 4 lb. 14 oz. **22.** 8. **23.** 6. **24.** $99\frac{3}{4}$. **25.** 30.

Page 192. — **26.** 15.626. **27.** 26.46. **28.** 40. **29.** 18.3. **30.** 460.5. **31.** 40. **32.** 5.18. **33.** 11. **34.** 32. **35.** $2\frac{2}{15}$. **36.** $\frac{7}{9}$. **37.** $1\frac{1}{3}$. **38.** $\frac{9}{10}$. **39.** $2\frac{7}{10}$. **40.** $\frac{16}{105}$. **41.** $\frac{7}{9}$. **42.** $\frac{5}{12}$. **43.** $\frac{2}{33}$. **44.** $12\frac{1}{18}$. **45.** $8\frac{3}{4}$. **46.** $\frac{51}{104}$. **47.** $3\frac{67}{77}$. **48.** $17\frac{7}{8}$. **49.** $\frac{144}{187}$. **50.** 32 qt. **51.** 594 yd. **52.** 5. **53.** 3 bu. 7 qt. **54.** 13. **55.** 162 lb. 9 oz. **56.** 38. **57.** 16 da. **58.** 11¢.

Page 194. — **4.** 2880 bd. ft. **5.** $10.80.

Page 195. — **6.** $429. **7.** $21.60. **8.** $113.10. **10.** $2.975. **11.** $51.75.

Page 196. — **13.** $280.80. **14.** 13,400. **15.** $224. **16.** 160 bd. ft.

Page 197. — **2.** 40,320. **3.** 6300. **4.** 125,433. **5.** 15,120. **6.** $113.40. **7.** $1413.

Page 198. — **10.** $666\frac{2}{3}$ cu. yd. **11.** 21. **12.** 226. **13.** $1220.40. **14.** $25\frac{1}{3}$ cu. yd.; $129.20.

Page 199. — **15.** $11.20. **16.** 37,240. **17.** 12,150. ART. **290.** — **1.** 18 gal. **2.** $75. **3.** $180. **4.** $23.01. **5.** $28.62. **6.** $21.39.

Page 200. — **2.** $2.77.

Page 201. — **3.** $9.60. **4.** $9.60. **5.** $5.36. ART. **296.** — **1.** 60 yd. **2.** $120.

Page 202. — **3.** $32. **4.** 10 yd. **5.** $26. **6.** $10. **7.** Carpeting. $275.

Page 203. — **3.** 3600 gal. **4.** 1800 gal. **5.** 12,000 gal. **6.** 24,300 gal. **7.** 1,080,000 gal. **8.** 172,800 gal. **11.** 2604+ bu. **12.** 385.78+ bu. **13.** 1446.7, nearly. **14.** 1012.68+ bu. **15.** 40 ft.

Page 204. — **16.** 128 cu. ft. **17.** $3\frac{1}{2}$ cords. **18.** $3\frac{1}{3}$ ft. **19.** $\frac{5}{8}$ of a cord. **20.** 12 ft. ART. **300.** — **1.** 42 T. **2.** $525.

Page 205. — **3.** $405. **4.** $232.75. **5.** 29.4 bu. **6.** $126. **7.** 3 A. **8.** $234. **9.** $37.50. **10.** 45 A. **11.** 6.75 T.; $121.50. **12.** 80 rd.

Page 206. — **2.** 40 rd.

Page 207. — **3.** 100 rd. **5.** $175. **6.** 8.4 gal. **8.** 5 yr.

Page 208. — **10.** $14.40. **11.** $.40, gain. **13.** $37.50.

Page 209. — **15.** 23 bbl. **17.** 1252.35 T. **19.** $27; $36; $42.

Page 210. — **21.** $1\frac{1}{11}$ da. **22.** $9. **23.** 144 mi. **24.** 20 da. **25.** $20. **26.** $19.80. **27.** 1100 bu. **28.** 525 lb. **29.** 3840 lb. **30.** $12\frac{1}{2}$ yd.

Page 211. — **31.** 66. **32.** 1850 bu.; $1757.50. **33.** 5.94 T. **34.** $21.70. **35.** $36. **36.** $5. **37.** $.38. **38.** $15. **39.** $52.40. **40.** $1950.

Page 212. — **41.** 78 bbl. **42.** $1056. **43.** 12. **44.** A, $4; B, $8. **45.** $2. **46.** $22.50. **47.** 160. **48.** $67\frac{8}{16}$ in. **49.** $19.20. **50.** 10 da.

Page 213. — **51.** $143. **52.** 120. **53.** A, $38.40; B, $51.84; C, $40.50. **54.** $56\frac{1}{4}$ hr. **55.** $838.20. **56.** $107\frac{1}{7}$ bu. **57.** $18.52.

Page 214. — **58.** $16.25, gain. **59.** $600. **60.** $1800. **61.** $4\frac{1}{5}$ mo. **62.** $2, loss. **63.** $1.35. **64.** 10%, gain. **65.** $3. **66.** 32,400 lb.; $648; $41.84.

Page 215. — **67.** $741.40. **68.** $42. **69.** $23.15. **70.** 180. **71.** 240. **72.** 1800. **73.** $25.98.

Page 216. — **1.** 4. **2.** 560 of each. **3.** 15,325; 613. **4.** $383,125.; $4. **5.** 74,810,790 T.

Page 217. — **6.** 39,888,000 T. **7.** $2.63. **8.** 193,520 sq. mi. **9.** $3,872,000. **10.** $224,053,788.67.

Page 220. — **3.** $4. **4.** 900 horses. **5.** 1155 men. **6.** $\frac{1}{20}$. **7.** 152,932 bu. **8.** $30.12. **9.** 534 yd. **10.** 1638 men. **11.** $625. **12.** 0. **13.** 504 ft. **14.** $4375. **15.** Wheat, 480 bu.; corn, 800 bu.; oats, 320 bu. **16.** $1449.

Page 221. — **17.** $104. **18.** 27. **19.** $127.50. **20.** 672,000 gal. **21.** Hats, $1870; suitings, $4505; underwear, $1275; furnishings, $850. **22.** Bedrooms, $375; parlor and library, $675; halls, $82.50; kitchen and pantry, $67.50; dining room, $300. **23.** Readers, 40,000; arithmetics, 25,000; spellers, 15,000; histories, 12,000; grammars, 8000.

Page 222. — **2.** 20%. **3.** 25%. **4.** $12\frac{1}{2}$%. **5.** 40%. **6.** 25%. **7.** 80%. **8.** 10%.

Page 223. — **9.** 99%. **10.** 54%. **11.** 20%. **12.** 50%. **13.** $37\frac{1}{2}$%. **14.** 13%. **15.** 60%. **16.** 40%.

Page 224. — **2.** 2000. **3.** $3816. **4.** 2800. **5.** 19,900. **6.** 4300. **7.** 21,744. **8.** 8547. **9.** 19,144. **10.** 3552.

Page 225. — **11.** 1365. **12.** 55,584. **13.** $3733\frac{1}{3}$. **14.** 2700. **15.** 12,592. **16.** 66,120. **17.** 1740 bu. **18.** $755. **19.** 550. **20.** $25,620.

Page 226. — **3.** $200. **4.** 1500. **5.** 3600. **6.** $750. **7.** $250. **8.** $15,000. **9.** 360 A. **10.** 320. **11.** 300 bu. **12.** 210 lb.

Page 227. — **1.** 80,500 T. **2.** $1,328,250; $171,062.50. **3.** 750 T. of ore; 250 T. each of coke and limestone.

Page 228. — **4.** 40,250; 1610. **5.** $317\frac{4}{33}$ mi. **6.** 1400. **7.** $285,046,800. **8.** 14,359,696 long tons. **9.** $2,036,960. **10.** $832,000,000. **11.** $4159\frac{1}{11}$ mi.

Page 230. — **1.** $18.10; $705.90. **2.** $25.48; $702.52. **3.** $11.625. **4.** $1116.25. **5.** $5000. **6.** $7351.50. **7.** (1) $135; (2) $2479.50; (3) $1042.20; (4) $2632.50; (5) $746.10; (6) $4905; (7) $1935; (8) $1687.50; (9) $256.50; (10) $5657.40; (11) $4888.80; (12) $613.80. **8.** $29,977; $2997.70.

Page 231. — **1.** $30. **2.** $64.50. **3.** $8094. **4.** $25. **5.** $1004. **6.** $120. **7.** $558.75. **8.** $3797.50.

Page 233. — **2.** $50; $16\frac{2}{3}\%$. **3.** $552.50. **4.** $66\frac{2}{3}\%$. **5.** Loss, $2\frac{1}{2}\%$. **6.** 40%. **7.** $33\frac{1}{3}\%$. **8.** $12\frac{1}{2}¢$. **9.** $66\frac{2}{3}\%$. **10.** 10¢. **11.** $14\frac{2}{7}\%$. **12.** 20%.

Page 234. — **13.** 20%. **14.** $11; $36\frac{4}{11}\%$. **15.** $120. **16.** 3¢. **17.** Gain, $25. **18.** $4. **19.** 25%. **20.** $33\frac{1}{3}\%$. **21.** 35¢. **22.** 24¢. **23.** $1.26. **24.** $3.60. **25.** $5.10.

Page 235. — **1.** $48. **2.** 25%. **3.** $33\frac{1}{3}\%$. **4.** 35¢. **5.** 20%. **6.** 50%. **7.** $6500. **8.** $131.25. **9.** $8.09.

Page 236. — **10.** $3.30. **11.** $40. **12.** $50. **13.** $230; 46%. **14.** Phosphoric acid, 120 lb.; ammonia, $52\frac{1}{2}$ lb.; potash, 90 lb. **15.** Acid phosphate, 60%; fish scrap, 35%; muriate of potash, 5%. **16.** $58.33\frac{1}{3}$. **17.** 28%. **18.** 25¢.

Page 237. — **19.** (*a*) $19.80; (*b*) $25.20; (*c*) $6.75. **20.** (*a*) $22.50; (*b*) $30.21; (*c*) $11.25; (*d*) $18. **21.** 74%. **22.** 25%. **23.** $23.375. **24.** $518.40. **25.** 65,849,489.

Page 238. — **1.** $27,000; $1260. **2.** 100 hr. **3.** $504.

Page 239. — **4.** $1833\frac{1}{3}$. **5.** 72%. **6.** 93%; 7%. **7.** 57%. **8.** $5,437,500; $36. **9.** 74,200,000. **10.** 1,658,160.

Page 240. — **11.** 60 sq. yd. **12.** $5\frac{1}{4}$ sq. yd. **13.** 12,468,000 yd. **14.** $22\frac{1}{2}\%$. **15.** 550 A. **16.** $297.50; $11,970,000. **17.** Agriculture, $1,939,981,680; mines, $320,144,760; forests, $119,457,000; fisheries, $9,556,560. **18.** $5,984,550,000.

Page 241. — **19.** Farmers, 16,500,000 lb.; factories, 283,500,000 lb. **20.** 71.8%. **21.** 90%. **22.** 113,050,000. **23.** $93,630,000. **24.** 25.1%. **25.** $1,460,000,000.

Page 242. — **1.** 4981. **2.** 50,686. **3.** 464,968. **4.** 5,235,626.

Page 243. — **3.** 75. **4.** 408. **5.** 417. **6.** 1092. **7.** 8878.

Page 244. — **2.** 1442. **3.** 3354. **4.** $152.65.

Page 245. — **5.** 129,300. **6.** 84,400. **7.** 128,700. **8.** 66,100. **9.** 70,300. **10.** 48,100. **11.** 32,400. **12.** 35,600. **13.** 54,100. **14.** 429.9. **15.** 2241. **16.** 28,820.

Page 246. — **4.** 59.64. **5.** 113.13. **6.** 19.28. **7.** 128.7. **8.** 178.88. **9.** 25.68. **10.** 310.85. **11.** 12.888. **12.** .3702. **13.** 19.12. **14.** 11.22. **15.** 14.94. **16.** 21.87. **17.** 11.64. **18.** 4.1. **19.** .2512. **20.** 3.615. **21.** .05832. **22.** 7.036. **23.** 9.4.

Page 247. — **3.** 20,850. **4.** 195,000. **5.** 331,000. **6.** 132,475. **7.** 223,575. **8.** 488,375. **9.** 335,325. **10.** 585,150. **11.** 2,318,750. **12.** 721,700. **13.** 108,600. **14.** 318,800. **15.** 617,475. **16.** 523,200. **17.** 100.88. **18.** 64.36. **19.** 94.32. **20.** 117.33. **21.** 224.56. **22.** 100.5. **23.** 56.84. **24.** 41.79. **25.** 79.04. **26.** 105. **27.** $240. **28.** $130. **29.** $42. **30.** 24 yd. **31.** 6 da. **32.** 2; 87½.

Page 248. — **33.** 5%. **34.** $\frac{1}{20}$. **35.** 500. **36.** $62.50. **37.** $58. **38.** 12 lb. **39.** 96. **40.** $4. **41.** 80 lb. **42.** $325. **43.** $930. **44.** 87½.

Page 252. — **2.** 5. **3.** 3.

Page 253. — **4.** $3.

Page 254. — **10.** A, $400; B, $100. **11.** Father, 45 yr.; son, 15 yr. **12.** 8; 64. **13.** Horse, $150; carriage, $300. **14.** 9. **15.** 16; 48.

Page 255. — **16.** 24. **17.** 96. **18.** 20; 30. **19.** One, $2; other, $5. **20.** Men, 30; women, 60; children, 180. **21.** A, 12; B, 4. **22.** Ralph, $30; Keller, $50. **23.** Elephants, 3; monkeys, 21; horses, 63. **24.** $22,950. **25.** 80,000. **26.** Will, 8; Tom, 16; Frank, 24. **27.** 1st, $48; 2d, $24; 3d, $12. **28.** 3.

Page 256. — **29.** 7. **30.** 20 rd. **31.** One, 24 mi.; other, 48 mi. **32.** 15 mi. **33.** Monday, 5; Tuesday, 15; Wednesday, 60. **34.** First, 25¢; second, 35¢; third, 45¢. **35.** $25,000.

Page 257. — **3.** $4000. **4.** 1232 lb. **5.** 200 lb. **6.** 98%.

Page 258. — **1.** $540,000. **2.** 30%. **3.** 3060. **4.** 80%. **5.** 6,181,350. **6.** 81,280,000. **7.** 11,314,176. **8.** $3.36; $16.81.

Page 260. — **2.** $49. **3.** $30. **4.** $5. **5.** $112.50. **6.** $47.25. **7.** $56. **9.** $5.25. **10.** $12.80. **11.** $16.15. **12.** $25.28. **13.** $45.28. **14.** $38.22. **15.** $90.20. **16.** $15.70. **17.** $40. **18.** $9.70.

Page 261. — **19.** $10.83. **20.** $170; $1670. **21.** $189.42; $1414.42. **22.** $265.96; $3515.96. **23.** $126.61; $2001.61. **24.** $174.42; $1614.42. **25.** $46.92; $766.92. **26.** $38.94; $389.44. **27.** $50.67; $236.27. **28.** $236.50; $2036.50. **29.** $214.58; $2714.58. **30.** $221.34; $3077.34. **31.** $117.55; $1247.55. **32.** $226.23; $2976.73. **33.** $201.01; $10,251.51. **34.** $1787.67. **35.** $1513.05. **36.** $786.88.

Page 262. — **2.** $13.82. **3.** $24.49. **4.** $36.42. **5.** $47.73. **6.** $60.29. **7.** $65.94. **8.** $83.52. **9.** $208.29. **10.** $259.57. **11.** $160.77.

Page 263. — **12.** $29.70. **13.** $55.86. **14.** $67.39. **15.** $129.90. **16.** $90.09. **17.** $258.33. **18.** $287.72. **19.** $261.56. **21.** $7.06; $572.06. **22.** $10.40; $790.40. **23.** $8.96; $835.96. **24.** $7.71; $932.71. **25.** $6.83; $1031.83. **26.** $30.71; $1705.71. **27.** $28.83; $1758.83. **28.** $12.98; $832.98. **29.** $12; $762. **30.** $8.68; $628.68.

Page 264. — **2.** $6.50. **3.** $9.55. **4.** $21.36. **5.** $4.43. **6.** $26.18. **7.** $57.26. **8.** $5.75. **9.** $7.91.

Page 265. — **10.** $27.90. **11.** $340.74.

Page 266. — **2.** 6%. **3.** 6%. **4.** 7%. **5.** 4%.

Page 267. — **2.** 2½ yr. **3.** 3 yr. **4.** 6 mo. **5.** 25 yr. **7.** 90 da. **8.** 80 da.

Page 268. — **2.** $166.67. **3.** $500. **4.** $340. **5.** $2100. **6.** $600.

Page 270. — **4.** $303. **5.** $515. **6.** $694.69. **7.** $462.19.

Page 272. — **2.** $572.09. **3.** $106.63. **4.** $903.47. **5.** $135.09. **6.** $1398.75.

Page 273. — **2.** $5.19. **3.** $185.21.

Page 274. — **4.** $109.91. **5.** $280.61. **6.** $82.01. **7.** $77.29.

ART. **375.** — **2.** $3665.03. **3.** $11,412.57. **4.** $99.05. **5.** $1095.78. **6.** $914.25. **7.** $976.16. **8.** $338.04.

Page 276. — **1.** $9666; 66.74%, nearly. **2.** Freight, $1,449,127,899.25; passenger, $572,187,840.05; other sources, $51,828,608.70. **3.** 1,755,600; $790,020.

Page 277. — **4.** 99,484 T.; $2,984,520. **5.** $260,186,000. **6.** $178,087,734. **7.** 2.1%. **8.** 40%. **9.** (*a*) $1,522,332.38, $1,598,188.81; (*b*) $75,856.43; (*c*) 4.98 + %; (*d*) 5.3 + %; 4.64 + %.

Page 279. — **1.** $268.46. **2.** $540.25. **3.** $253.70.

Page 282. — **2.** $2.79; $276.21. **3.** $4555.11. **4.** $7.50. **5.** $.77. **6.** $5.80. **7.** $25.13. **8.** $3.18.

Page 283. — **9.** $495.08. **10.** $2970. **11.** $593.70.

Page 284. — **2.** $603.07. **3.** $452.88. **4.** $225.75. **5.** $243.24. **6.** $1252.56. **7.** $559.74. **8.** $178.72.

Page 288. — **2.** $2501.25. **3.** $3504.38. **4.** $2001.50.

Page 289. — **5.** $244.85. **6.** $1231.23. **7.** $749.25. **9.** $147.32. **10.** 5210 francs. **11.** $141.75. **12.** $95.97. **13.** $392.24. **14.** $6458.25.

Page 292. — **1.** 50. **2.** $18. **3.** $6.60. **4.** 31,614 ft.; 6 mi., nearly. **5.** 27,972 ft.; $5\frac{11}{40}$ mi. **6.** 5 da. **7.** $1.10. **8.** 256,680 mi.

Page 293. — **9.** $1,693,541; $1,278,464; $415,076. **10.** 41¢; 45¢. **11.** $10.50. **12.** $390. **13.** $68.25. **14.** $337.36. **15.** $242.08. **16.** $28,875.

Page 294. — **17.** 19. **18.** 52 lb.; $57\frac{9}{13}$%. **19.** $44\frac{4}{9}$%; $8\frac{1}{3}$%. **20.** 256; 16 lb. **21.** 85; 73. **22.** 14; 12. **23.** $2450.

Page 295. — **24.** $960. **25.** $31.05. **26.** 160 bd. ft. **27.** 1728 bu. **28.** $5.97. **29.** $72.65. **30.** 18.3%. **31.** 23,040. **32.** 12,672 T.; $354,816. **33.** $260.76.

Page 296. — **34.** $82\frac{1}{2}$ cords. **35.** 10 yd. **36.** B's; $.50. **37.** 3¢. **38.** $318\frac{1}{2}$ lb. **39.** $2450. **40.** 71. **41.** 75. **42.** 200 ft. **43.** 40,400 sq. ft. **44.** 60%.

Page 297. — **45.** 15%. **46.** $58.80. **47.** 20%; $33\frac{1}{3}$%; 24%. **48.** $40.25; $14. **49.** $2750. **50.** $14.54. **51.** $866.25. **52.** $1621.62. **53.** $4\frac{1}{2}$%. **54.** $450. **55.** $429.79.

Page 299. — **2.** $62.50. **3.** $48. **4.** $281.25. **5.** $30. **6.** $156.25. **7.** 2%. **8.** $2400. **9.** $16,720. **10.** $3975. **11.** $75. **12.** $468.75. **13.** $312.50.

Page 300. — **1.** $64.47. **2.** $70.32.

Page 301. — **3.** $383.40; $7668. **4.** $8100. **5.** $242.40; $4848. **6.** $3533.95. **7.** $3942. **8.** $1920. **9.** $77.

Page 303. — **3.** $73.50. **4.** $45. **5.** Schools, $19.20; pensions, $6.40; general purposes, $16; $41.60. **6.** $44.18.

Page 304. — **7.** 50¢ on $100; $32. **8.** $42. **9.** $400. **10.** $5200.

Page 305. — **1.** $45. **2.** $9.38. **3.** $125. **4.** $1400. **5.** $100.80. **6.** $525. **7.** $350. **8.** $1800. **9.** $1920; $56.

Page 306. — **1.** $7481.25. **2.** 45¢. **3.** $47,436.39; $26.35. **4.** $4457.05; $4345.62. **5.** $61,783.72. **6.** $180,000,000.

Page 309. — **3.** $14,205. **4.** $8482.50. **5.** $2880. **6.** $8021.25. **8.** 40. **9.** 160.

Page 310. — **12.** $1260. **13.** $600. **15.** $30,000. **16.** $24,000. **17.** $250. **18.** 8%. **19.** $230,000.

Page 312. — **2.** $\frac{5}{6}$. **3.** $\frac{12}{5}$. **4.** $\frac{15}{16}$. **5.** $\frac{16}{15}$. **6.** 12. **7.** $\frac{3}{4}$. **8.** $\frac{1}{10}$. **9.** $\frac{1}{4}$. **10.** $\frac{1}{3}$. **11.** $\frac{7}{50}$. **12.** 3.1416, nearly.

Page 315. — **1.** 15. **2.** 1. **3.** 24. **4.** 24. **5.** 25. **6.** $22\frac{1}{2}$. **7.** 3. **8.** 25. **9.** 64. **10.** 39.

Page 317. — **4.** $18. **5.** 225 mi. **6.** 700 bu. **7.** 414 bu. **8.** 11.76 lb. **9.** 17 bu. **10.** $137.50. **11.** $81\frac{1}{4}$ ft. **12.** 555 ft. **13.** 1800 yd. **14.** 9 mo. **15.** $300. **16.** 10 oz.

Page 318. — **17.** 6 wk. **18.** $108. **19.** $558. **20.** 9 mo. **21.** 180 mi. **22.** $4.50. **23.** 20 da. **24.** 8910. ART. **441.** — **2.** $180; $120.

Page 319. — **3.** A, 24 yr.; B, 32 yr. **4.** 135; 315. **5.** 54; 81; 135. **6.** Smith, $97.50; Brown, $162.50; Jones, $227.50; Robertson, $292.50. **7.** A, $315; B, $378. **9.** $275; $594; $990. **10.** A, $16\frac{2}{3}$%, $250; B, $83\frac{1}{3}$%, $1250.

Page 320. — **1.** 5,200,000 bd. ft.; $52,000. **2.** 520. **3.** $468,800,000. **4.** $33\frac{1}{3}$%; $360,000.

Page 321. — **5.** $28\frac{8}{97}$%. **6.** $11\frac{1}{4}$%. **7.** 313 da. **8.** $570,117,697.50; $190,039,232.50.

Page 324. — **2.** 1 hr. 35 min. 2.3 sec.

Page 325. — **3.** 59 min. $44\frac{1}{3}$ sec. **4.** 1 hr. 5 min. 7.3 sec. **5.** 54 min. 33.1 sec. **6.** 5 hr. 17 min. 36.7 sec. **7.** 2 hr. 4 min. 6.3 sec. A.M. **8.** Moved up 37 min. 30 sec. **9.** 4 hr. 44 min. 14 sec. faster. **10.** 3 hr. 13 min. 49.1 sec. earlier.

Page 326. — **2.** 26° 50′ 45″. **3.** 35° 4′. **4.** 101° 29′ 30″. **5.** 79° 24′ 45″. **6.** 33° 7′. **7.** 32° 10′ 27″. **8.** 69° 33′ W. **9.** 22° 30′ E.

Page 328. — **2.** Local time is 4 min. 6.3 sec. later (faster). **3.** Standard time 8 min. 15.7 sec. faster; 9 min. 33.2 sec. slower; 9 min. 42.8 sec. faster. **4.** 11 hr. 1 min. 1 sec. A.M. **5.** 30 min. 56 sec. faster.

Page 329. — **1.** 160 A. **2.** 80 A.

Page 330. — **3.** 80 A. **4.** 80 A. **5.** 360 A.; 1120 rd. **6.** $35,840; 10 mi. **7.** $12,000. **8.** 2280; 328,320. **9.** $384. **10.** $3000.

Page 333. — **2.** 15,030 m. **3.** .057 Km. **4.** .825 m. **5.** 9 m. 8 dm. **6.** 9 Km. 8 Hm. 7 Dm. 5 dm. 4 cm.

Page 335. — **2.** 616 qcm. **3.** 179.52 ares. **4.** $2115. **5.** $5625. **6.** 51.4 m. ART. **470.** — **1.** 53.625 st. **2.** 50 cu. m. **3.** 34 min. 22.5 sec.

Page 336. — **1.** 1,024,000 liters. **2.** 30,800 Kg.

Page 337. — **3.** 55,328 Hl; 5532.8 metric tons. ART. **477.** — **1.** 260.946 mi. **2.** 984¼ ft. **3.** 187 lb.

Page 339. — **11.** 529. **12.** 2025. **13.** 12.25. **14.** 20,736. **15.** 15,625. **16.** 57,600. **17.** 1728. **18.** 17,576. **19.** 54.872. **20.** $\frac{4}{5}$. **21.** $\frac{9}{25}$. **22.** $\frac{9}{64}$.

Page 341. — **3.** 25. **4.** 36. **5.** 49. **6.** 42. **7.** 56. **8.** 72. **9.** 55. **10.** 96. **11.** 108.

Page 345. — **1.** 35. **2.** 68. **3.** 43. **4.** 86. **5.** 76. **6.** 125. **7.** 246. **8.** 267. **9.** 401. **10.** 369. **11.** .37. **12.** .308. **13.** 6.42. **14.** 73.6. **15.** 9.89. **16.** $\frac{1}{14}$. **17.** $\frac{1}{17}$. **18.** $\frac{1}{18}$. **19.** $\frac{1}{21}$. **20.** $1\frac{1}{3}$. **21.** 1.414. **22.** 1.732. **23.** 2.236.

Page 346. — **1.** 80 rd. **2.** 160 rd. **3.** 115 in. **4.** 96 ft. **5.** 16.1 Dm. **6.** 67 cm. **7.** 57 yd. **8.** 41.5 cm. **9.** 9.88 m. **10.** 40 rd. **11.** 30 rd. **12.** 480 rd. **13.** 16 ft. square. **14.** 92 ft. **15.** 5 yd. **16.** 515.5 mi.

Page 348. — **3.** 13 in. **4.** 17 in. **5.** 25 m. **6.** 41 in. **7.** 37 in. **8.** 65 Km. **9.** 33 in. **10.** 45 ft.

Page 349. — **11.** 33.94 in. **12.** 11 ft. **13.** 80 rd. **14.** 7.79 in. **15.** 17 ft. **16.** 65 mi. **17.** 18 ft. **18.** 63.6 + ft. **19.** 85.9 ft.

Page 350. — **1.** 8⅓%. **2.** $60. **3.** 2982.8125 sq. mi.; $89,723,000; $63,951,500. **4.** 54.615 + mi.

Page 352. — **2.** 392$\frac{4}{7}$ cu. in. **3.** 7$\frac{23}{36}$ cu. ft. **4.** 22,000 cu. ft. **5.** 64,968¾ gal.

Page 353. — **2.** 353$\frac{4}{7}$ sq. in. **3.** 24$\frac{4}{5}$ gal. **4.** 1833⅓ sq. ft.

Page 355. — **2.** 63 ccm. **3.** 38 cu. in. **4.** 462.888 cu. in. **5.** 1.299 cu. m. **6.** 66 cu. in. **7.** 78$\frac{4}{7}$ cu. in. **8.** 49,280 ccm.

Page 356. — **1.** 510 sq. ft. **2.** 150$\frac{6}{7}$ sq. ft. **3.** $33. **4.** 16½ sq. yd. **5.** 301$\frac{5}{7}$ qcm.

Page 357. — **2.** 523$\frac{17}{21}$ cu. in. **3.** 7 ^{1} cu. ft. **4.** 10½ gal. **5.** 5.

Page 358. — **1.** 804$\frac{4}{7}$ sq. ft. **2.** 314$\frac{2}{7}$ sq. in. **3.** 982$\frac{1}{7}$ sq. ft.

Page 359. — **1.** 3.09 T.; $253.84. **2.** $2855 **3.** $25.92. **4.** 18,327,273, nearly.

Page 360. — **5.** $2768.28. **6.** $10,564. **7.** 29. **8.** 12.

Page 361. — **1.** 13.6 yr. **2.** $80.75. **3.** 800. **4.** 75,635. **5.** 449,145 T.; 8293. **6.** 800%; 12½%. **7.** 6300.

Page 362. — **8.** 6172.6 bu., nearly. **9.** $5542; $12,520.50. **10.** $2700. **11.** $67.50; 100%. **12.** $6. **13.** 6000 gal. **14.** $390.

Page 363. — **15.** $2.10. **16.** $2800. **17.** 138,750. **18.** $87\frac{1}{8}$, $90\frac{1}{2}$, $95\frac{1}{2}$; $87\frac{7}{8}$, 96, $85\frac{3}{8}$, 93, 99, 85; $91\frac{1}{11}$. **19.** $529\frac{11}{14}$. **20.** Men, $14.48 each; boy, $7.24. **21.** $900.

Page 364. — **22.** 119. **23.** 313 : 348. **24.** 14 sq. in. **25.** 4.28 mi. **26.** $1\frac{1}{8}$ hr. **27.** $106\frac{3}{8}$ perches. **28.** 5445 T. **29.** $258\frac{1}{3}$ ft. **30.** 371.25 gr. **31.** $58\frac{1}{14}$ lb.

Page 365. — **32.** $3\frac{14}{15}$ lb. **33.** $33,939 to nearest dollar; $542,636 to nearest dollar. **34.** $124.74. **35.** $195.84. **36.** 6,625,000 ft., or 1254.7+ mi. **37.** $2187\frac{3}{8}$ yd. **38.** $909\frac{1}{11}$ kilos. **39.** 35.27+ liters. **40.** $2868.15. **41.** $3\frac{3}{4}$ A.

Page 366. — **42.** $48,536.89. **43.** $366.34. **44.** $28\frac{7}{11}$ ft. **45.** Length, 105 yd.; width, 21 yd. **46.** 5 A. **47.** 60 sq. in.

Page 367. — **48.** $800. **49.** 21%. **50.** $26\frac{2}{3}$ ft. **51.** Mines, 670,280,000 T.; agriculture, 124,670,000 T.; manufactures, 174,330,000 T.; lumber and forest, 162,630,000 T.; animals, 35,620,000 T.; unclassified, 132,470,000 T. **52.** $700; $675; $675. **53.** $15.75; $21.47. **54.** $35.28; $19.95.

Page 368. — **55.** The first by $6\frac{1}{4}$%. **56.** $12\frac{1}{2}$%; $16\frac{2}{3}$%; $33\frac{1}{3}$%. **57.** $3200. **58.** $6\frac{1}{4}$%. **59.** 25 yr. **60.** $106.33. **61.** 5%. **62.** $20. **63.** $5622.48. **64.** $2029.12. **65.** $700; $14.

Page 369. — **66.** $3450. **67.** $105. **68.** 3125 bu. **69.** $874.14. **70.** 2:30 P.M.; 125 mi. **71.** 200 nights. **72.** 680.4 ft. **73.** 120. **74.** 8%.

Page 370. — **75.** $975.05. **77.** $21\frac{9}{11}$ min. after 4 o'clock. **78.** $38\frac{2}{11}$ min. after 1 o'clock. **79.** $10\frac{1}{2}$ hr. **80.** $1\frac{7}{8}$ da. **81.** 18 da. **82.** $\frac{10}{11}$ hr. **83.** $42\frac{2}{3}$ mi. an hr.

Page 371. — **84.** $12, gain. **85.** 180. **86.** $2.80. **87.** $33\frac{1}{3}$%. **88.** $122\frac{2}{9}$%. **89.** 20%; $25,000; $33,333$\frac{1}{3}$. **90.** 29.08%, nearly; 14.1+%. **91.** 50 mi.

Page 372. — **92.** 23 m., very nearly. **93.** 35.34 + ft. **94.** $78. **95.** 6%. **96.** 500 cg. **97.** $18.09. **98.** $24,737.78; $30.92. **99.** $49.09.

Page 373. — **100.** $4.80. **101.** First, 1¢ better. **102.** 169.16+ m. **103.** 16,927.3+ gal.; 537.3+ bbl. **104.** $725.50. **106.** $1600. **107.** 20 mi.

Page 374. — **108.** $30,000. **109.** $14,000. **110.** 30 mi. **111.** 23° 45′ W. **112.** $16.85; $27.19; $30.89; $15.17. **113.** $9.73; $12.34; $8.82. **114.** (*a*) 17.2¢, 27.8¢, 31.5¢, 15.5¢; (*b*) 9.9¢, 12.6¢, 9¢.

Page 375. — **115.** 6. **116.** 268,190,476,190$\frac{10}{21}$ cu. mi.; 201,142,857$\frac{1}{7}$ sq. mi. **117.** 66 ccm. **118.** 61.12+ yd. **119.** 2 da., very nearly. **120.** 2. **121.** 441,000. **122.** 8,960,914$\frac{3}{7}$ lb. **123.** 14 ft. **124.** 12$\frac{1}{2}$%.

Page 376. — **125.** 275%; 296.15%; 74.36%; 23.83%; 230.19%. 8422.48%; 175.63%; 90.72%; 81.53%; 126.39%. **126.** 40%; 8$\frac{1}{2}$%. **127.** 17.14+%; 12.45+%; 15.8+%; 1.74+%; 19.15+%; 7.63+%; 3.21+%; 22.85−%.

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